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Impact of Univariate Error Distribution Assumption toward Multivariate GARCH Parameter Estimation Performance

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Abstract

Study on the relationship between volatilities and co-volatilities of several financial data is very important in investigating the volatility impact among the variables. This study aims to examine such pheonomenon using Multivariate Generalized Autoregressive Conditional Heterocedastic (GARCH) model. However, one needs to specify the GARCH model with univariate specification with care before modeling the multivariate GARCH. For numerical ease, the underlying univariate error distribution is always assumed to follow normal distribution. However, the financial time series is always detected to exhibit a fatter tails distribution, therefore this study attempts to examine the effects of using different error assumptions, namely Student-t distribution, Generalized Error Distribution (GED), skewed Normal distribution, skewed Student-t distribution and skewed GED on the performance of parameter estimations of the multivariate GARCH. For empirical application, three selected indices from ASEAN countries include, Indonesia, Singapore and Malaysia were selected. As to ensure that the indices of JCI (Jakarta Composite Indices), STI (Singapore Strait Times Index) and FBMKLCI (FTSE Bursa Malaysia Kuala Lumpur Composite Index) are well defined in modeling the multivariate GARCH, several integration study related to these indices are mentioned. The results show that the skewed error distribution assumptions outperform the non-skewed distribution, suggesting that the skewed error distribution assumption at univariate level may lead to better performance of Multivriate GARCH parameters.

Keywords: *Multivariate GARCH, DCC-GARCH, Parameters Estimation, Error distribution.*

1 Introduction

It is worth to study the relationship of stock markets between countries as it gives wide foresight towards the future markets environment related to the selected regions. One way to look at the integration between markets is to model the variable using multivariate Generalized Autoregressive Conditional Heterocedastic (GARCH). Multivariate (GARCH) models have received numerous interests in the literature review as it gives the possibilities to investigate the relationship among many markets. One of the most popular multivariate GARCH is Dynamic Conditional Correlation (DCC)-GARCH was discussed in Engle [1]. DCC-GARCH models received so much attention as it has several advantages on parsimony basis and the flexibility in univariate GARCH specifications.

Before we proceed with the study of the relationship between countries' stock markets one of the problems that may occur in DCC-GARCH modelling is the error distribution assumption. Error distribution assumption seem to affect the parameters' estimation performance of GARCH model. Orskaug [2] modeled DCC-GARCH with different error distribution such as Gaussian, Student's-t and skewed Student t-distribution towards European, American and Japanese stock markets. The DCC-GARCH with skewed Student's t-distribution was found to give the best result as compared to others distribution according to the performance of goodness of fit tests. However, when asymmetry effects exist in the data series, Orskaug [2] suggested to use other univariate GARCH specification such as Exponential GARCH (EGARCH), Quadratic GARCH (QGARCH) and GJR-GARCH to tackle such problems. Asymmetric effect might come from the news impact on volatility.

Simonata [3] examined time-aggregated stock return using multivariate distribution. The estimation and simulation results were based on Dow Jones Industrial Average index which contain 30 stocks portfolio. For the variance specification, they used and concluded that QGARCH model with leptokurtic innovations and constant correlation provide an adequate fit to the time series return of the stocks in the Dow Jones Industrial average stock index. Fioruci et al. [4] proposed to use Bayesian approach in modeling Multivariate GARCH with DCC-GARCH under various error distributions such as Gaussian, Student t, Generalized Error Distribution (GED), Skew Normal, Skewed Student's-t, and Skewed GED. An empirical application was conducted towards daily indices of stock markets in Frankfurt (DAX), Paris (CAC40) and Tokyo (Nikkei). The distributions were compared using the deviance information criterion (DIC).They concluded that the skew-t multivariate distribution is the best compared to other distribution. Fioruci et al. [4] proposed to investigate the further application of skewed Student's-t multivariate distribution in capturing the fat tails in addition to Student's-t and GED.

Study between these selected indices is not new. Cointegration studies between countries have been done by researchers such as Narayan and Smyth [5] with the case study of New Zealand and US market and also Fernández-Serrano and Sosvilla-Rivero [6] where they found that the economy of Korea and Japan were related from April 1987. The integration of financial markets across national borders is not a new phenomenon but still, there are still lack of studies with respect to market volatility in our local environment. For example, Lau et al. [7] examined the relationship among the ASEAN-5 economies namely, Jakarta Stock Exchange (JSX-Indonesia), Bursa Malaysia (KLCI-Malaysia), The Philippines Stock Exchange (PSE-Philippines), Stock Exchange of Thailand (SET-Thailand), and Singapore Exchange (SGX-Singapore and the study revealed that the markets seemed to relate to each other after the post crisis. The study is found to be in line with the previous findings of Chen et al. [8], and recent study by Karim and Karim [9]. This may due to the formation of Investment Union for ASEAN-5 that resulted in the markets moving in union manners.

Tsukuda et al. [10] examined on the bond markets integration in East Asian. Their research focused on investigating the strength of relationship of Japanese market towards ASEAN+3 (Indonesia, Malaysia, Philippines, Thailand, Singapore, Hong Kong, China and South Korea) with US market as the global benchmark. They used DCC-GARCH and concluded that the level of dependency between the markets remain low except for Hong Kong and Singapore market. Even though there are many efforts have been done on markets integration across the ASEAN and external market, the markets still showed low integration among them [10].

This study focuses on investigating the integration between ASEAN-3 (Malaysia, Singapore and Indonesia) composite indices which are FTSE Bursa Malaysia Kuala Lumpur Composite Index (FBMKLCI), Jakarta Composite Index (JCI) and Singapore Strait Times Index (STI) after the post crisis of 1997-1998 using DCC-GARCH model. These three countries are selected as they give the highest intraday price indices within ASEAN countries. Besides looking at the integration of these three major indices, we also investigate the effect of different univariate GARCH distribution assumptions towards DCC-GARCH parameters estimation performance.

2 Methodology

This section discusses on techniques that have been performed in this study. All the techiques are coded using R-programming of rugarch and rmgarch packages.

2.1 Univariate GARCH

For univariate GARCH specification, each index is setup with standard GARCH (1,1) with different error distribution assumptions; normal, Student's-t distribution, generalized error distribution (GED), skewed normal, skewed Student's-t distribution and skewed GED. For univariate GARCH, the function is defined by;

$$r_t = \mu_t + \varepsilon_t \quad (1)$$
$$\varepsilon_t = h_t^{1/2} z_t \quad (2)$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \ldots + \beta_p h_{t-p} \quad (3)$$

where, r_t is log return of an asset at time t and ε_t is innovation process. For GARCH general equation, h_t is the square of the volatility or can be interpreted as the conditional variance at time t, conditional on the history while z_t is the sequence of independent and identically distributed (iid) standardized, random variables with unit variance. $\alpha_0 > 0, \alpha_q \geq 0$ $(q = 1,\ldots, q)$ and $\beta_p \geq 0$ $(p = 1,\ldots, p)$.

2.2 Multivariate GARCH

Dynamic conditional correlation (DCC) model was proposed by Engle [1] as extension of constant conditional correlation (CCC) that first being introduced by Bollerslev [11] as the constant conditional correlation may not holds in empirical application. The DCC-GARCH took form as;

$$H_t = D_t R_t D_t \quad (4)$$

where $D_t = diag(h_{h1t}^{1/2},\ldots,h_{nt}^{1/2})$ and h_{iit} can be defined as any univariate GARCH model and;

$$R_t = diag(q_{11,t}^{-1/2} \ldots q_{NN,t}^{-1/2}) Q_t diag(q_{11,t}^{-1/2} \ldots q_{NN,t}^{-1/2}) \quad (5)$$

where the $N \times N$ symmetric positive definite matrix $Q_t = (q_{ij,t})$ is given by:

$$Q_t = (1-\alpha-\beta)\overline{Q} + \alpha u_{t-1} u'_{t-1} + \beta Q_{t-1} \quad (6)$$

2.3 Data profile

Three major ASEAN composite indices for the past five year have been selected which are Jakarta Composite Index (JCI), Singapore Strait Times Index (STI) and FTSE Bursa Malaysia Kuala Lumpur Composite Index (FBMKLCI). These indices consist of daily data from 28th September 2009 until 26th September 2014. Logarithmic returns are calculated to ensure the stationarity of each data series.

2.4 Diagnostic Checking

Next, the normality assumption of the multivariate distribution of these three series are checked as our main objective is to explore the effect of different distribution on multivariate GARCH parameter estimation. Histogram and Jarque-Bera tests are used for univariate model while, the Mardia's Multivariate Normality and Henze-Zirkler's Multivariate Normality tests are used for multivariate model. This is done in order to check whether the data series violates the normality assumption. These graphical representation and tests will measure the existence of excessive kurtosis and the skewness level in the data.

In addition, it is important to employ diagnostic checking, includes autocorrelation coefficient function (ACF) and Ljung-Box portmanteau test in GARCH modeling in order to check for the presence of heterocedastic innovation and serial correlation, respectively, in the data series. Ljung-Box test has the null hypothesis of independency and the ARCH data series are expected to have independency in return while serial correlation in its squared returns. For the ARCH effect test, Lagrange Multiplier test is used.

2.5 Model Comparison

The performance of univariate distributions on multivariate GARCH parameter estimation are evaluated using Log likelihood, Akaike's Information Criteria (AIC) and Bayesian Information Criteria. The best model should return the lowest value of AIC and BIC and also have the maximum value of Log likelihood compared to other models specifications.

3 Results and findings

This section discusses the results and findings from modelling the multivariate GARCH using different univariate distribution assumptions.

3.1 Descriptive Statistics

Fig. 1 shows the pattern of the prices for the three indices. It can be seen that the highest price index is JCI while STI comes second. It seems that JCI and STI have the same pattern while FBMKLCI has a slight increase of price index for the past year.

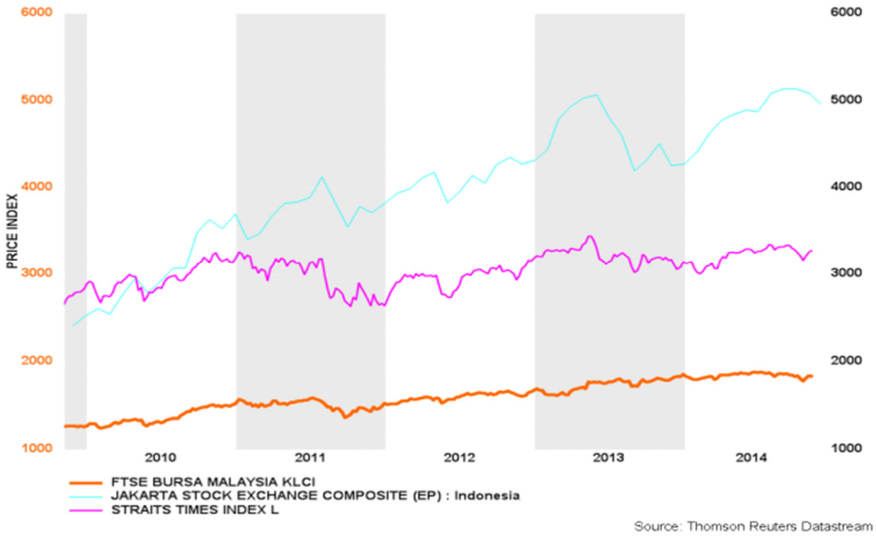

Fig. 1: Price indices of ASEAN-3

For further analysis, investigation of these three indices, each exchange markets log return and squared returns were presented in Fig. 2 and Fig. 3. The characteristics of the volatilities can be determined based on these two figures. STI is the most volatile series as compared to JCI and FBMKLCI. Furthermore, these three series showed volatility clustering at the same period of time frame between observation 400-600 and also 900-1000.

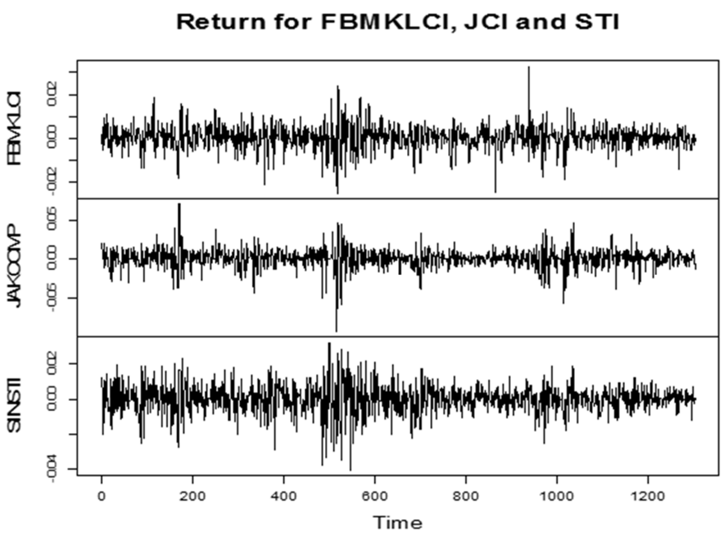

Fig. 2: Return for FBMKLCI, JCI and STI

Summary on returns series are presented in Table 1.

Table 1: Summary of return series

Exchange	Mean	Median	Skewness	Kurtosis
FBMKLCI	0.0003	0.0002	-0.2700	3.4300
JCI	0.0005	0.0005	-0.6600	6.4800
STI	0.0002	0.0001	-0.4300	2.5100

All the indices show negative skewness which suggest the left skewed distribution while JCI series produce the highest kurtosis value. From the first overview, it can be concluded that the distributions are not normally distributed for the return series. Further investigation using normality statistical testing such as histogram and Jarque-Bera test performed for univariate distribution and Mardia's Multivariate Normality Test and Henze-Zirkler's Multivariate Normality Test for multivariate distribution in order to get more accurate results.

3.2 Normality Checking

A table, Table 2 summarizes the Jarque-Bera test and confirms these distributions are not normal because it rejects the null hypothesis of the normally distributed as the p-value for all the series are less than 0.05.

Table 2: Jarque-Bera Test for FBMKLCI,JCI and STI

Series	χ^2	p-value
FBMKLCI	660.8402	0.0000
JCI	2386.121	0.0000
STI	385.3488	0.0000

Next, the normality assumptions of the multivariate distribution of these three series are checked as our main objective is to explore how the distribution assumption effect on multivariate GARCH parameters estimation. Tests for multivariate distribution such as Mardia's Multivariate Normality Test and also Henze-Zirkler's Multivariate Normality Test are used the indicator. Table 3 show the results of both tests and these confirm the multivariate normal assumption is not met as it rejects the null hypothesis of multivariate normally distributed when the p-value is less than 0.05.

Table 3: Multivariate Normality Tests

Multivariate Normality Test	p-value	Decision
Mardia's	0.0000	H_0 is rejected, the data is not normally distributed.
Henze-Zirkler's	0.0000	H_0 is rejected, the data is not normally distributed.

Based on the above statistical testing, it shows the possible state of the financial time series characterized as volatility clustering, leptokurtic and skewed distribution for its univariate and multivariate distribution. This is actually stylized facts of the financial time series and our exploratory here proves the facts. Therefore, it is necessary to assume another choice of distribution assumption when modeling ASEAN-3 data series.

3.3 Diagnostic Checking

Ljung-Box portmanteau test is used to see the autocorrelation in the log returns and squared returns for all the price indices. The test has the null hypothesis of independency and if the p-value is less than 0.05, the null hypothesis is rejected. FBMKLCI and JCI show the presence of autocorrelation for both returns and squared returns since all respected lags give the significant p-value. However, STI shows independence for the returns but not squared returns. To ensure the existence of ARCH effect in the data series, an ARCH test is performed and based on **Table 4**, the null hypothesis of there is no ARCH effect is rejected for all the series as the p-values are less than 0.05.

Table 4: ARCH test before GARCH modeling

Series	χ^2	p-value	Decision
FBMKLCI	145.4728	0.0000	H_0 is rejected, there is ARCH effect in the data series.
JCI	184.3867	0.0000	H_0 is rejected, there is ARCH effect in the data series.
STI	237.9255	0.0000	H_0 is rejected, there is ARCH effect in the data series.

3.4 Results

In Table 5**,** it is seen that the univariate GARCH with skewed GED error distribution assumption produce less insignificant parameters as compared to Normal, Student's-t, GED, skewed Normal and skewed Student's-t distribution as only mean parameter for STI is not significant.

As for the Log-Likelihood values, model with the largest value is more preferable and for AIC and BIC, the model with the lowest value is selected for a better model. The results in Table 5 show that, skewed normal gives the highest Log Likelihood values followed by normal an skewed GED error distribution assumption. However, for the AIC and BIC, skewed Student's-t distribution gives the small values.

Table 5: DCC GARCH model estimation with different univariate error distribution assumption

		Normal	**Student-t**	**GED**	**Skewed Normal**	**Skewed Student-t**	**Skewed GED**
FBMKLCI	Mu	0.0004 (0.0002)	0.0005** (0.0001)	0.0003** (0.0001)	0.0003 (0.0002)	0.0003* (0.0001)	0.0002** (0.0001)
	AR(1)	0.2569 (0.1373)	0.0772 (0.1814)	0.0733* (0.0332)	0.2137 (0.1591)	0.0297 (0.2009)	0.0506 (0.0304)
	MA(1)	-0.1443 (0.1383)	0.0067 (0.1768)	-0.0202** (0.0032)	-0.1140 (0.1626)	0.0480 (0.1956)	0.0116 (0.0070)
	Omega	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000* (0.0000)
	Alpha	0.0912 (0.0556)	0.0783** (0.0274)	0.0756** (0.0281)	0.0934 (0.0558)	0.0741* (0.0234)	0.0749** (0.0215)
	Beta	0.8461** (0.1001)	0.8883** (0.0406)	0.8855** (0.0440)	0.8478** (0.0993)	0.8993** (0.0319)	0.8879** (0.0315)
	Shape		4.5055** (0.6240)	1.1151** (0.0781)		4.6345** (0.6387)	1.1463** (0.0891)
	Skew				0.9037** (0.0480)	0.9195** (0.0312)	0.9607** (0.0325)
JCI	Mu	0.0007** (0.0002)	0.0010** (0.0001)	0.0010** (0.0001)	0.0006** (0.0002)	0.0008** (0.0002)	0.0007** (0.0001)
	AR(1)	-0.2008 (0.2660)	0.7530** (0.0529)	0.7546** (0.0089)	0.7406** (0.0706)	0.7231** (0.0581)	0.7271** (0.0151)
	MA(1)	0.2369 (0.2632)	-0.8217** (0.0443)	-0.8172** (0.0128)	-0.8128** (0.0598)	-0.8037** (0.0480)	-0.8021** (0.0172)
	Omega	0.0000 (0.0000)	0.0000* (0.0000)	0.0000* (0.0000)	0.0000 (0.0000)	0.0000* (0.0000)	0.0000** (0.0000)
	Alpha	0.1089** (0.0324)	0.1168** (0.0314)	0.1090** (0.0296)	0.0973** (0.0244)	0.1085** (0.0273)	0.1012** (0.0224)
	Beta	0.8686** (0.0405)	0.8533** (0.0370)	0.8580** (0.0372)	0.8774** (0.0326)	0.8618** (0.0328)	0.8658** (0.0274)
	Shape		4.2616* (0.5418)	1.1086** (0.0677)		4.5590** (0.6087)	1.1500** (0.0766)
	Skew				0.8332** (0.0404)	0.8694** (0.0339)	0.9281** (0.0224)
STI	Mu	0.0002 (0.0001)	0.0003* (0.0001)	0.0003 (0.0002)	0.0003 (0.0002)	0.0003 (0.0002)	0.0002 (0.0002)
	AR(1)	0.7934* (0.0665)	0.7734** (0.0836)	-0.6342** (0.0110)	-0.5075** (0.1285)	-0.5794** (0.1530)	-0.6229** (0.0215)
	MA(1)	-0.7755** (0.0681)	-0.7599** (0.0802)	0.6192** (0.0110)	0.4936** (0.1286)	0.5635** (0.1551)	0.6040** (0.0231)
	Omega	0.0000* (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000* (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
	Alpha	0.0741** (0.0175)	0.0678** (0.0162)	0.0716** (0.0169)	0.0719** (0.0163)	0.0668** (0.0157)	0.0670** (0.0160)
	Beta	0.9149**	0.9240**	0.9198**	0.9172**	0.9248**	0.9212**

		(0.0190)	(0.0180)	(0.0183)	(0.0180)	(0.0174)	(0.0175)
	Shape		8.5186** (1.9200)	1.4166** (0.0873)		9.4055** (2.3760)	1.4744** (0.0887)
	Skew				0.8794** (0.0306)	0.9082** (0.0310)	0.9205** (0.0338)
DCC	Alpha	0.0186* (0.0093)	0.0180 (0.0094)	0.0188* (0.0091)	0.0194* (0.0088)	0.0178 (0.0093)	0.0183* (0.0093)
	Beta	0.9571** (0.0272)	0.9613** (0.0269)	0.9578** (0.0267)	0.9568** (0.0254)	0.9627** (0.0259)	0.9595** (0.0271)
Log Likelihood		14194.3	14191.49	14193.08	**14197.17**	14191.16	14192.1
Information Criteria	AIC	-21.735	-21.726	-21.729	-21.735	**-21.721**	-21.723
	BIC	-21.644	-21.623	-21.626	-21.632	**-21.606**	-21.608

4 Conclusion

Based on the above result, all the related parameters must to remain significant especially the GARCH parameters to ensure the volatility information are well captured.

In conclusion, even though Log Likelihood and information criterion give inconsistent results, skewed error distribution assumption still outperform the non-normal. Thus, the use of skew error distribution assumption can provide more efficient models and the characteristics of heterocedastic error can be presented in more precise manner. However, this study is limited only on the standard GARCH(1,1) without suggesting a comparison between symmetric and asymmetric GARCH and this can become the next subjects to be further investigate.

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