Evaluation of Multiobjective Particle Swarm Optimization for Optimizing Tank Model's Parameters

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Abstract

Many hydrologists proclaimed Tank model is able to achieve comparable or better forecasting results than more sophisticated models even with its simple concept and computation. With the development of Artificial Intelligence (AI) in recent years, various Global Optimization Methods (GOMs) had been adopted to calibrate Tank model parameters automatically. However, these GOMs are only able to search optimal result for a single objective function. The calibration and validation processes need to be repeated for each objective function in searching the optimal solution and this consumes a lot of time and effort. Hence, multiobjective particle swarm optimization (MOPSO) is adapted in this study to allow PSO be able to deal with a few objective optimization functions simultaneously. The selected study area is Bedup basin, Samarahan, Sarawak, Malaysia. Input data used for model calibration are hourly and daily rainfall and runoff. Two sets of objective functions are investigated. The first set of optimization function consists of ordinary least square (OLS) and root mean square error (RMSE). Where else the second set objective function consists of OLS, RMSE and coefficient and correlation (R). The accuracy of the simulation results are measured using R and Nash-Sutcliffe Coefficient (E^2) . Results revealed that the performance of MOPSO with 3 objective functions is slightly better than MOPSO with 2 objective functions

for both hourly and daily Tank model. Results also proved that MOPSO is able to discover a set of optimal nondominated solution through the true Pareto optimal solutions for the test problems considered.

Keywords: Hydrological Tank model, Particle Swarm Optimization (PSO), Multiobjective Optimization, Pareto Optimal solutions.

1 Introduction

Tank Model was firstly introduced by Sugawara and Funiyuki in 1956. Tank model represents the catchment surface and the underlying system of soil strata using a series of tanks and simple equations. Many hydrologists proclaimed Tank model is simple in concept and computation, while achieving forecasting results comparable or better than more sophisticated models such as Sacramento Model and the Linear Perturbation Model.

The main challenge in the development of Tank model is searching the optimal parameters set. In recent years, hydrologist used automatic calibration technique to calibrate the model parameters automatically (Sorooshian & Dracup, 1980; James & Burges, 1982; Sorooshian & Gupta, 1983; Hendrickson *et al.*, 1988; Franchini, 1996). Among the Global Optimization Methods (GOMs) used, Kuok (2010) found that the most reliable and promising auto-calibration method is Particle Swarm Optimization (PSO).

In the past, PSO has proven its ability to solve single objective optimization problems (Abido, 2002; Wachowiak *et al.*, 2004; Tasgetiren *et al.*, 2004; Mishra, 2005). In order to search the most suitable optimization function, Cooper *et al.* (1997) calibrated Tank model with ordinary least squares (OLS), Nash coefficient (NAS), root mean square error of the peaks (PKS) and root mean square error of the average flows (FLW) separately. Cooper *et al.* (1997) repeated the experiments for twenty five times for each single objective function and this consumed a lot of time and effort. Therefore, multiobjective particle swarm optimization (MOPSO) is adapted in this study to allow PSO to deal with multiobjective optimization functions simultaneously. Till to date, little work was done on multiobjective optimization problems.

PSO is suitable for solving multiobjective optimization problems due to its ability for searching multiple Pareto optimal solutions simultaneously and perform better global search within the search space (Zitzler, 1999). PSO is simple in concept and easy to implement. Besides, the convergence speed is high and it is able to compute efficiently. Concurrently, PSO also found to be flexible and built with well-balanced mechanism for enhancing and adapting global and local exploration abilities (Abido, 2007).

In recent years, only a few researchers applied MOPSO in hydrology. Alexandre and Darrel (2006) applied MOPSO for finding nondominated Pareto solutions

when minimizing deviations from outflow water quality targets. Janga and Nagesh (2007) used MOPSO approach to generate Pareto-optimal solutions for reservoir operation problems.

2 Study Area

The selected study area is Bedup basin, about 47.5km² in area and located approximately 80km from Kuching City, Sarawak, Malaysia. Bedup basin is mainly covered with shrubs, low plant and forest. The elevation are varies from 8m to 686m above mean sea level (JUPEM, 1975). There is no significant land use change in the past 30 years. The length of Bedup river is approximately 10km. Main soil type of Bedup basin is clayey soils and part of it is covered with coarse loamy soil.

Bedup river is located at upper stream of Batang Sadong, where the tide is not reachable. Rating curve equation for Bedup basin is represented by Equation 1 (DID, 2012).

$$Q=9.19(H)^{1.9}$$
 (1)

where Q is the discharge (m³/s) and H is the stage discharge (m). These observed runoff data were used to compare the model runoff.



Fig. 1: Locality map of Bedup basin, Sub-basin of Sadong basin, Sarawak

Fig.1 shows the location of Bedup basin, which is at south west of the land of Borneo. Fig. 1b shows the boundary of Sadong basin, and its rainfall and water level gauging stations that were installed by Department of Irrigation and Drainage (DID) Sarawak. Fig. 1c presents the 5 current rainfall gauging stations available within Bedup basin named as Bukit Matuh (BM), Semuja Nonok (SN), Sungai Busit (SB), Sungai Merang (SM) and Sungai Teb (ST), and one river stage gauging station at Sungai Bedup.

For calibration and validation purposes, input data fed into Tank model are hourly and daily areal rainfall data that obtained through Thiessen Polygon Analysis. Area weighted precipitation for BM, SN, SB, SM, ST are 0.17, 0.16, 0.17, 0.18 and 0.32 respectively. The calibrated Tank model will then carry out computation to simulate the hydrograph at Bedup outlet.

3 Methodology

3.1 Headings and format Multiobjective Particle Swarm Optimization (MOPSO)

The basic concept of MOPSO is solving an optimization problem by optimizing several possibly conflicting objectives simultaneously using an external repository and a mutation operator. The reason for optimizing a set of objective functions simultaneously is that no one can consider better than any other with respect to all objective functions. In MOPSO, the notion of preference must be initially established in order to determine which one dominates another. The preference widely used by multiobjective optimizers is Pareto preference and the optimal solutions discovered are Pareto-optimal solutions. In Pareto preference, the set of nondominated obtained is referred to the Pareto set.

There are two main differences between MOPSO and PSO. The first significant difference is a set of nondominated solutions in MOPSO are replacing the single global best individual in the single objective PSO. The second difference is there might be no single local best individual for each particle of the swarm in MOPSO. Hence, choosing the global best and local best to guide the swarm particles becomes nontrivial task in MOPSO (Abido, 2007). In addition, elitism is also considered by copying any nondominated solution obtained to an external set in order to keep the new nondominated solutions obtained during generations. The external set is updated regularly to hold only the nondominated solutions.

Generally, there are 11 steps of computational procedures for MOPSO as listed below. Details of computational procedures can refer to Abido (2007) and the basic MOPSO procedure is illustrated in Fig.2.

Step 1 - Initialization Step 2 - Time updating Step 3 - Weight updating

- Step 4 Velocity updating
- Step 5 Position updating
- Step 6 Nondominated local set updating
- Step 7 Nondominated global set updating
- Step 8 External set updating
- Step 9 Local best and global best updating
- Step 10 Stopping criteria



Fig. 2: Basic MOPSO procedure

MOPSO consists of three major elements namely:

- a) Nondominated local set, $S_j^*(t)$: $S_j^*(t)$ stores the nondominated solutions obtained by the *j*th particle up to the current time. As the *j*th particle moves through the search space, the new position is added and the set is updated to keep only the nondominated solutions. If nondominated local set size exceeds a certain prespecified value, average linkage based hierarchical clustering algorithm (Morse, 1980) will be employed to reduce the nondominated local set size.
- b) Nondominated global set, $S^{**}(t)$: $S^{**}(t)$ stores the nondominated solutions obtained by the j^{th} particle up to the current time. The union of all $S_j^*(t)$ is formed, and the nondominated solutions out of this union are members in $S^{**}(t)$. An average linkage based hierarchical clustering

algorithm is employed to reduce the nondominated global set to a manageable size.

c) *Local best,* $X_j^{*}(t)$ and *Global best,* $X_j^{**}(t)$: $X_j^{*}(t)$ and $X_j^{**}(t)$ are the individual distances between members in $S_j^{*}(t)$ and $S^{**}(t)$ respectively, that are measured in the objective space. $X_j^{*}(t)$ and $X_j^{**}(t)$ are selected as the local best and the global best of the *j*th particle respectively when they give the minimum distance in $S_j^{*}(t)$ and $S^{**}(t)$ respectively.

3.2 Tank Model Parameters

Four vertically connected storage vessels (4-Tank) was selected in this study. The 4 tanks are named as TS1, TS2, TS3 and TS4 (refer to Fig. 3). Every tank has one or more side and bottom outlets. Side outlet flow will happen when the water level in each tank is higher than the height of side outlets. The output from the bottom outlet of the TS1 (located the top) could be regard as infiltration. Meanwhile, the outputs from the bottom outlets for the rest of the tanks (TS2, TS3 and TS4) could be regarded as percolation.



Fig. 3: Schematic of Tank model used in this study

The side outlet coefficients of 4-Tank model are C1, C2, C4, C6 and C8. The bottom outlet coefficients consist of C3, C5 and C7, and X1, X2, X3, X4 and X5

are height of side outlets. Prior to calibration, parameters X3, X4 and X5 were found to have little impact to model output and the values obtained are always near to 0. Hence, X3, X4 and X5 values are set to 0 in this study. Thereafter, the remaining 10 parameters calibrated automatically by MOPSO are C1, C2, C3, C4, C5, C6, C7, C8, X1 and X2. The descriptions of 10 parameters are tabulated in Table 1.

No	Coefficient	Identification
1	C1	Side outlet coefficients No.1 for TS1
2	C2	Side outlet coefficients No.2 for TS1
3	C3	Bottom outlet coefficient from TS1 to TS2
4	C4	Side outlet coefficients for TS2
5	C5	Bottom outlet coefficient from TS2 to TS3
6	C6	Side outlet coefficients for TS3
7	C7	Bottom outlet coefficient from TS3 to TS4
8	C8	Side outlet coefficients for TS4
9	X1	Height of side outlets No.2 for TS1
10	X2	Height of side outlets No.1 for TS1

Table 1: The description of the 10 parameters for Tank model

The total discharge, Q was calculated using Equation 2.

$$Q = C1Q1 + C2Q2 + C4Q3 + C6Q4 + C8Q5$$
(2)

3.3 Objective Functions

This study evaluates two sets of multiobjective functions. The first set consists of ordinary least squares (OLS) and root mean squared error (RMSE). Meanwhile, the second set composed of OLS, RMSE and coefficient of correlation (R). The aim is to investigate the effect of number of objective functions to the accuracy of optimization results.

OLS is a method for estimating the unknown parameters in a linear regression model. OLS minimizes the sum of squared vertical distances between the observed responses in the dataset, and the responses predicted by the linear approximation. Meanwhile RMSE is used to quantify the difference between an estimator and the true value of the quantity being estimated. RMSE also measures the average of the square of the "error." Besides, R measures the strength of the linear relationship between two variables that is defined in terms of the dataset covariance of the variables divided by dataset standard deviations.

3.4 Model Calibration and Validation

The input data to hourly Tank model calibration are hourly average areal rainfall and discharge data from 8 to 12 Aug 1998. The calibrated model will be validated

with 11 sets of hourly data tabulated in Table 2. Meanwhile, daily average areal rainfall and discharge data from 1 Aug 1998 to 31 Dec 1998 are used for daily Tank model calibration. Thereafter, the optimal daily Tank model will be further validated with 11 sets of daily data as presented in Table 3.

In order to find the optimal parameters set, hourly and daily Tank models will be investigated with:

- 1. Different probability of mutation (pMut) including 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8.
- 2. Different number of nondominated solutions in archive (nondomCtr) 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8.

Item	Period
1	1-7 Jan 1999
2	5-8 Apr 1999
3	5-8 Feb 1999
4	8-12 Aug 1998
5	9-12 Sep 1998
6	15-18 Mac 1999
7	20-24 Jan 1999
8	26-31 Jan 1999
9	5-8 Apr 2000
10	18-21 Jan 2000
11	9-12 Oct 2003

Table 2: Validation data sets for hourly model

Table 3. Validation	data sets	for	vlieb	model
	uala sels	101	uany	model

Item	Period			
1	1 Aug 1992 to 31 Dec 1992			
2	1 Aug 1993 to 31 Dec 1993			
3	1 to 31 Jan 1992 , 1 Apr 1992 to 31 Jul 1992			
4	1 Jan 2000 to 30 Jun 2000			
5	1 Jan 2002 to 30 Jun 2002			
6	1 Jan 2003 to 30 Jun 2003			
7	1 Jan 1990 to 31 May 1990			
8	1 Jan 1993 to 31 May 1993			
9	1 Jul 2000 to 31 Dec 2000			
10	1 Jul 2002 to 31 Dec 2002			
11	1 Jul 2003 to 31 Dec 2003			

4 **Results and Discussions**

For hourly Tank, the optimal configuration and optimal parameters for 2 and 3 objective functions are tabulated in Table 4.

Table 4: Optimal configuration and parameters of hourly Tank model for 2 and 3 objective functions

	2 objective functions	3 objective functions				
Optimal Configurations						
Numbers of particles in the	100	100				
population						
Maximum number of generations	1000	1000				
pMut	0.3	0.2				
nondomCtr	0.2	0.2				
Optimal Parameters						
C1	0.04493	0.041006				
C2	0.001212	0.000876				
X1	0.04858	0.038918				
X2	0.049373	0.005549				
С3	0.006541	0.002268				
C4	0.001747	0.01457				
C5	0.035347	0.043932				
C6	0.039015	0.024109				
C7	0.039142	0.043001				
C8	0.037816	0.042907				

For hourly calibration set, the best R, E^2 and peak error obtained are found to be 0.7954, 0.7735 and 9.24% respectively for 2 objective functions. As the optimal parameters validated with 11 sets of hourly data, the average R, E^2 and peak error are yielding to 0.8780, 0.7530 and 15.85% respectively. Meanwhile for 3 objective functions, the optimal R, E^2 and peak error for hourly model calibration are found to be 0.8535, 0.8704 and 0.43% respectively. The average R, E^2 and peak error obtained for 11 sets of validation data are improved to 0.8522, 0.8082 and 10.55% respectively using the optimal parameter set. Validation results for 2 and 3 objective functions for hourly Tank model are tabulated in Table 5.

Table 5: Validation results for hourly tank model optimized with 2 and 3 objective functions.

	2 objective functions			3 objective functions		
Storm Event	R	E ²	peak	R	E ²	peak
			error%			error%
1-7 Jan 1999	0.8310	0.8356	21.34	0.8882	0.8090	13.23
5-8 Apr 1999	0.8349	0.5796	24.26	0.7942	0.5651	21.97

5-8 Feb 1999	0.9075	0.6364	19.07	0.8357	0.8307	6.03
8-12 Aug 1998	0.8450	0.7903	9.24	0.7590	0.7752	0.43
9-12 Sep 1998	0.9631	0.9033	18.24	0.9453	0.9571	11.83
15-18 Mac 1999	0.9173	0.6589	18.71	0.8803	0.8621	13.26
20-24 Jan 1999	0.9138	0.9025	3.88	0.8846	0.9126	8.10
26-31 Jan 1999	0.8444	0.5723	14.49	0.8507	0.7783	4.58
5-8 Apr 2000	0.8358	0.9465	4.59	0.8332	0.8679	5.79
18-21 Jan 2000	0.9042	0.6350	18.62	0.8924	0.7279	19.52
9-12 Oct 2003	0.8608	0.8221	21.90	0.8105	0.8041	11.31
Average	0.8780	0.7530	15,85	0.8522	0.8082	10.55

Table 6 presents the optimal configuration and optimal parameters for 2 and 3 objective functions for daily Tank Model.

Table 6: Optimal configuration and parameters of daily Tank model for 2 and 3 objective functions.

	2 objective functions	3 objective functions				
Optimal Configurations						
Numbers of particles in the	100	100				
population						
Maximum number of generations	1000	1000				
pMut	0.4	0.3				
nondomCtr	0.3	0.3				
Optimal Parameters						
C1	0.751597	0.212304				
C2	0.617975	0.016427				
X1	0.151455	0.140361				
X2	0.564359	0.812545				
C3	0.617435	0.618418				
C4	0.292717	0.191117				
С5	0.71284	0.313945				
C6	0.992902	0.007968				
C7	0.447575	0.95198				
C8	0.761768	0.080796				

The best R and E^2 obtained for daily model calibration using 2 objective functions are found to be 0.7407 and 0.6550 respectively. As the optimal parameters validated with 11 sets of data, the average R and E^2 are yielding to 0.6543 and 0.6872 respectively. For 3 objective functions, the optimal R and E^2 for daily model calibration are found to be 0.8161 and 0.7478 respectively. Meanwhile, the average R and E^2 obtained are improved to 0.7415 and 0.6944 respectively using the optimal parameter set when validated with 11 sets of daily data. Validation

results for 2 and 3 objective functions for daily Tank model are tabulated in Table 7.

Storm Event	2 objective f	unctions	3 objective fu	nctions
	R	E ²	R	E ²
1 Aug 1992 to 31 Dec 1992	0.6510	0.6637	0.7372	0.5904
1 Aug 1993 to 31 Dec 1993	0.6267	0.6515	0.7149	0.6623
1 to 31 Jan 1992 , 1 Apr 1992 to	0.7176	0.8221	0.7987	0.7424
31 Jul 1992				
1 Jan 2000 to 30 Jun 2000	0.6472	0.7993	0.7378	0.5445
1 Jan 2002 to 30 Jun 2002	0.8334	0.8131	0.8920	0.8229
1 Jan 2003 to 30 Jun 2003	0.5118	0.5047	0.6688	0.7132
1 Jan 1990 to 31 May 1990	0.5846	0.5990	0.6577	0.5251
1 Jan 1993 to 31 May 1993	0.6760	0.8503	0.7378	0.8475
1 Jul 2000 to 31 Dec 2000	0.6711	0.5869	0.7688	0.5952
1 Jul 2002 to 31 Dec 2002	0.5982	0.5568	0.6849	0.7681
1 Jul 2003 to 31 Dec 2003	0.6793	0.7116	0.7576	0.8273
Average	0.6543	0.6872	0.7415	0.6944

Table 7: Validation results for daily tank model optimized with 2 and 3 objective functions.

As hourly Tank Model optimized with 2 objective functions, the average validation results discovered are R=0.8780 and E^2 =0.7530. Concurrently, average R= 0.8522 and E^2 =0.8082 are obtained when optimized with 3 objective functions as illustrated in Fig. 4. Results also indicate that the average peak error for 2 objective functions is 15.85% and 10.55% for 3 objective functions. This revealed that the performance of MOPSO optimized with 3 objective functions is able to produce more accurate parameters compared to MOPSO optimized with 2 objective functions for hourly Tank model.

Meanwhile for daily Tank model, the average validation results discovered are R= 0.6543 and E²=0.6872 when optimized with 2 objective functions. Meanwhile, average R= 0.7415 and E²=0.6944 are obtained when optimized with 3 objective functions as illustrated in Fig. 4. This revealed that the performance of MOPSO optimized with 3 objective functions is able to produce more accurate parameters compared to MOPSO optimized with 2 objective functions.

Besides, when daily Tank model optimized with 2 objective functions, part of the simulated hydrograph are overestimated, and part of it are underestimated than the observed data. In contrast, most of simulated hydrograph are slightly underestimated than the observed runoff when daily Tank model optimized with 3 objective functions.



Fig. 4: Comparison of Hourly and Daily Tank Model optimized with 2 and 3 objective functions

5 Conclusion

This study has successfully applied MOPSO to calibrate and optimize hourly and daily Tank model's parameters with 2 and 3 objective functions. Optimal R=0.7954, E^2 =0.7735 and peak error=9.24% are discovered for model calibration when hourly Tank model calibrated simultaneously with 2 objective functions. Meanwhile, when validated with 11 sets of data, an average R=0.8780, E^2 =0.7530 and peak error of 15.85% are obtained. Concurrently, when hourly Tank model optimized simultaneously with 3 objective functions, the optimal R, E^2 and peak error are found to be 0.8535, 0.8704 and 0.43% for model calibration, and average R=0.8522 E^2 =0.8082 and peak error of 10.55% for hourly model validation.

Besides, adaptation of 2 objective functions has yielded R and E² to 0.7407 and 0.6550 respectively for daily model calibration. An average R=0.6543 and E²=0.6872 are discovered after validating 11 sets of daily data. Meanwhile, the optimal R and E² are found to be 0.8161 and 0.7478 respectively when adapting 3 objective functions for daily model calibration, and average R=0.7415 and E²=0.6944 are obtained for daily model validation. These results revealed that performance of MOPSO is slightly more accurate and robust when adapting 3 objective functions than 2 objective functions.

The results also proved that MOPSO has the ability to solve an optimization problem by optimizing several possibly conflicting objectives simultaneously using an external repository and a mutation operator. Besides, the newly developed MOPSO has proven its capability to discover a set of optimal

nondominated solution through the true Pareto optimal solutions for the test problems considered.

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