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Ranking of Generalized Exponential Fuzzy Numbers using Integral Value Approach

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Abstract

Ranking of fuzzy numbers play an important role in decision making, optimization, forecasting. In fuzzy decision making problems fuzzy numbers must be ranked before an action is taken by a decision maker. Chen and Li have proposed on "Representation, ranking, and distance of fuzzy number with exponential membership function using grade mean integration method" by ranking index for ranking exponential fuzzy numbers which does not depend on the height of fuzzy number. But in the related literature, it is shown that ranking index depends upon the height of fuzzy number. In this paper, using integral value approach of T.S. Liou and M.J. Wang, a ranking formula is introduced for comparing the exponential fuzzy numbers which depends on height of fuzzy number. Also it is proved that the ranking function for exponential fuzzy numbers is not linear.

Keywords: Ranking function, Integral value index, Generalized exponential fuzzy numbers

1 Introduction

Fuzzy set theory [19] is a powerful tool to deal with real life situations. Real numbers can be linearly ordered by ≥ or ≥, however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient approach for ordering the fuzzy numbers is by the use of a ranking function

$\Re : F(R) \rightarrow R,$ where $F(R)$ is a set of fuzzy numbers defined on real line, which maps each fuzzy number into the real line, where a natural order exists. Thus, specific ranking of fuzzy numbers is an important procedure for decision-making in a fuzzy environment and generally has become one of the main problems in fuzzy set theory.

The method for ranking was first proposed by Jain [10]. Yager [18] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0,1]. In Kaufmann and Gupta [11], an approach is presented for the ranking of fuzzy numbers. Campos and Gonzalez [3] proposed a subjective approach for ranking fuzzy numbers. Liou and Wang [14] developed a ranking method based on integral value index. Cheng [7] presented a method for ranking fuzzy numbers by using the distance method. Kwang and Lee [13] considered the overall possibility distributions of fuzzy numbers in their evaluations and proposed a ranking method. Chen and Li [6] showed how to treat the defuzzification, ranking and distance of fuzzy numbers with exponential membership function by modified concept of Graded Mean Integration representation method and derived a ranking formula which does not depend upon the height of the fuzzy number. Modarres and Nezhad [15] proposed a ranking method based on preference function which measures the fuzzy numbers point by point and at each point the most preferred number is identified. Chu and Tsao [8] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point.

Deng and Liu [9] presented a centroid-index method for ranking fuzzy numbers. Chen and Chen [4] presented a method for ranking generalized trapezoidal fuzzy numbers. Wang and Lee [17] also used the centroid concept in developing their ranking index. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some $\alpha-$levels of trapezoidal fuzzy numbers. Chen and Chen [5] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Alias et al. [2] presented fuzzy analytic hierarchy process technique which will be used to rank alternatives to find the most reasonable and efficient use of river system. Ramli and Mohamad [16] presented a comprehensive survey of different ranking methods of fuzzy numbers. Kumar et al. [12] presented RM approach for ranking of generalized trapezoidal fuzzy numbers.

In this paper, using integral value approach [14], a ranking formula is introduced for comparing the exponential fuzzy numbers which depends on height of fuzzy number. Also it is proved that the ranking function for exponential fuzzy numbers is not linear.

This paper is organized as follow: In section 2 some basic definitions, arithmetic operations between two generalized exponential fuzzy numbers and definition of ranking function for comparing fuzzy numbers are reviewed. In section 3 the ranking formula for generalized exponential fuzzy numbers is derived and also it is proved that ranking function is not linear for generalized exponential fuzzy numbers. In the section 4 ranking formula is illustrated with examples. In the last section conclusions are discussed

2 Preliminaries

In this section some basic definitions and arithmetic operations are reviewed [6, 11].

2.1 Basic definitions

In this subsection some basic definition are reviewed.

Definition 2.1 [11] The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set A.
The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 [11] A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}} : R \to [0,1]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a,b]$ and strictly decraesing on $[c,d]$.

(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b,c]$, where $a \le b \le c \le d$.

Definition 2.3 [6] A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a generalized exponential fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} w\exp\left\{-\frac{(b-x)}{(b-a)}\right\} & ,a\le x\le b \\ w & ,b\le x\le c \\ w\exp\left\{-\frac{(x-c)}{(d-c)}\right\} & ,c\le x\le d \end{cases}$$

where $0<w\le 1$. This type of generalized exponential fuzzy number is denoted as $\tilde{A}=(a,b,c,d;w)$.

2.2 Arithmetic operations

In this subsection, arithmetic operations between two generalized exponential fuzzy numbers, defined on universal set of real numbers R, are discussed [6].

Let $\tilde{A}_1=(a_1,b_1,c_1,d_1;w_1)$ and $\tilde{A}_2=(a_2,b_2,c_2,d_2;w_2)$ be two generalized exponential fuzzy numbers then

(i) $\tilde{A}_1\oplus\tilde{A}_2=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2;\text{minimum}(w_1,w_2))$

(ii) $\tilde{A}_1\Theta\tilde{A}_2=(a_1+d_2,b_1+c_2,c_1+b_2,d_1+a_2;\text{minimum}(w_1,w_2))$

(iii) $\lambda\tilde{A}_1=\begin{cases}(\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1;w_1),\lambda>0\\(\lambda d_1,\lambda c_1,\lambda b_1,\lambda a_1;w_1),\lambda<0\end{cases}$

2.3 Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function [14], $\Re:F(R)\to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

(i) $\tilde{A}\succ\tilde{B}$ iff $\Re(\tilde{A})>\Re(\tilde{B})$
(ii) $\tilde{A}\prec\tilde{B}$ iff $\Re(\tilde{A})<\Re(\tilde{B})$
(iii) $\tilde{A}\sim\tilde{B}$ iff $\Re(\tilde{A})=\Re(\tilde{B})$

3 Ranking Formula for Generalized Exponential Fuzzy Numbers

Chen and Li [6] proposed a ranking index for ranking exponential fuzzy numbers which does not depend on the height of fuzzy number. But in the literature [11], it is shown that ranking index depends upon the height of fuzzy number. In this paper, using integral value approach [14], a ranking formula is introduced for comparing the exponential fuzzy numbers which depends on height of fuzzy number. Also it is proved that the ranking function for exponential fuzzy numbers is not linear.

Let $\tilde{A} = (a,b,c,d;w)$ be a generalized exponential fuzzy number then $L(x) = w\exp\left\{-\frac{(b-x)}{(b-a)}\right\}$ and $R(x) = w\exp\left\{-\frac{(x-c)}{(d-c)}\right\}$, where $L(x)$ and $R(x)$ are left and right reference functions [6] of generalized exponential fuzzy number $\tilde{A}$ [6] $\Rightarrow x = L^{-1}\left(w\exp\left\{-\frac{(b-x)}{(b-a)}\right\}\right) \Rightarrow L^{-1}(\alpha) = b-(b-a)\log\frac{\alpha}{w}$, where $\alpha = w\exp\left\{-\frac{(b-x)}{(b-a)}\right\}$.

Therefore left inverse function of $L(x)$ is given by $L^{-1}(\alpha) = b-(b-a)\log\frac{\alpha}{w}$,

Similarly the right inverse function of $R(x)$ is $R^{-1}(\alpha) = c+(d-c)\log\frac{\alpha}{w}$.

Now putting the values of left inverse and right inverse functions in

$$\Re(\tilde{A}) = \frac{1}{2}\int_0^w \left\{L^{-1}(\alpha)+R^{-1}(\alpha)\right\} d(\alpha)$$

$$\Rightarrow \Re(\tilde{A}) = \frac{1}{2}\int_0^w \left\{b-(b-a\log\frac{\alpha}{w}+c+(d-c)\log\frac{\alpha}{w}\right\} d(\alpha) \Rightarrow \Re(\tilde{A}) = \frac{w}{2}(a+d)$$

3.1 Proposed method to compare two generalized exponential fuzzy numbers

In this subsection, a method is proposed for comparing two generalized exponential fuzzy numbers.

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized exponential fuzzy numbers then $\tilde{A}$ and $\tilde{B}$ can be compared by using the following steps:

Step 1. Find $w = \text{minimum}(w_1 + w_2)$

Step 2. Find $\Re(\tilde{A}) = \frac{w}{2}(a_1 + d_1)$ and $\Re(\tilde{B}) = \frac{w}{2}(a_2 + d_2)$

Now

(i) $\tilde{A} \succ \tilde{B}$ iff $\Re(\tilde{A}) > \Re(\tilde{B})$

(ii) $\tilde{A} \prec \tilde{B}$ iff $\Re(\tilde{A}) < \Re(\tilde{B})$

(iii) $\tilde{A} \sim \tilde{B}$ iff $\Re(\tilde{A}) = \Re(\tilde{B})$

Proposition 3.1 Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers and k_1, k_2 be two real numbers then the ranking function < is not a linear function for generalized fuzzy numbers i.e.

$$\Re(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2) \neq k_1\Re(\tilde{A}_1) + k_2\Re(\tilde{A}_2).$$

Proof:- Let k_1 and k_2 be two positive real numbers then

$$\Re(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2) = \Re((k_1a_1, k_1b_1, k_1c_1, k_1d_1) \oplus (k_2a_2, k_2b_2, k_2c_2, k_2d_2))$$

$$= \Re(k_1a_1 + k_2a_2, k_1b_1 + k_2b_2, k_1c_1 + k_2c_2, k_1d_1 + k_2d_2)$$

$$= \frac{\text{minimum}(w_1, w_2)}{2}(k_1a_1 + k_2a_2 + k_1d_1 + k_2d_2)$$

$$= \frac{\text{minimum}(w_1, w_2)}{2}\left(\frac{w_1}{w_1}(k_1a_1 + k_1d_1) + \frac{w_2}{w_2}(k_2a_2 + k_2d_2)\right)$$

$$= \frac{\text{minimum}(w_1, w_2)}{2}\left(\frac{k_1\Re(\tilde{A}_1)}{w_1} + \frac{k_2\Re(\tilde{A}_2)}{w_2}\right)$$

$$\neq k_1\Re(\tilde{A}_1) + k_2\Re(\tilde{A}_2).$$

In this proposition the result is proved for positive real numbers. Similarly it can be proved that above result is true for all real numbers.

$$\Re(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2 \oplus ... \oplus k_n\tilde{A}_n) = \frac{\text{minimum}(w_1, w_2, ..., w_n)}{2}\left(\frac{k_1\Re(\tilde{A}_1)}{w_1} + \frac{k_2\Re(\tilde{A}_2)}{w_2}, ..., \frac{k_n\Re(\tilde{A}_n)}{w_n}\right)$$

where $k_1, k_2, ..., k_n \in R$ and $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n$ are n generalized fuzzy numbers.

Hence $\Re$ is not a linear function for generalized fuzzy number.

Remark 3.1 If $w_1 = w_2 = ... = w_n = 1$ then above result reduces to

$$\Re(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2 \oplus ... \oplus k_n\tilde{A}_n) \neq k_1\Re(\tilde{A}_1) + k_2\Re(\tilde{A}_2) + ... + k_n\Re(\tilde{A}_n).$$

4 Illustrated Examples

In this section the proposed method is illustrated by some numerical examples.

Example 4.1 Let $\tilde{A}_1 = (1, 2, 5, 7; 0.3)$ and $\tilde{A}_2 = (-1, 3, 8, 9; 0.1)$ be two generalized exponential fuzzy numbers then $\tilde{A}_1$ and $\tilde{A}_2$ may be compared as follow:

Step 1 minimum $(0.3, 0.1) = 0.1$

Step 2 $\Re(\tilde{A}_1) = 0.4$ and $\Re(\tilde{A}_2) = 0.4$

Since $\Re(\tilde{A}_1) = \Re(\tilde{A}_2) \Rightarrow \tilde{A}_1 \sim \tilde{A}_2$

Example 4.2 Let $\tilde{A}_1 = (1, 2, 3, 6; 0.1)$ and $\tilde{A}_2 = (3, 5, 6, 7; 0.2)$ be two generalized exponential fuzzy numbers then $\tilde{A}_1$ and $\tilde{A}_2$ may be compared as follow:

Step 1 minimum $(0.1, 0.2) = 0.1$

Step 2 $\Re(\tilde{A}_1) = 0.35$ and $\Re(\tilde{A}_2) = 0.5$

Since $\Re(\tilde{A}_1) < \Re(\tilde{A}_2) \Rightarrow \tilde{A}_1 \prec \tilde{A}_2$

Example 4.3 Let $\tilde{A}_1 = (1, 3, 5, 9; 0.3)$ and $\tilde{A}_2 = (-1, 4, 5, 7; 0.5)$ be two generalized exponential fuzzy numbers then $\tilde{A}_1$ and $\tilde{A}_2$ may be compared as follow:

Step 1 minimum $(0.3, 0.5) = 0.3$

Step 2 $\Re(\tilde{A}_1) = 1.5$ and $\Re(\tilde{A}_2) = 0.9$

Since $\Re(\tilde{A}_1) > \Re(\tilde{A}_2) \Rightarrow \tilde{A}_1 \succ \tilde{A}_2$

5 Conclusion

A new method for comparing generalized exponential fuzzy numbers is introduced. Also it is proved that ranking function is not linear for generalized exponential fuzzy numbers. The proposed ranking method can be used tosolve real life applications such as decision making, forecasting.

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