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Tuning the Fractional-order PID-Controller for Blood Glucose Level of Diabetic Patients

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Abstract

The conducted research aims to attain the optimal approach for a certain control unit, namely the Proportional-Integral-Derivative-controller (or simply PID-controller), which typically monitors the blood sugar level of a diabetic patient. This would be implemented through introducing a fractional-order PID-controller (or $PI^{\gamma}D^{\rho}$-controller) instead of the already classical one. Two optimization algorithms, Particle Swarm Optimization (PSO) and Bacteria Foraging Optimization (BFO) algorithms, together with two different approximations, the Continued Fraction Expansion (CFE) and the Outstaloup's approaches, will be used to complete the design process. Several numerical comparisons will be performed to reach the best approach to meet the optimal needs of this industrial application.

Keywords: *$PI^{\gamma}D^{\rho}$-controller, Diabetic patients, Blood Glucose, Laplacian operator, Oustaloup's approach, continued fractional expansion approach.*

1 Introduction

Recently, the number of people with diabetes has increased significantly, especially adults. Diabetes is a chronic disease that occurs when the pancreas is unable to produce sufficient and adequate amounts of insulin. Insulin is the hormone that regulates the level of sugar in the blood, and moreover helps the glucose flow into the body's cells for energy. Scientists have shown that diabetes is the cause of many diseases of the body, such as kidney failure, heart attacks, strokes, and remove limb. Diabetes has several types, type 1, type 2 and gestational diabetes, including type 1 Diabetes, which often appears during childhood. The body cannot produce insulin, as for type 2, it is the most common, which often appears in adults over the age of 40, due to the slow appearance of symptoms on the patient for several years, finally gestational diabetes, it is the one that affects some pregnant women, as blood glucose levels are above the normal level [1, 2].

In order to know and sense the disturbance of the sugar level in the blood, a certain device containing a unit of control known usually as PID-controller is used. This would inform the medical practitioner automatically with an appropriate amount of insulin needed to control the level of glucose in the blood [1, 2]. Typically, the construction of the PID-controller relies on an integro-differential equation that possesses three parameters, K_p , K_i , K_d. It is well-known in the industrial field that optimal tuning of these parameters would yield a significant improvement in the performance of the device under consideration. Such tuning can be performed using several schemes, including Particle Swarm Optimization (PSO) algorithm, Bacteria Foraging Optimization (BFO) algorithm, Genetic Algorithm (GA), Ziegler–Nichols (ZN) tuning method, and many others.

More recently, it has been shown that the PID-controller can be further improved using the notion of fractional calculus, see [3, 4, 5]. Such notion, which relies on dealing with the derivative and the integral in their fractional-order forms, allow to turn the classical controller into the so-called fractional-order PID-controller, which consequently implies that the classical integro-differential equation would be turn into its fractional-order form, see [5, 6, 7]. This would add two extra parameters to the required tuning process; the fractional-order integral value (γ) and the fractional-order derivative value (ρ), which consequently requires a proper dealing with the two-corresponding fractional-order Laplacian operators, s^γ and s^ρ. Actually, these two operators can be approximated using several numerical approaches. The Continued Fractional Expansion (CFE) scheme, Oustaoup's approximation, Matsuda's approximation, El-Khazali's approximation are some of these approaches. The advantage of such approaches lies in approximately converting the fractional-order Laplacian operators (s^γ and s^ρ) into their corresponding integer-order rational transfer functions. The same optimization techniques can be also employed to obtain the optimal tuning of the five parameters (K_p , K_i , K_d, γ, ρ) for this newly constructed controller, the fractional-order PID-controller (or simply the $PI^\gamma D^\rho$-controller).

In this work, we intend to implement two optimization algorithms, the PSO and BFO algorithms, for the purpose of tuning the fractional-order PID-controller, and hence make the performance of blood glucose monitoring system more accurate and responsive. Once the fractional-order Laplacian operators, s^{γ} and s^{ρ} , will be needed to be approximated, we will use the CFE and Oustaloup's approaches. However, the remaining of this paper is arranged as follows: In the next section, we provide a brief overview about the Fractional-order PID–controller, while the design process of such controller for the blood glucose test system is illustrated in Section 3, followed by the last section that summarizes the whole work.

2 Fractional-order PID-Controller

The major construction of the fractional-order PID-controller was established by Podlubny et al. in 1997 in [8]. This was carried outby adding two additional parameters, namely (γ and ρ) to the main parameters (K_p , K_i , K_d) of the PID-controller. It was clearly verified that this construction is faster and more responsive than the classical one. In general, the PID-controller is extracted by applying the following fractional-order integro-differential equation:

$$y(t) = K_p e(t) + K_i J^{\gamma} e(t) + K_d D^{\rho} e(t) \quad (1)$$

where J^{γ} is the Riemann-Liouville operator of order γ, D^{ρ} is the Caputo operator of order ρ, and $e(t)$ is the error signal. By using the forward Laplace transform of (1), we get:

$$Z(s) = \frac{Y(s)}{E(s)} = K_p + \frac{K_i}{s^{\gamma}} + K_d s^{\rho} \quad (2)$$

where $E(s) = L\{e(t)\}$ is the Laplace transforms of $e(t)$. The main task of this work is to make the proposed controller applied efficiently within the blood glucose device. For attaining this purpose, we apply the PSO and BFO algorithms to find the optimal values of the five parameters of the Fractional-order controller. For those whom interested in understanding how these two algorithms work, they may refer to several references [5, 9, 10, 11]. In the optimality theory, there is an urgent need to build the so-called fitness function within the algorithms, in which reducing its value represents the desired goal of the optimization algorithm, so as to obtain optimal values for the fractional-order PID–controller. Herein, we intend to consider a certain fitness function that relies on four criteria; the steady-state error, settling time, rise time, and peak overshoot [10, 11]. However, the following is how such fitness function can be expressed [10, 11]:

$$V = \left(1 - e^{\beta}\right)(M_P - E_{ss}) + e^{-\beta}(T_s - T_r) \quad (3)$$

where β is the scaling factor, M_P is the peak overshoot, E_{ss} is the steady state error, T_s is the settling time, and T_r is the rise time. However, the overall tuning process of the $PI^{\gamma}D^{\rho}$-controller using the PSO and BFO algorithms could be described by the block diagram shown in Figure 1.

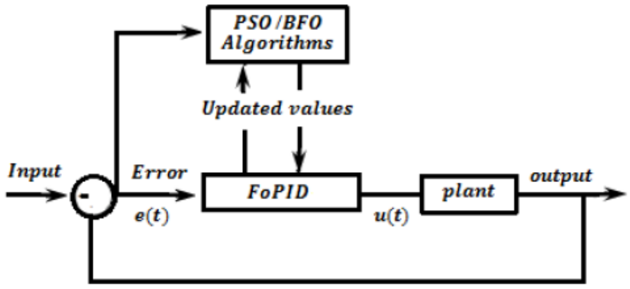

Fig. 1: Block diagram of PSO/BFO running to tune the $PI^{\gamma}D^{\rho}$ - controller

In order to go forward through the proposed tuning scheme, we will need to approximate the two fractional-order Laplacian operators, s^{γ} and s^{ρ} , where $0 < \gamma, \rho < 1$. In this regard, we will use two significant approaches; the CFE and Oustaloup's approaches. In particular, these two approaches have an ability to generate two more acceptable equivalent formulations to the two aforesaid Laplacian operators. Such formulations are of the form of integer-order rational transfer functions. However, to obtain a complete overview about the two under consideration approaches, the reader may refer to the references [6, 12, 13, 14].

3 Design the Fractional-order PID-Controller

It was reported in [1, 2] that the transfer function, which indicates the ratio between the two Laplace transforms of the output and the input for the blood glucose level system, is given by:

$$G(s) = \frac{1}{s^3+6s^2+5s} \tag{4}$$

In this regard, we will attempt to reduce the value of the fitness function given in (3) by applying the PSO and BFO algorithms. The two yielded fractional-order operators (s^{γ} and s^{ρ}) will be then approximated using the CFE and the Oustaloup's methods. This would approximately construct four fractional-order PID-controllers $C_i(s)$, which would imply also four closed-loop systems $H_i(s)$, where $\mathrm{i} = 1,2,3,4$. These closed-loop systems will be compared with each other to attain the best controller from the proposed ones. However, the overall results of the improvements are highlighted in the next manner.

- **The $PI^{\gamma}D^{\rho}$-PSO-controller via CFE approach:**

After executing PSO algorithm, we gain the following Fractional-order PID-controller:

$$C_1(s) = 2.69683 + \frac{55}{s^{0.911}} + 51s^{0.52043} \tag{5}$$

The two Laplacian operators, $s^{0.911}$ and $s^{0.52043}$, can be therefore approximated using the CFE approach to be as follows:

$$s^{0.911} = \frac{246.9621s^5+2.6421e+3s^4+5.6074e+3s^3+2.9951e+3s^2+332.0787s+1}{s^5+332.0787s^4+2.9951e+3s^3+5.6074e+3s^2+2.6421e+3s+246.962} \quad (6)$$

$$s^{0.52043} = \frac{12.2762s^5+180.8438s^4+499.3263s^3+351.6941s^2+57.5560s+1}{s^5+57.5560s^4+351.6941s^3+499.3263s^2+180.8438s+12.27621} \quad (7)$$

Thus, the $PI^{\gamma}D^{\rho}$-PSO-controller $C_1(s)$ would be in the form:

$$C_1(s) = \frac{\begin{array}{c}1.553e5s^{10}+3.999e6s^9+3.606e7s^8+1.453e8s^7+3.132e8s^6\\+4.018e8s^5+3.2198s^4+1.523e8s^3+3.826e7s^2+4.272e6s+1.668e55\end{array}}{\begin{array}{c}247s^{10}+1.686e4s^9+2.445e5s^8+1.378e6s^7+3.509e6s^6+4.353e6s^5\\+2.659e6s^4+7.767e5s^3+9.732e4s^2+4258s+12.28\end{array}} \quad (8)$$

This, consequently, implies the close-loop system $H_1(s)$, which would be in the following form:

$$H_1(s) = \frac{\begin{array}{c}1.553e5s^{10}+3.999e6s^9+3.606e7s^8+1.453e8s^7+3.132e8s^6\\+4.018e8s^5+3.2198s^4+1.523e8s^3+3.826e7s^2+4.272e6s+1.668e5\end{array}}{\begin{array}{c}247s^{13}+1.834e4s^{12}+3.469e5s^{11}+3.085e6s^{10}+1.7e7s^9+6.836e7s^8+1.916e8s^7\\+3.517e8s^6+4.199e8s^5+3.263e8s^4+1.528e88s^3+3.828e7s^2+4.272e6s+1.668e\end{array}} \quad (9)$$

- **The $PI^{\gamma}D^{\rho}$-BFO-controller via CFE approach:**

Herein, to obtain another Fractional-order PID-controller, we intend to execute the BFO algorithm this time. In summary, we obtain the following form:

$$C_2(s) = 6.3127 + \frac{4.7229}{s^{0.9151}} + 17.4946s^{0.5781} \quad (10)$$

The two operators $s^{0.9151}$ and $s^{0.5781}$ can be approximated using the CFE approach to be as follows:

$$s^{0.9151} = \frac{262.5893s^5+2.8005e+003s^4+5.9273e+3.1564e+3s^2+348.3569s+1}{s^5+348.3569s^4+3.1564e+3s^3+5.9273e+3s^2+2.8005e+3s+262.5893} \quad (11)$$

$$s^{0.5781} = \frac{16.9101s^5+236.9145s^4+628.9111s^3+425.6896s^2+66.1069s+1}{s^5+66.1069s^4+425.6896s^3+628.9111s^2+236.9145s+16.9101} \quad (12)$$

This would make (10) to be rewritten in the following form:

$$C_2(s) = \frac{\begin{array}{c}7.935e4s^{10}+2.046e6s^9+1.829e7s^8+7.105e7s^7+1.376e8s^6\\+1.425e8s^5+8.376e7s^4+2.883e7s^3+5.714e6s^2+5.634e5s+2.11e4\end{array}}{\begin{array}{c}262.6s^{10}+2.016e4s^9+3.028e5s^8+1.752e6s^7+4.556e6s^6+5.762e6s^5\\+3.585e6s^4+1.068e6s^3+1.365e5s^2+6128s+16.91\end{array}} \quad (13)$$

Hence, the closed-loop system will be in the form:

$$H_2(s) = \frac{\begin{array}{c}7.935e4s^{10}+2.046e6s^9+1.829e7s^8+7.105e7s^7+1.376e8s^6\\+1.425e8s^5+8.376e7s^4+2.883e7s^3+5.714e6s^2+5.634e5s+2.11e4\end{array}}{\begin{array}{c}262.6\,s^{13}+2.174e4s^{12}+4.251e5s^{11}+3.749e6s^{10}+1.86e7s^9+6.02e7s^8+1.32e8s^7\\+1.89e8s^6+1.67e8s^5+8.992e7s^4+2.955e7s^3+5.745e6s^2+5.635e5s+2.11e4\end{array}} \quad (14)$$

- **The $PI^{\gamma}D^{\rho}$-PSO-controller via Oustaloup's approach:**

In this instance, we execute the PSO algorithm to obtain a third fractional-order PID-controller, which would be in the following form:

$$C_3(s) = 28.8018 + \frac{66}{s^{0.821}} + 57.9553s^{0.976} \tag{15}$$

Using Oustaloup's approach will turn the two operators $s^{0.821}$ and $s^{0.976}$ into the following integer-order rational transfer functions:

$$s^{0.821} = \frac{43.85s^5+973.9s^4+2957s^3+1388s^2+100.8s+1}{s^5+100.8s^4+1388s^3+2957s^2+973.9s+43.85} \tag{16}$$

$$s^{0.976} = \frac{89.54\,s^5+1724\,s^4+4538s^3+1847s^2+116.2\,s+1}{s^5+116.2s^4+1847s^3+4538s^2+1724s+89.54} \tag{17}$$

Therefore, $C_3(s)$ in (15) will be turned in

$$C_3(s) = \frac{\begin{matrix}2.289e5s^{10}+9.624e6s^9+1.308e8s^8+6.544e8s^7+1.535e9s^6\\+2.002e9s^5+1.695e9s^4+7.52e8s^3+1.51e8s^2+1.107e7s+2.618e5\end{matrix}}{\begin{matrix}43.85s^{10}+6069s^9+1.971e5s^8+2.343e6s^7+1.012e7s^6+1.768e7s^5\\+1.167e7s^4+3.117e6s^3+3.026e5s^2+1.075e4s+89.54\end{matrix}} \tag{18}$$

This consequently yields the following closed-loop system:

$$H_3(s) = \frac{\begin{matrix}2.289e5s^{10}+9.624e6s^9+1.308e8s^8+6.544e8s^7+1.535e9s^6\\+2.002e9s^5+1.695e9s^4+7.52e8s^3+1.51e8s^2+1.107e7s+2.618e5\end{matrix}}{\begin{matrix}43.85s^{13}+6332s^{12}+233750s^{11}+3.785e6s^{10}+3.478e7s^9+2.209e8s^8+8.227e8s^7\\+1.697e9s^6+2.08e9s^5+1.712e9s^4+7.536e8s^3+1.511e8s^2+1.107e7s+2.618e5\end{matrix}} \tag{19}$$

- **The $PI^{\gamma}D^{\rho}$-BFO-controller via Oustaloup's approach:**

Similarly, we execute here the BFO algorithm to obtain the last fractional-order PID controller. This controller has the form:

$$C_4(s) = 8.3518 + \frac{17.6913}{s^{0.8676}} + 10.5999s^{0.906} \tag{20}$$

In this case, we use the Oustaloup approach to approximate the two operators $s^{0.8676}$ and $s^{0.9061}$, which would be in the following two forms:

$$s^{0.8676} = \frac{54.35s^5+1156s^4+3363s^3+1513s^2+105.2s+1}{s^5+105.2s^4+1513s^3+3363s^2+1156s+54.35} \tag{21}$$

$$s^{0.906} = \frac{64.89s^5+1332s^4+3741s^3+1624s^2+109s+1}{s^5+109s^4+1624s^3+3741s^2+1332s+64.89} \tag{22}$$

Actually, the above two Laplacian operators can convert (8) to be written in the following form:

$$C_4(s) = \frac{\begin{matrix}3.786e004s^{10}+1.625e6s^9+2.287e7s^8+1.218e8s^7+3.154e8s^6\\ +4.608e008s^5+4.125e8s^4+1.846e8s^3+3.687e7s^2+2.678e6s+6.295e4\end{matrix}}{\begin{matrix}54.35s^{10}+7080s^9+2.176e5s^8+2.449e6s^7+1.002e7s^6+1.659e7s^5\\ +1.039e7s^4+2.629e6s^3+2.42e5s^2+8158s+64.89\end{matrix}} \quad (23)$$

Hence, the corresponding closed-loop system would be in the form:

$$H_4(s) = \frac{\begin{matrix}3.786e4s^{10}+1.625e6s^9+2.287e7s^8+1.218e8s^7+3.154e8s^6\\ +4.608e8s^5+4.125e8s^4+1.846e8s^3+3.687e7s^2+2.678e6s+6.295e4\end{matrix}}{\begin{matrix}54.35s^{13}+7406s^{12}+2.604e5s^{11}+3.828e6s^{10}+2.743e7s^9+1.118e8s^8+2.819e8s^7\\ +4.633e8s^6+5.287e8s^5+4.271e8s^4+1.859e8s^3+3.692e7s^2+2.679e6s+6.295e4\end{matrix}} \quad (24)$$

To spotlight the dissimilarities between all previous design methods, some numerical results of the closed-loop transfer functions given in $H_1(s)$, $H_2(s)$, $H_3(s)$ and $H_4(s)$ are exhibited in Figure 1, Figure 2, Figure 3, Figure 4, and Table 1.

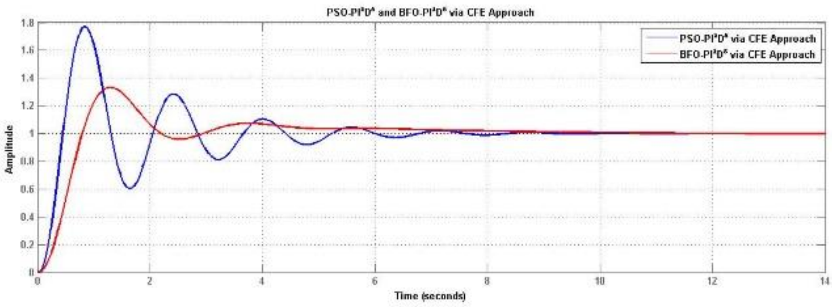

Fig.2: Step responses of $H_1(s)$ & $H_2(s)$.

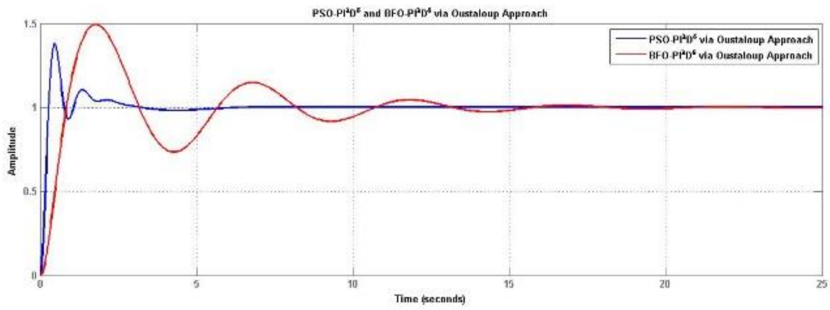

Fig.3: Step responses of $H_3(s)$ & $H_4(s)$.

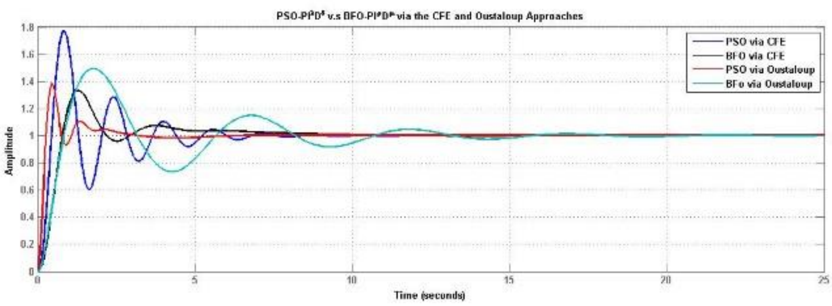

Fig.4: Step responses of $H_1(s)$, $H_2(s)$, $H_3(s)$ & $H_4(s)$.

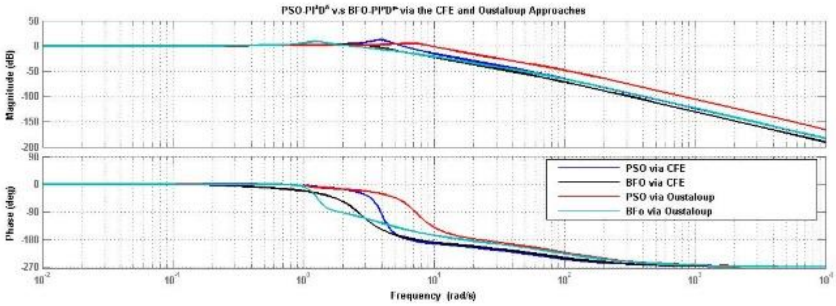

Fig.4: Bode diagrams of $H_1(s)$, $H_2(s)$, $H_3(s)$ & $H_4(s)$.

Table 1: Step responses of $H_1(s)$, $H_2(s)$, $H_3(s)$ & $H_4(s)$

Step response	$H_1(s)$	$H_2(s)$	$H_3(s)$	$H_4(s)$
Rise Time	0.28180	0.51710	0.18170	0.57550
Settling Time	7.22640	7.80250	2.62810	14.8656
Settling Min	0.60260	0.92860	0.93150	0.73440
Settling Max	1.77110	1.33160	1.37960	1.49400
Overshoot	77.1078	33.1579	37.9555	49.4014
Undershoot	0.00000	0.00000	0.00000	0.00000
Peak	1.77110	1.33160	1.37960	1.49400
Peak time	0.84000	1.28560	0.45400	1.78430

5 Conclusion

Four $PI^{\gamma}D^{\rho}$–controllers of the blood glucose level system have been designed by two different algorithms; Particle Swarm Optimization (PSO) algorithm and Bacteria Foraging Optimization (BFO) algorithm via two different approaches of the fractional-order integro-differential Laplacian operators s^{γ} and s^{ρ}, namely the Continued Fractional Expansion (CFE) approach and the Oustaloup's approach, where $0 < \gamma, \rho < 1$. Based on the numerical results gained from several performed comparisions, we can conclude that, although all proposed controllers are often competing to each other in providing high performance response specifications of all their corresponding closed-loop systems, there are some slight improvements of the step responses achieved using PSO algorithm over than that of using BFO algorithm. In particular, it is clearly shown that the best controller among of all proposed controllers is the $PI^{\gamma}D^{\rho}$–controller which has been established by executing PSO algorithm through Oustaloup's approach. This is because such controller has provided the closed-loop system with the fastest step response and fastest settling time without too much overshoot among of all other controllers.

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