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HFLTS-DEA Model for Benchmarking Qualitative Data

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Abstract

In Data Envelopment Analysis (DEA), performance evaluation is generally assumed to be based on a set of quantitative data. In many real-world environments, however, it is necessary to take into account the presence of qualitative factors in assessing the performance of decision-making units. The easiest thing to do is to give an assessment to the input and output values of each Decision-Making Unit (DMU) in the form of scale eg 1 is the best and 5 is the worst. The benchmarking process in qualitative data often creates problems when at the same time some people are giving judgments. Some people who provide an assessment may be able to provide different assessments, perhaps also hesitancy when assessing. The advantage of the Hesitant Fuzzy Linguistic Term Sets (HFLTS) model is that it can provide value for each Input and Output of DMU based on a qualitative and sometimes hesitancy-based assessment. The value provided by HFLTS will be used for the benchmarking process with Data Envelopment Analysis (DEA). Some qualitative data measurements involving Likert scale and Ordinal Data approaches have disadvantages when there are some assessors that provide judgment and can not model the computational trust considering hesitancy, vagueness, and uncertainty. The results of this study indicate that in HFLTS-DEA, the assessor can perform a good assessment in the form of qualitative data on the input and output of each DMU and then there will be available HFLTS evaluation results for use in the benchmarking process with DEA.

Keywords: *Computational Trust, Benchmarking, Data Envelopment Analysis, Hesitant Fuzzy Linguistic Term Sets, Qualitative Data*

1 Introduction

A wide range of problem-setting issues that can be used by DEA, especially in non-profit cases, qualitative factors are often present[1]. In general, the application of DEA requires information in the form of quantitative data. Therefore, qualitative inputs or outputs must be converted into quantitative data to suit the DEA structure [2]. Recent studies have begun to address qualitative measures such as education levels [3], actual deaths from expected deaths in health care [4] in the aerospace industry [5]. To solve this problem, there were several approaches to arranging a series of data such as the imprecise model [6] and the project model [7]. Lee and Kim have proposed the DEA-SERVQUAL method which is basically used to measure service quality[8]. SERVQUAL is a multi-dimensional scale of five dimensions and 22 items to measure customer service quality perceptions[9]. The benchmarking process with SERVQUAL has the limitation of not having a guideline that clearly states what will be compared and on what parameters must be improved to improve service quality [8]. Cronin and Taylor suggested SERVPERF[10], which directly evaluates perceptions of customer performance. The SERVPERF level is more effective than the SERVQUAL weight, because this method can reduce almost half the number of items used for measurement on the SERVQUAL method.

Traditionally, DEA assumes that input and output variables are known in advance, ignoring critical data uncertainty[11]. Researchers try to create a model that allows the DEA to adopt uncertain theory. The first method is to use a probability theory that is commonly used in DEA Stochastic. Sengupta [12] generalized the stochastic DEA model using the expected value. Fuzzy theory is another theory that has been used to address uncertainty in DEA. As one of the DEA initiators, Cooper et al. [6] presented how to deal with inaccurate data such as bound data, serial data, and data delimited by DEA. Kao and Liu have developed a method for finding member functions of fuzzy efficiency values when some inputs or inputs are fuzzy numbers[13]. Puri and Yadav have put forward the Fuzzy DEA method, where for input and output using qualitative data will be processed using the Fuzzy method [14]. Cheng et al. using the Hybrid DEA-AdaBoost in selecting suppliers for fuzzy variables and multiple targets[15]. Hesitant Fuzzy Sets (HFS) was first introduced by Torra. The HFS model gives us a natural model of decision making that takes into account the trusted level [16]. Ashtiani and Azgomi have proposed the Hesitant Fuzzy Linguistic Term Sets (HFLTS) method which provides a model that can perform on qualitative data where there are circumstances of hesitancy, vagueness, and uncertainty which often make it difficult especially in decision making involving many assessors [17].

In this study we propose the HFLTS-DEA model, in which HFLTS can combine assessment of some assessors for qualitative input and output by considering computational trusts. The value of the HFLTS will be used by DEA to measure the efficiency of a Decision-Making Unit (DMU). The rest of the paper is organized as follows. In Section 2, The Related Work, In Section 3, Problem Formulations or Methodology. In Section 4, the proposed method. In Section 5, Result, Analysis and Discussions. Finally, the paper reaches a conclusion in Section 6.

2 Related Work

The benchmarking process for quantitative data is easier because it can be measured. Measurement of qualitative data will be more difficult, especially if it contains elements of uncertainty and hesitant. The benchmarking process carried out on qualitative data has been carried out by a number of researchers. Lee and Worthington have used the DEA Network Model to conduct a benchmarking process against the quality of university research publications in relation to the benchmarking process of higher education[18]. The use of the DEA Network Model has been carried out by a number of other researchers in the benchmarking process against the university library[19] and also research and development[20]. The Use of the DEA Network is actually to overcome the limitations of the Standard DEA Model in handling measurements on qualitative data, where each input and output is denoted by a node. Where the weight at each input and output node will be distinguished based on the type of data. The existence of the DEA Model Network can do the benchmarking process on qualitative data, but still cannot overcome the conditions that contain fuzzy, uncertainty, and hesitancy. Research conducted by Jafarzadeh et al. have used Fuzzy Quality Function Development and DEA in the process of benchmarking against qualitative data in uncertainty[21].

The use of Fuzzy in the benchmarking process has been carried out by a number of researchers such as: Ghapanchi et al.[22] proposed the Fuzzy DEA method, Huang et al. explained about the Fuzzy AHP method[23], and Tavana et al. put forward the Fuzzy DEA TOPSIS method[24]. Research conducted by a number of researchers although it can overcome the element of uncertainty but has not been able to overcome hesitancy from assessors in providing an assessment. The research conducted by Kao and Lin gave results that in giving assessment is easier to do by using linguistic variables provided by fuzzy[25]. In research involving fuzzy efficiency there are several values that can be given to a qualitative data in a linguistic variable and therefore it is necessary to pay attention to the hesitancy aspects of decision making in the provision of qualitative data[26]. Fuzzy Hesitant can be used for assessment due to the presence of doubt in giving an assessment among several values [27].

3 Problem Formulations or Methodology

3.1 HFLTS-DEA Correspondence

Taking the two advantage of HFLTS, A benchmarking process that involves many assessors and computational trusts in providing an assessment, this study proposes a HFLTS-DEA to improve DEA capability in measuring qualitative data. Qualitative data measurements cannot be done as easily as quantitative data that directly have a measurable value. Quality measurement is usually expressed in linguistic variables such as: Good and excellent or in the form of linguistic value ranges such as lower than moderate which can mean low and very low. This is coupled with hesitations in decision making related to computational trust. The existence of this can be more easily done by obtaining the exact value to be used in the benchmarking process with DEA, using HFLTS. The HFLTS-DEA correspondence can be seen in Figure 1.

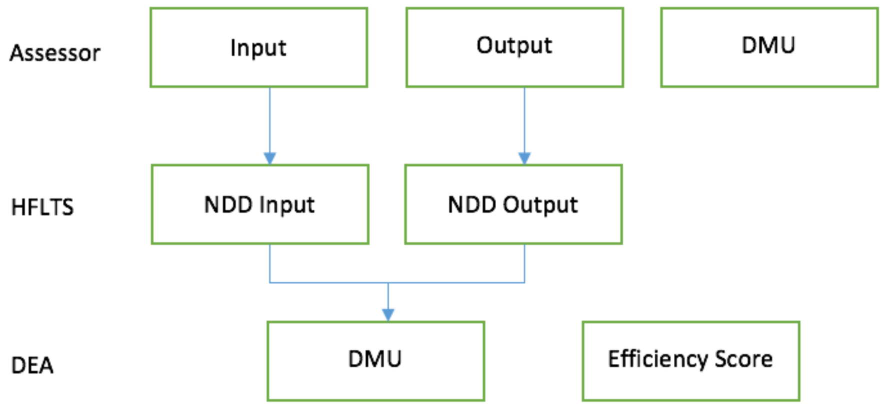

Fig. 1: HFLTS-DEA Correspondence

In Figure 1 it can be seen that in the evaluation of efficiency with DEA, the assessor conducts an assessment process for each DMU based on the inputs and outputs produced by each DMU. The problem that arises is if there are inputs and outputs that contain qualitative data so that the HFLTS method is needed to carry out the assessment process with the DEA. Where each input and output containing qualitative data will be assessed based on Non-Dominance Choice Degree (NDD) values generated by HFLTS. NDD values generated in qualitative data will be assessed together with quantitative data from each DMU by using DEA to obtain efficiency scores from each DMU.

3.2 HFLTS-DEA Model

In general HFLTS-DEA begins with a number of assessors providing an assessment of linguistic terms to the inputs and outputs of each DMU. HFLTS will process collecting the linguistic expressions and then collecting linguistic terms for HFLTS will be conducted. After that, calculating the envelope for hesitant terms will be done and then aggregation process can be done to calculate linguistic interval vector. The next step will be established preference relation degrees and followed by calculating NDD to generate exact values for input and output of each DMU. Once generated the input and output values of each DMU will proceed with the benchmarking process with the bound output of DEA. The benchmarking results will determine whether a DMU is efficient or not. General Scheme of the proposed approach can be seen in Figure 2.

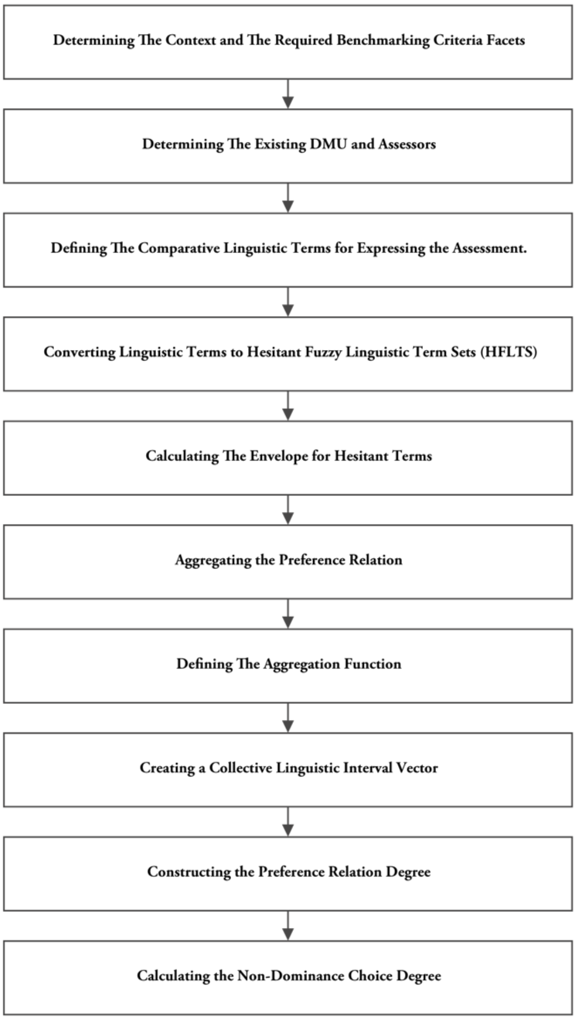

Fig. 2: Stage of The Proposed Method

In Figure 2, it can be seen that the HFLTS-DEA Model consists of 10 (ten) stages, and a more detailed discussion of the stages of the HFLTS-DEA Model can be seen in Sections 4 and 5.

4 The Proposed Method

4.1 Hesitant Fuzzy Linguistic Term Sets (HFLTS)

As for the steps in Hesitant Fuzzy Linguistic Term Sets (HFLTS) are as follows[17].
1. Determining The Context and The Required Benchmarking Criteria Facets

The first step is to determine the Context and benchmarking criteria for each input and output. Once obtained, then the next step is to define the priority of each criteria in the following set.

$$P_{r}{}_{Criteria}^{Context} = \{pr_1 \ldots, pr_N\} \tag{1}$$

2. Determining The Existing DMU and Assessors

We assume that a set of assessors $R = \{r_1, \ldots, r_M\}$ $(M \geq 2)$ express their opinions about a set of existing DMU represented as $Tr = \{tr_1, \ldots, tr_T\} (T \geq 2)$. The Assessors provide their assessment about the DMU by using the paired comparison matrix as follows:

$$P_f^i = \begin{bmatrix} p_{f11}^i & \cdots & p_{f1T}^i \\ \vdots & \ddots & \vdots \\ p_{fT1}^i & \cdots & p_{fTT}^i \end{bmatrix} \tag{2}$$

Where p_f^i denotes the preference relation matrix of r_i toward the benchmarking criteria facet f and p_{flk}^i $(1 \leq l \leq n, 1 \leq k \leq n)$ represents the preference degree of t_l over t_k corresponding of the benchmarking facet f and from the subjective viewpoint of r_i.

3. Defining The Comparative Linguistic Terms for Expressing the Assessment.

In general, in the assessment process for qualitative problems, the expression given is in linguistic form which is a characteristic of the fuzzy method, where the linguistic terms are defined as follows.

$$S = \{neither, very\ low, low, medium, high, very\ high, absolute\} \tag{3}$$

There for an assessors r_i can state the results of their assessment in the form of a preference matrix as follows:

$$P_f^i = \begin{bmatrix} - & Very\ High & Low \\ Medium & - & Absolute \\ Very\ Low & Neither & - \end{bmatrix} \tag{4}$$

Rodriguez et al. Have used Context-Free Grammar in the use of a more complex process of linguistic disclosure [28].

$$V_N = \left\{ \begin{matrix} \langle Primary\ Term \rangle, \langle Composite\ Term \rangle, \langle Unary\ Relation \rangle, \\ \langle Binary\ Relation \rangle, \langle Conjunction \rangle \end{matrix} \right\}$$

$$V_T = \{Lower\ than, Greater\ than, At\ Least, At\ Most, \\ Between, and, s_0, \ldots, s_g\}$$

$$i \in V_N$$

$$P =$$

$$\{\ I ::=\ \langle Primary\ Term \rangle : \langle Composite\ Term \rangle$$

$$\langle Composite\ Term \rangle ::==$$

$$\langle Unary\ Relation \rangle \langle Primary\ Term \rangle | \langle Binary\ Relation \rangle$$

$$\langle Primary\ Term \rangle \langle Conjunction \rangle \langle Primary\ Term \rangle$$

$$\langle Primary\ Term \rangle ::=\ s_0 | s_1 | \ldots | s_g$$

$$\langle Unary\ Relation \rangle ::=\ Lower\ Than | Greater\ Than | At\ Least | At\ Most$$

$$\langle Binary\ Relation \rangle ::=\ Between$$

$$\langle Conjunction \rangle ::=\ And\} \tag{5}$$

An Assessor may state his/her assessment in as follows:

$$P_f^i = \begin{bmatrix} - & Lower\ than\ High & Low \\ Greater\ than\ Medium & - & At\ most\ Medium \\ At\ least\ High & Between\ Very\ Low\ and\ Medium & - \end{bmatrix} \tag{6}$$

4. Converting Linguistic Terms to Hesitant Fuzzy Linguistic Term Sets (HFLTS)

The conversion function should be performed by using a transformation function as E_{G_H}.

Assume that E_{G_H} to be transformation function that converts the linguistic expressions $l \in S_l$ obtained by using the grammar G_H to HFLTS. Therefore, this transformation function is formally defined as:

$$E_{G_H}: S_l \rightarrow H_s \tag{7}$$

Where S_l is the result of measurements produced by G_H and H_S denotes the HFLTS.

5. Calculating The Envelope for Hesitant Terms

The envelope process in the HFLTS method can be denoted by *env* (H_s), this eqaution stating the linguistic interval in the HFLTS and can be defined as follows:

$$env(H_s) = [H_{S^-}, H_{S^+}], H_{S^-} \leq H_{S^+} \tag{8}$$

Where H_{S^-} and H_{S^+} are defined as follows:

$$H_{S^+} = max(s_i) = s_j \in H_s \text{ and } s_i \leq s_j \forall i$$

$$H_{S^-} = min(s_i) = s_j \in H_s \text{ and } s_i \geq s_j \forall i \tag{9}$$

6. Aggregating the Preference Relation

To obtain a Collective Preference from each DMUs, language intervals must be collected using the linguistic aggregation operator. This process will be carried out based on the upper and lower limits of each linguistic interval. For aggregation at the lower limit will produce a pessimistic value and for aggregation at the upper limit will produce optimistic values. Both of these values are needed to form the optimistic and pessimistic matrix as follows:

$$P_C^+ = \begin{bmatrix} (S_r, \alpha)_{11}^+ & \dots & (S_r, \alpha)_{1+}^+ \\ \vdots & \ddots & \vdots \\ (S_r, \alpha)_{T1}^+ & \dots & (S_r, \alpha)_{TT}^+ \end{bmatrix} \tag{10}$$

7. Defining The Aggregation Function

Determining the weight of each parameter in the benchmarking process is based on the aggregation process using the following equation:

$$\omega_i = Vagueness\ x\ Certainty\ x\ Trustworthiness \tag{11}$$

In accordance with Equation 11, there are stages where a parameter is needed for determining vagueness based on the assessment given by the assessor. If we assume that the envelope of HFLTS is defined as $[s_i, s_j]$ $\{0 \leq i \leq g, 0 \leq j \leq g\}$, then, the vagueness parameter is defined as follows:

$$Vagueness = \frac{g-|(i-j)|}{g} \tag{12}$$

Assume that the basic linguistic terms as below:

$$S = \{Very\ Doubtful, Doubtful, Neutral, Sure, Very\ Sure, Absolutely\ Sure\} \tag{13}$$

To obtain a concrete value from the certainty expression, it can be done using the following equation:

$$Certainty_i = \Delta^{-1}(S_i, \propto) \tag{14}$$

Trustworthiness states the level of trust from the assessment given by each assessor to each DMU. The distance between the assessment results of each assessor can be measured using the Hesitant Normalized Hamming Distance as follows:

$$d(DMU, Assessors) = \frac{1}{l}\sum_{j=1}^{l} |h^{Direct}(x_j) - h^{Evaluation}(x_j)| \tag{15}$$

8. Creating a Collective Linguistic Interval Vector

The collective linguistic interval vector are calculated as follows:

$$p_i^+ = \Delta(\delta(\Delta^{-1}(s_t, \propto)_{ij}^+))\ \ \forall j \epsilon \{1, \ldots,)T\}$$

$$p_i^- = \Delta(\delta(\Delta^{-1}(s_t, \propto)_{ij}^-))\ \ \forall j \epsilon \{1, \ldots,)T\} \tag{16}$$

A Collective linguistic interval vector for each DMU as follows:

$$V^t = (p_1^t, p_2^t, \ldots, p_T^t) \tag{17}$$

9. Constructing the Preference Relation Degree

By using the following equation we can calculate the preference degree of each DMU:

$$P(A > B) = \frac{max(0,a_2-b_1)-max(0,a_1-b_2)}{(a_2-a_1)+(b_2-b_1)} \quad (18)$$

$$P(B > A) = \frac{max(0,b_2-a_1)-max(0,b_1-a_2)}{(a_2-a_1)+(b_2-b_1)}$$

10. Calculating the Non-Dominance Choice Degree

The NDD of the each DMU is defined as follows:

$$NDD_f = min\left\{1 - p^S_{f\,ji}, j = 1\ldots n, j \neq i\right\} \quad (19)$$

4.2 Data Envelopment Analysis with Upper Bound on Output

In general, CCR Method uses fractional programming which can be stated as follows[29]:

$$Maximize \ \propto = \frac{\sum_{r=1}^{k} u_r y_{r0}}{\sum_{s=1}^{l} v_s x_{s0}} \quad (20)$$

Limit or constraint function:

$u_r, v_s \geq 0$; r = 1, ..., k; s = 1,.., l

Where:
$\propto = Efficiency\ Object\ s$
$k = Output\ Object\ s\ Observed$
$y_{is} = The\ Number\ of\ Outputs\ i\ produced\ by\ Object\ s$
$x_{is} = The\ nUmber\ of\ Inputs\ i\ Used\ by\ Object\ s$
$u_i = The\ Output\ Weight\ i\ Produced\ by\ Object\ s$
$y_i = The\ Input\ Weight\ i\ Given\ by\ Object\ s$

The linear programming model for generating bounded intervals is as follows:

$$Maximize\ \beta \ = \sum_{r=1}^{k} u_r y_{rj}$$

$$\sum_{i=1}^{m} w_{id}\, x_{id}^{l} - \sum_{r=1}^{s} u_{rd} y_{rd}^{u} \geq 0, j = 1,2,\ldots,n$$

$$\sum_{i=1}^{m} w_{id}\, x_{id}^{l} = 1$$

$$w_{id}, u_{rd} \geq \in, \forall_{id}, rd \quad (21)$$

In Eq. (21), it can be seen that the existing DMU is still in an unfavorable condition because the input used is still greater than the output. This situation can be corrected by using linear programming that is oriented to bounded output as can be seen in Eq. (22).

$$Maximize\ \beta\ =\ \sum_{r=1}^{k} u_r y_{rj}$$

$$\sum_{i=1}^{m} w_{id}\, x_{id}^{l} - \sum_{r=1}^{s} u_{rd} y_{rd}^{u} \geq 0, j = 1,2,\dots,n$$

$$\theta_d^l * \sum_{r=1}^{s} u_{rd} y_{rd}^{l} - \sum_{i=1}^{m} w_{id} x_{id}^{u} = 0$$

$$\sum_{i=}^{m} w_{id} x_{id}^{u} = 1$$

$$w_{id}, u_{rd} \geq \in, \forall_{id}, rd \tag{22}$$

Where θ is the upper bound interval. θ itself can be obtained by using Eq. (23).

$$\sum_{i=}^{m} w_{id} x_{id}^{u} = 1$$

$$w_{id}, u_{rd} \geq \in, \forall_{id}, rd \tag{23}$$

5 Results, Analysis and Discussions

5.1 HFLTS Process

To facilitate us in understanding how the HFLTS method works in benchmarking inputs and inputs that are qualitative data, in this study will be given a benchmarking example of a study program at the University of Malikussaleh. In general the inputs are: Number of Students, Number of Lecturers, academic service quality, and academic atmosphere. While the output is: Number of Research, Number of Graduates, and Student Satisfaction. Where there are 2 inputs that is qualitative data, namely: academic service quality and academic atmosphere and 1 output that is qualitative data, that is: student satisfaction. In the study it is assumed that 3 criteria will be assessed by 3 assessors using HFLTS for 3 DMU. The steps are as follows.

1. Determining the results of the assessment in linguistic form

The assessment process stated linguistically can be seen as follows.

a. Neither
b. Very Poor
c. Poor
d. Medium
e. Good
f. Very Good
g. Absolut

2. Determining the assessment results in linguistic form

Suppose the results of the assessment of the first assessor in the preference matrix is as follows.

$$P^1_{Service}=\begin{bmatrix} - & Between\ very\ poor\ and\ poor & Lower\ than\ medium \\ High & - & At\ least\ medium \\ At\ most\ Good & Between\ poor\ and\ Medium & - \end{bmatrix}$$

$$P^1_{Atmosphere}=\begin{bmatrix} - & Greater\ than\ medium & Poor \\ Very\ Poor & - & At\ least\ Good \\ At\ least\ medium & Between\ high\ and\ very\ high & - \end{bmatrix} \quad (24)$$

$$P^1_{Satisfaction}=\begin{bmatrix} - & Good & Poor \\ Very\ Good & - & At\ most\ poor \\ Between\ medium\ and\ very\ good & Greater\ than\ medium & - \end{bmatrix}$$

Suppose the results of the assessment of the second assessor in the preference matrix is as follows.

$$P^2_{Service}=\begin{bmatrix} - & Very\ poor & Lower\ than\ poor \\ Between\ good\ and\ very\ good & - & very\ good \\ Greater\ than\ medium & At\ most\ poor & - \end{bmatrix}$$

$$P^2_{Atmosphere}=\begin{bmatrix} - & Greater\ than\ medium & Very\ good \\ Greater\ than\ good & - & At\ least\ medium \\ Between\ very\ poor\ and\ medium & Between\ medium\ and\ very\ good & - \end{bmatrix} \quad (25)$$

$$P^2_{Satisfaction}=\begin{bmatrix} - & Between\ good\ and\ very\ good & Medium \\ Very\ poor & - & At\ least\ medium \\ Lower\ than\ medium & Between\ very\ poor\ and\ poor & - \end{bmatrix}$$

Suppose the results of the assessment of the third assessor in the preference matrix is as follows.

$$P^3_{Service}=\begin{bmatrix} - & Very\ poor & Very\ good\ lower\ than\ poor \\ Very\ poor & - & Very\ good \\ Greater\ than\ medium & medium & - \end{bmatrix}$$

$$P^3_{Atmosphere}=\begin{bmatrix} - & Poor & Very\ good \\ Greater\ than\ good & - & Greater\ than\ medium \\ Between\ very\ poor\ and\ medium & Very\ good & - \end{bmatrix} \quad (26)$$

$$P^3_{Satisfaction}=\begin{bmatrix} - & High & Poor \\ Poor & - & Medium \\ Lower\ than\ medium & Between\ very\ poor\ and\ poor & - \end{bmatrix}$$

3. Presents the results of the results of the assessment in linguistic envelopes

The next step after obtaining the results of the assessment of the assessors 1, 2, and 3. Then the next step is to combine the results of the assessment and presents it in the form of envelopes linguistic as can be seen in Table 1.

Table 1: Envelopes Linguistic

	Service	**Atmosphere**	**Satisfaction**
R1	$\begin{bmatrix} - & [vp,p] & [n,m] \\ [g,g] & - & [m,a] \\ [n,g] & [p,m] & - \end{bmatrix}$	$\begin{bmatrix} - & [g,a] & [p,p] \\ [vp,vp] & - & [g,a] \\ [m,a] & [g,vg] & - \end{bmatrix}$	$\begin{bmatrix} - & [g,g] & [p,p] \\ [vg,vg] & - & [n,p] \\ [m,vg] & [g,a] & - \end{bmatrix}$
R2	$\begin{bmatrix} - & [vp,vp] & [vp,vp] \\ [g,vg] & - & [vg,vg] \\ [g,a] & [n,p] & - \end{bmatrix}$	$\begin{bmatrix} - & [g,a] & [vg,vg] \\ [vg,vg] & - & [m,a] \\ [vp,m] & [m,vg] & - \end{bmatrix}$	$\begin{bmatrix} - & [g,vg] & [m,m] \\ [vp,vp] & - & [m,a] \\ [n,p] & [vp,p] & - \end{bmatrix}$
R3	$\begin{bmatrix} - & [vg,vg] & [n,n] \\ [vp,vp] & - & [vg,vg] \\ [g,a] & [m,m] & - \end{bmatrix}$	$\begin{bmatrix} - & [p,p] & [vg,vg] \\ [m,m] & - & [g,a] \\ [vp,m] & [vg,vg] & - \end{bmatrix}$	$\begin{bmatrix} - & [g,g] & [p,p] \\ [p,p] & - & [m,m] \\ [n,p] & [vp,p] & - \end{bmatrix}$

4. Express the degree of certainty of the assessment

The assessor may also state the degree of certainty in providing an assessment in the previous stage (if not granted, the assumed value is 2). The degree of certainty of the assessment if given then it is done in the form of linguistic value.

a. Very Doubtful = 0
b. Doubtful = 1
c. Neutral = 2
d. Sure = 3
e. Very Sure = 4
f. Absolutely = 5

For example, suppose the level of assurance that is filled can be seen in Table 2.

Table 2: Degree of Certainty

	Service	Atmosphere	Satisfaction
R_1	$\begin{bmatrix} - & [d,s] & [s,as] \\ [vd,d] & - & [d,n] \\ [vs,vs] & [vs,as] & - \end{bmatrix}$	$\begin{bmatrix} - & [n,s] & [vd,as] \\ [vd,vd] & - & [d,n] \\ [vs,as] & [d,as] & - \end{bmatrix}$	$\begin{bmatrix} - & [s,s] & [d,d] \\ [vd,n] & - & [vd,d] \\ [n,as] & [s,s] & - \end{bmatrix}$
R_2	$\begin{bmatrix} - & [n,s] & [d,as] \\ [vd,vd] & - & [d,s] \\ [s,vs] & [vs,vs] & - \end{bmatrix}$	$\begin{bmatrix} - & [d,n] & [n,vs] \\ [d,d] & - & [vd,n] \\ [n,vs] & [s,as] & - \end{bmatrix}$	$\begin{bmatrix} - & [n,vs] & [d,n] \\ [d,vd] & - & [d,d] \\ [d,n] & [vd,vs] & - \end{bmatrix}$
R_3	$\begin{bmatrix} - & [vd,n] & [s,vs] \\ [d,d] & - & [vd,vd] \\ [n,n] & [s,s] & - \end{bmatrix}$	$\begin{bmatrix} - & [sd,vd] & [s,s] \\ [d,d] & - & [vd,vd] \\ [as,as] & [s,s] & - \end{bmatrix}$	$\begin{bmatrix} - & [d,n] & [s,as] \\ [s,s] & - & [as,as] \\ [n,n] & [d,d] & - \end{bmatrix}$

5. Determining the Value of Optimistic and Pessimistic Assessment

5.1. Determining the Value of Optimistic Assessment

To perform an optimistic assessment it can be done using equations (10), (11), and (13). Suppose we will determine the optimistic value of DMU 1 to DMU 2 derived from the assessors 1, 2, and 3. Then we can see Table 1 and Table 2. The optimistic assessment results for DMU 1 to 2 for Service, Atmosphere, and Satisfaction are as follows.

a. Assessment of the 1st Assessor (DMU 1 to 2 for service)

   [VP,P], this mean Optimistic is P, and Pessimistic is VP

b. Assessment of the 2nd Assessor (DMU 1 to 2 for service)

   [VP,P], this mean Optimistic is VP, and Pessimistic is VP

c. Assessment of the 3rd Assessor (DMU 1 to 2 for service)

   [VG,VG], this mean Optimistic is VG, and Pessimistic is VG

The degree of certainty results for DMU 1 to 2 for Service, Atmosphere, and Satisfaction are as follows.

a. Degree of certainty of the 1st Assessor (DMU 1 to 2 for service)

   [D,S], this mean Optimistic is S, and Pessimistic is D

b. Degree of certainty of the 2nd Assessor (DMU 1 to 2 for service)

   [N,S], this mean Optimistic is S, and Pessimistic is N

c. Degree of certainty of the 3rd Assessor (DMU 1 to 2 for service)

   [VD,N], this mean Optimistic is N, and Pessimistic is VD

So to calculate the optimistic value of DMU 1 to 2 for services derived from the assessors 1, 2, and 3 are as follows:

$$P^{+}_{C_{Service_{12}}} = \Delta\left(\frac{1}{\sum_{i=1}^{3} wi}\left(w_1\Delta^{-1}(p,0) + w_2\Delta^{-1}(vp,0) + w_3\Delta^{-1}(vg,0)\right)\right) \quad (27)$$

The values of W_1, W_2, and W_3 are calculated as follows.

$$W_1 = Vagueness_1 \chi\ Certainty_1 \chi Trustworthiness_1$$

$$W_1 = \frac{6-1}{6} x\ \Delta^{-1}(s,0)\ x\ 0.5 = 1.25$$

$$W_2 = \frac{6-(1-1)}{6} x\ \Delta^{-1}(s,0)\ x\ 0.5 = 1.5 \quad (28)$$

$$W_3 = \frac{6-(5-5)}{6} x\ \Delta^{-1}(s,0)\ x\ 0.5 = 1$$

Having obtained values W_1, W_2, and W_3 then we can calculate the optimistic value of DMU 1 to 2 for service criteria.

$$P^{+}_{C_{Service_{12}}} = \Delta\left(\frac{1}{3.75}(1.25x2 + 1.5\ x\ 1 + 1x5)\right) = \Delta(2.39) \quad (29)$$

$$P^{+}_{C_{Service_{12}}} = (p, 0.39)$$

The same way can be used to calculate the optimistic values of various DMU combinations for each input and output. So the final result of optimistic value calculation is shown in Table 3.

Table 3: Optimistic Value

	Service	**Atmosphere**	**Satisfaction**
P^{+}_{C}	$\begin{bmatrix} - & [p,0.39] & [vp,0. \\ [p,0.5] & - & [vg,0 \\ [vg,0.42] & [m,-0.29] & - \end{bmatrix}$	$\begin{bmatrix} - & [a,0] & [g,-0 \\ [g,0] & - & [a,-0. \\ [g,-0.08] & [a,-0.01] & - \end{bmatrix}$	$\begin{bmatrix} - & [g,0.4] & [p,0. \\ [m,0.2] & - & [m,0. \\ [g,-0.34] & [m,0.33] & - \end{bmatrix}$

5.2. Determining the Value of Pessimistic Assessment

The equation for calculating pessimistic value is the same as calculating the optimistic value. We must calculate the values of W_1, W_2, and W_3 as follows:

$$P^{-}_{C_{Service_{12}}} = \Delta\left(\frac{1}{\sum_{i=1}^{3} wi}\left(w_1\Delta^{-1}(vp,0) + w_2\Delta^{-1}(vp,0) + w_3\Delta^{-1}(vg,0)\right)\right) \quad (30)$$

How to calculate the value of W_1, W_2, and W_3 is the same as in Optimistic calculation. Vagueness and trustworthiness values for W_1, W_2, and W_3 are the same as in Optimistic. The only difference in the calculation of certainty, the Pessimistic calculation used is the Pessimistic scale value.

In the Optimistic Calculation, Vagueness value for W_1 is 5/6, for W_2 is 1, and for W_3 is 1. The Trustworthiness value is 0.5. The Certainty value for W_1 is to take the pessimistic value is d (scale value = 1), the certainty value for W_2 is n (scale value = 2), and the certainty value for W_3 is vd (scale value = 0). Then the values W1, W2, and W3 are as follows.

$$\begin{aligned} W_1 &= \frac{5}{6}\ x\ 1\ x\ 0.5\ =\ 0.41 \\ W_2 &= 1\ x\ 2\ x\ 0.5\ =\ 1 \\ W_3 &= 1\ x\ 0\ x\ 0.5\ =\ 0 \end{aligned} \tag{31}$$

Pessimistic value for DMU 1 to 2 for Service is as follows:

= 1 / 1.41 * (0.41 * 1 + 1 * 1 + 0 * 5)

= 1 / 1.41 * (1.41) = 1.41

= [VP, 0}

The same calculation can be done to calculate the pessimistic value of alternative combinations for each criterion. So the final result of calculation of pessimistic value is can be seen in Table 4.

Table 4: Pessimistic Value

	Service	Atmosphere	Satisfaction
P_C^+	$\begin{bmatrix} - & [vp,0] & [n,0.17] \\ [vp,0] & - & [g,0.27] \\ [m,-0.3] & [p,-0.3] & - \end{bmatrix}$	$\begin{bmatrix} - & [g,0] & [vg,0] \\ [vp,0] & - & [g,0] \\ [p,-0.38] & [g,0.14] & - \end{bmatrix}$	$\begin{bmatrix} - & [g,0] & [p,0.2] \\ [p,-0.25] & - & [m,0] \\ [vp,0.19] & [m,0.14] & - \end{bmatrix}$

6. Establish a Vector Interval Linguistic for Each Criterion

Before building a vector interval linguistic for each criterion, we must construct vector optimistic and pessimistic for each criterion. In this process, the optimistic and pessimistic data values on the same row in the matrix will be combined. Vector optimistic and pessimistic for each criterion can be seen in Table 5.

Table 5: Vector Optimistic and Pessimistic for Each Criterion

	P_1^+	P_2^+	P_3^+	P_1^-	P_2^-	P_3^-
Service	$(p,-0.235)$	$(g,-0.125)$	$(g,0.065)$	$(vp,-0.415)$	$(m,-0.365)$	$(p,0.2)$
Atmosphere	$(vg,-0.125)$	$(vg,-0.005)$	$(g,0.455)$	$(g,0.5)$	$(p,0.5)$	$(m,-0.12)$
Satisfaction	$(m,0.1)$	$(m,0.175)$	$(m,0.495)$	$(m,0.1)$	$(p,0.375)$	$(p,0.165)$

Having obtained an optimistic and pessimistic merging matrix for each criterion we can construct interval linguistic vector for each criterion as can be seen in Table 6.

Table 6: Linguistic Interval Vector for Each Criterion

	V^T
Service	$([(vp,-0.415),(p,-0.235)],[(vp,-0.415),(p,-0.235)],[(p,0.2),(g,0.065)])$
Atmosphere	$([(g,0.5),(vg,-0.125)],[(p,0.5),(vg,-0.005)],[(m,-0.12),(g,0.455)])$
Satisfaction	$([(m,0.1),(m,0.1)],[(p,0.375),(m,0.175)],[(p,0.165),(m,0.495)])$

7. Determining the Preference Relation Degree for Each Criteria of the Alternative

To be able to determine the preference relation degree value for each criterion by using equation (18). For example, we will combine the 2nd and 3rd DMUs on the service.

A represents ([m, -0.365), [g, -0.125) and B represents ([p, 0.2], [g, 0.065)

The merging process can be seen in Table 7.

Table 7: Merging Process

A (2nd DMU)	**B (3rd DMU)**
m,-0.35 = 2.635	p,0.2 = 2.2
g,-0.125 = 3.875	g,0.065

$$P_{13}^{Service} = \frac{max(0,\ 3.875-2.2)-max(0,\ 2.635-4.065)}{(3.875-2.635)+(4.065-2.2)}$$

$$P_{13}^{Service} = \frac{1.675}{3.0465} = 0.549$$

The result of the merger becomes the 2nd column row element to 3 in the preference relation degree matrix. The same way can be used for other elements to obtain a preference relation degree matrix as can be seen in Table 8.

Table 8: Preference Relation Degree Matrix

	Service	Atmosphere	Satisfaction
P_D	$\begin{bmatrix} - & 0 & 0 \\ 1 & - & 0.549 \\ 1 & 0.451 & - \end{bmatrix}$	$\begin{bmatrix} - & 0.82 & 1 \\ 0.18 & - & 0.519 \\ 0 & 0.481 & - \end{bmatrix}$	$\begin{bmatrix} - & 0.9 & 0.7 \\ 0.1 & - & 0.474 \\ 0.3 & 0.526 & - \end{bmatrix}$

8. Determining the Non-Dominance Choice Degree (NDD)

To be able to determine the value of NDD then we must calculate PNDD with equation (32).

$$Max(P_{f_{ji}} - P_{f_{ij}}, 0) \quad (32)$$

For example we want to obtain the 1st row element of the 2nd column in service, then the data used is the 1st column row element data and the 2nd column row element data in the preference relation degree matrix.

The 1st row element of column 2 = 0
The 2nd row element of column 1 = 1
Then Max (0-1, 0) = Max (-1, 0) = 0
So the PNDD matrix formed can be seen in Table 9.

Table 9: PNDD Matrix

	Service	Atmosphere	Satisfaction
P_{NDD}	$\begin{bmatrix} - & 0 & 0 \\ 1 & - & 0.098 \\ 1 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 0.64 & 1 \\ 0 & - & 0.038 \\ 0 & 0 & - \end{bmatrix}$	$\begin{bmatrix} - & 0.8 & 0.4 \\ 0 & - & 0 \\ 0 & 0.052 & - \end{bmatrix}$

Having obtained the value of PNDD, then we can get the value of NDD by using the following equation.

$$NDD_f = min\left\{1 - P^S_{f_{ji}}, j = 1 \ldots n, j \neq 1\right\} \quad (33)$$

For example, we want to calculate NDD service for the 1st DMU, then we can use the data element on the 1st row of column 1, row 2nd column 1

The 1st row element of column 1 = is not taken into account

The 2nd row element of column 1 = 1

The 3rd row element of the 1st column = 1

The NDD element in the service for the 1st DMU is

min (1-1, 1-1) = min (0,0) = 0

NDD calculation results can be seen in the following matrix.

Table 10: NDD Matrix

	DMU 1	DMU 2	DMU 3
***NDD**Service*	0	1	0.902
***NDD**Atrmosphere*	1	0.36	0
***NDD**Satisfaction*	1	0.2	0.6

5.2 Result of HFLTS-DEA Process

The DMU used in this study is 19 courses at the University of Malikussaleh. The data of each DMU is as follows.

Table 11: DMU University of Malikussaleh

DMU	Input		Output	
	Number of Lecturers	Number of Students	Number of Research	Number of Graduates
Information Technology	17	588	5	610
Civil Engineering	26	747	5	533
Architectural Engineering	15	396	5	195
Industrial Engineering	17	467	5	300
Chemical Engineering	25	348	5	252
Mechanical Engineering	23	499	5	224
Electrical Engineering	19	420	5	326
Agribusiness	17	689	5	273
Agro-Technology	34	822	5	284
Aquaculture	10	501	5	204
Communication Science	11	719	5	273
Political Science	11	262	5	183
Sociology	13	487	5	204
Anthropology	9	173	5	116
Jurisprudence	50	1096	10	467
Medicine	30	278	4	257
Management	48	1265	5	1302
Economic Development	11	853	5	290
Accounting	23	1127	5	417

The above DMU data was obtained from University of Malikussaleh. The data of each DMU will be coupled with 2 inputs sourced from HFLTS namely: Academic Service Quality and Academic Atmosphere and 1 output sourced from HFLTS are: Student Satisfaction. So the result of DMU HFLTS-DEA is as follows.

Table 12: The Result of HFLTS-DEA

DMU	Input				Output		
	Number of Lecturers	Number of Students	Academic Service Quality	Academic Atmosphere	Number of Research	Number of Graduates	Student Satisfaction
Information Technology	17	588	0.75	0.8	5	610	0.87
Civil Engineering	26	747	0.32	0.67	5	533	0.43

Architectural Engineering	15	396	0.59	0.65	5	195	0.64
Industrial Engineering	17	467	0.81	0.56	5	300	0.76
Chemical Engineering	25	348	0.73	0.75	5	252	0.81
Mechanical Engineering	23	499	0.56	0.32	5	224	0.57
Electrical Engineering	19	420	0.66	0.68	5	326	0.68
Agribusiness	17	689	0.75	0.52	5	273	0.65
Agro-Technology	34	822	0.73	0.65	5	284	0.72
Aquaculture	10	501	0.59	0.57	5	204	0.61
Communication Science	11	719	0.67	0.87	5	273	0.9
Political Science	11	262	0.65	0.77	5	183	0.73
Sociology	13	487	0.66	0.69	5	204	0.71
Anthropology	9	173	0.65	0.45	5	116	0.67
Jurisprudence	50	1096	0.54	0.56	10	467	0.58
Medicine	30	278	0.67	0.71	4	257	0.73
Management	48	1265	0.68	0.78	5	1302	0.81
Economic Development	11	853	0.59	0.73	5	290	0.87
Accounting	23	1127	0.81	0.77	5	417	0.81

5.3 Discussion

Using (22), We can determine benchmarking for each DMU. The result can be seen in Table 13. For example for DMU1 (Department of Information Technology), the linear programming model can be written as follows.

Maximize 610 U1 + 5 U2+0.87 U3
Subject to
17 V1 + 588 V2+0.75 V3+ 0.8 V4 = 1
610 U1 + 5 U2+0.87 U3 - 17 V1 - 588 V2-0.75 V3 - 0.8 V4 <= 0
533 U1 + 5 U2+0.43 U3 - 26 V1 - 747 V2-0.32 V3-0.67 V4 <= 0
195 U1 + 5 U2+0.64 U3 - 15 V1 - 396 V2-0.59 V3-0.65 V4 <= 0
300 U1 + 5 U2+0.76 U3 - 17 V1 - 467 V2-0.81 V3-0.56 V4 <= 0
252 U1 + 5 U2+0.81 U3 - 25 V1 - 348 V2-0.73 V3-0.75 V4 <= 0
224 U1 + 5 U2+0.57 U3 - 23 V1 - 499 V2-0.56 V3-0.32 V4 <= 0
326 U1 + 5 U2+0.68 U3 - 19 V1 - 420 V2-0.66 V3-0.68 V4 <= 0
273 U1 + 5 U2+0.65 U3 - 17 V1 - 689 V2-0.75 V3-0.52 V4 <= 0
284 U1 + 5 U2+0.72 U3 - 34 V1 - 822 V2-0.73 V3-0.65 V4 <= 0
204 U1 + 5 U2+0.61 U3 - 10 V1 - 501 V2-0.59 V3-0.57 V4 <= 0
273 U1 + 5 U2+0.9 U3 - 11 V1 - 719 V2-0.67 V3-0.87 V4 <= 0
183 U1 + 5 U2+0.73 U3 - 11 V1 - 262 V2-0.65 V3-0.77 V4 <= 0

204 U1 + 5 U2+0.71 U3 - 13 V1 - 487 V2-0.66 V3-0.69 V4 <= 0
116 U1 + 5 U2+0.67 U3 - 9 V1 - 173 V2-0.65 V3-0.45 V4 <= 0
467 U1 + 5 U2+0.58 U3 - 50 V1 - 1096 V2-0.54 V3-0.56 V4 <= 0
257 U1 + 5 U2+0.73 U3 - 30 V1 - 278 V2-0.67 V3-0.71 V4 <= 0
1302 U1 + 5 U2+0.81 U3 - 48 V1 - 1265 V2-0.68 V3-0.78 V4 <= 0
290 U1 + 5 U2+0.87 U3 - 11 V1 - 853 V2-0.59 V3-0.73 V4 <= 0
417 U1 + 5 U2+0.81 U3 - 23 V1 - 1127 V2-0.81 V3-0.77 V4 <= 0
U1, U2, U3, V1, V2, V3, V4 >= 0
END

Based on the measurements with HFLTS-DEA, the efficiency of each as follows.

Table 13: Efficiency Score

Number	DMU	DEA Score
1	Information Technology	1
2	Civil Engineering	1
3	Architectural Engineering	0.95
4	Industrial Engineering	0.96
5	Chemical Engineering	0.99
6	Mechanical Engineering	1
7	Electrical Engineering	0.93
8	Agribusiness	0.91
9	Agrotechnology	0.81
10	Aquaculture	1
11	Communication Science	1
12	Political Science	1
13	Sociology	0.9
14	Anthropology	1
15	Jurisprudence	1
16	Medical	1
17	Management	1
18	Economic Development	1
19	Accounting	0.83

Based on Table 13 it can be seen that DMU 3, 4, 5, 7, 8, 9, 13, and 19 are inefficient marked with DEA Score which is not worth 1. It can be seen that HFLTS can give good measurement result on Qualitative Data so the HFLTS-DEA model can be used in the benchmarking process for qualitative data.

5 Conclusion

Based on the results of the research, there are several things obtained. First, the qualitative data assessment process can be used HFLTS. Second, HFLTS can be used in the assessment process that contains uncertainty and can take into account the computational trust. Third, HFLTS-DEA can perform benchmarking process in qualitative data well. Future research should be able to provide suggestions for efficiency improvement processes by considering the use of input and output resources generated.

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