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A Generalised Fuzzy Soft Set Based Student Ranking System

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Abstract

In this paper a new technique based on generalised fuzzy soft set has been introduced for the determination of class ranking of students. Some problems in student ranking has been solved which could not be solved using earlier available standard techniques.

Keywords: *Fuzzy set, generalised fuzzy soft set, membership function, soft set, student grade.*

1 Introduction

The practice of assigning grades or marks to measure the amount a student has learned is very common. Students are given different letter grades in different subjects and the overall performance or class rank is calculated by taking grade point averages of these subject grades. Alternatively, class rank is done on the basis of total marks scored in different subjects. But both the methods are not totally accurate. For many students may subject wise have a wide range of grades, which show that average grades may not show what a student really knows. This is true also in the case of marks. Again a proper careful assignment of grades to individual students is very important simply because, hindering a student's performance with a bad grade in the middle of the year can make them give up for the rest of the year. Once a student has received a bad grade they might lose faith in their academic ability. By giving a student poor grade does not always reflect their academic ability and their bad grades are not always based on what they have learned. Therefore, the standard grading system should be scrutinized.

A soft set is a parameterized family of subsets of universal set. The theory of soft sets was introduced by Molodtsov [3] in 1999. He has shown several potential applications of soft set in different fields including game theory, operations research, integration theory etc. Again in recent years a lot of work has been done in the field of soft set theory and it's generalizations. Maji & Roy [1,2] have introduced the idea of fuzzy soft sets and applied this set in solving decision making problems. The notion of similarity measurement between two fuzzy soft sets was introduced by Majumdar & Samanta [5]. In 2010, Majumdar & Samanta [4] has introduced the notion of generalised fuzzy soft sets and have applied this set in decision making.

In recent years several authors [6-10] have studied problems regarding educational measurement, particularly student assessments and grading. But most of the new methods are based on statistical techniques.
In this paper a generalized soft set based technique for determination of student grades has been developed. This technique has been tested on real data set and a comparison has been made with the conventional grading system.
The organization of the rest of the paper is as follows: In section 2, generalised fuzzy soft sets are discussed. A generalised fuzzy soft set based technique for determination of student ranking has been discussed in section 3. Section 4 concludes the paper.

2 Generalised Fuzzy Soft Sets

In this section we give recollect the definition of generalised fuzzy soft sets from [4] and study their properties.

Definition 2.1 Let $U = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. Let $F : E \to I^U$ and μ be a fuzzy subset of E, i.e. $\mu : E \to I = [0,1]$, where I^U be the collection of all fuzzy subset of U. Let F_μ be the mapping $F_\mu : E \to I^U \times I$ be a function defined as follows: $F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^U$. Then F_μ is called a generalised fuzzy soft set (GFSS in short) over (U, E).

Here for each parameter e_i , $F_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness.
The following is an example of a generalised fuzzy soft set.

Example 2.2 Let $U = \{x_1, x_2, x_3\}$ be a set of three shirts under consideration. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where $e_1 = bright$, $e_2 = cheap$, $e_3 = colorful$. Let $\mu : E \to I = [0,1]$ be defined as follows:- $\mu(e_1) = 0.1$, $\mu(e_2) = 0.4$, $\mu(e_3) = 0.6$.
We define a function $F_\mu : E \to I^U \times I$ be defined as follows:

$$F_\mu(e_1) = \left(\left\{ \frac{x_1}{0.7}, \frac{x_2}{0.4}, \frac{x_3}{0.3} \right\}, 0.1 \right), F_\mu(e_2) = \left(\left\{ \frac{x_1}{0.1}, \frac{x_2}{0.2}, \frac{x_3}{0.9} \right\}, 0.4 \right),$$

$$F_\mu(e_3) = \left(\left\{ \frac{x_1}{0.8}, \frac{x_2}{0.5}, \frac{x_3}{0.2} \right\}, 0.6 \right).$$

Then F_μ is a GFSS over (U, E).

In matrix form this can be expressed as $F_\mu = \left(\begin{array}{c|cccc} & x_1 & x_2 & x_3 & \mu \\ \hline e_1 & 0.7 & 0.4 & 0.3 & 0.1 \\ e_2 & 0.1 & 0.2 & 0.9 & 0.4 \\ e_3 & 0.1 & 1.0 & 0.7 & 0.6 \end{array} \right)$ and which will be called membership matrix of F_μ.

Definition 2.3 Let F_μ and G_δ be two GFSS over (U, E). Now F_μ is said to be a generalised fuzzy soft subset of G_δ if
(i) μ is a fuzzy subset of δ (ii) $F(e)$ is also a fuzzy subset of $G(e)$, $\forall e \in E$.
In this case we write $F_\mu \subseteq G_\delta$.

Example 2.4 Consider the GFSS F_μ over (U, E) given in example 2.2. Let G_δ be another GFSS over (U, E) defined as follows:

$$G_\delta(e_1) = \left(\left\{ \frac{x_1}{0.2}, \frac{x_2}{0.3}, \frac{x_3}{0.1} \right\}, 0.1 \right), G_\mu(e_2) = \left(\left\{ \frac{x_1}{0.0}, \frac{x_2}{0.1}, \frac{x_3}{0.7} \right\}, 0.3 \right)$$

$G_\delta(e_3) = \left(\left\{ \frac{x_1}{0.7}, \frac{x_2}{0.3}, \frac{x_3}{0.1} \right\}, 0.5 \right)$, where $\delta \in I^E$ be defined as above.

Then G_δ is a generalised fuzzy soft subset of F_μ.

Note 2.5 Let c be an involutive fuzzy complement and g be an increasing generator of c.
Let $*$ and $\circ$ be two binary operations on $[0,1]$ defined as follows:
$a * b = g^{-1}(g(a) + g(b) - g(1))$ and $a \circ b = g^{-1}(g(a) + g(b))$.

Then $*$ is a $t-norm$ and $\circ$ is a $t-conorm$. Moreover $(*,\circ,c)$ becomes a dual triple.
Henceforth in the rest of the paper we will take such an involutive dual triple to consider the general case.

Definition 2.6 Let F_μ be a GFSS over (U,E). Then the complement of F_μ, denoted by F_μ^c and is defined by $F_\mu^c = G_\delta$, where $\delta(a) = c(\mu(a)),\ \forall a \in [0,1]$ and $G(e) = c(F(e)),\ \forall e \in E.$

Note 2.7 Obviously $(F_\mu^c)^c = F_\mu$ as the fuzzy complement c is involutive in nature.

Definition 2.8 Union of two GFSS F_μ and G_δ, denoted by $F_\mu \tilde{\cup} G_\delta$, is a GFSS H_ν, defined as $H_\nu : E \to I^U \times I$ such that $H_\nu(e) = (H(e), \nu(e)),$ where $H(e) = F(e) \circ G(e)$ and $\nu(e) = \mu(e) \circ \delta(e).$

Definition 2.9 Intersection of two GFSS F_μ and G_δ, denoted by $F_\mu \tilde{\cap} G_\delta$, is a GFSS H_ν, defined as $H_\nu : E \to I^U \times I$ such that $H_\nu(e) = (H(e), \nu(e)),$ where $H(e) = F(e) * G(e)$ and $\nu(e) = \mu(e) * \delta(e).$

Definition 2.10 A GFSS is said to be a generalised null fuzzy soft set, denoted by Φ_θ, if $\Phi_\theta : E \to I^U \times I$ such that $\Phi_\theta(e) = (F(e), \theta(e)),$ where $F(e) = \bar{0}\ \forall e \in E$ and $\theta(e) = 0\ \forall e \in E.$

Definition 2.11 A GFSS is said to be a generalised absolute fuzzy soft set, denoted by $\tilde{A}_\alpha$ if $\tilde{A}_\alpha : E \to I^U \times I,$ where $\tilde{A}_\alpha(e) = (A(e), \alpha(e))$ is defined by $A(e) = \bar{1}\ \forall e \in E,$ and $\alpha(e) = 1\ \forall e \in E.$

Definition 2.12 Let $F = \{F_{\mu_i}^i, i \in \Delta\}$ be any collection of GFSS over (U,E) and $C \subseteq E^n$. Then an n-ary generalised fuzzy soft relation R on F is the mapping $R : C \to I^U \times I,$ defined by:

$$R(e_{i_1}, e_{i_2}, ...e_{i_n}) = \bigcap_{j=1}^{n} F_{\mu_j}^{i_j}(e_{i_j}), \text{ where } (e_{i_1}, e_{i_2}, ...e_{i_n}) \in C.$$

3 A GFSS based student ranking system

Consider the following problem that a set of ten students $x_1, x_2, ..., x_9, x_{10}$ from a class (Table 1). They are tested to detect their performance in three subjects, namely Mathematics, English & Biology. Full marks in each subject are 100. Their scores of the test and class ranking based on total marks are as follows:

Table 1: Marks table

Sub./students	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Mathematics(m)	80	65	79	45	87	98	87	36	54	32
English(e)	60	65	71	55	63	**42**	83	54	62	72
Biology(b)	65	70	80	60	70	65	80	45	54	66
Total Marks Obtained:	**205**	200	230	160	220	**205**	250	135	**170**	**170**
Class Rank:	4	5	2	7	3	4	1	8	6	6

Here the ranking is based on the total marks. But this is not always justified. For example x_1 & x_6 has same rank although x_6 has scored very poor marks in English. Again x_9 & x_{10} has been given equal ranks although x_{10} has got better marks than x_9 in two subjects, viz. English and Biology.

Here a new technique, based on generalised fuzzy soft set, has been proposed to rank the students more rationally than the one based on total marks. For this let the set of students $U = \{x_1, x_2, ..., x_{10}\}$ be our universal set and the subjects $E = \{m, e, b\}$ are our parameter set. Let $\mu : E \to I = [0,1]$ be a fuzzy subset of E and defined as follows:

$$\begin{array}{c|ccc} \mu & m & e & b \\ \hline & 0.9 & 0.7 & 0.8 \end{array}.$$

This μ determines the grade of difficulty associated to each subject. The choice of μ is important in ranking a student and may vary due to various factors such as difficulty of the subjects, geographic regions, socio-economic factors, gender etc. This will enable us to model different situations using the same equation. For this we may have to consult domain experts in this regard.

Now we will convert marks in three different subjects into three grades. For example x_1 got 80 in mathematics which is good marks in Mathematics with some grade $l \in [0,1]$, say. For this we define three fuzzy sets $M, E, B:[0,100] \to [0,1]$ as follows:

$$M(x) = \frac{x}{100}, 0 \le x \le 100 \quad \text{...(1)}$$

$$E(x) = (\frac{x}{100})^3, 0 \le x \le 100 \quad \text{...(2)}$$

$$B(x) = (\frac{x}{100})^2, 0 \le x \le 100 \quad \text{...(3)}$$

Using these fuzzy sets we convert marks in three subjects into corresponding grades. The choice of these fuzzy sets is taken arbitrarily for demonstration purpose only.

Figures of the above functions are shown in Figure 1 through Figure 3 as follows:

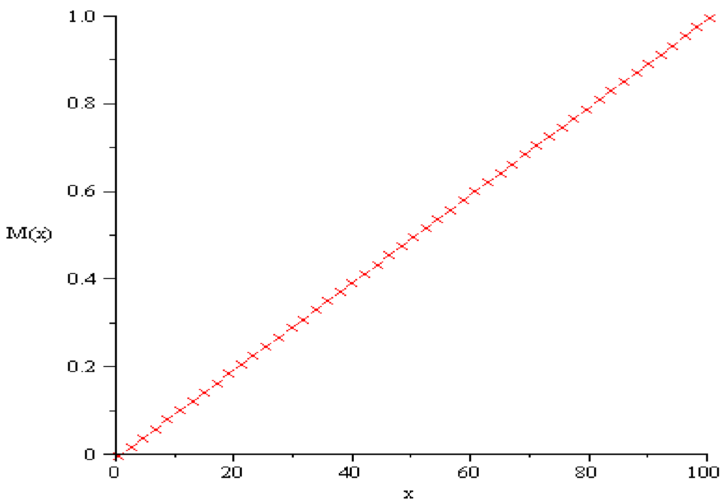

Figure 1: Membership function M of Mathematics

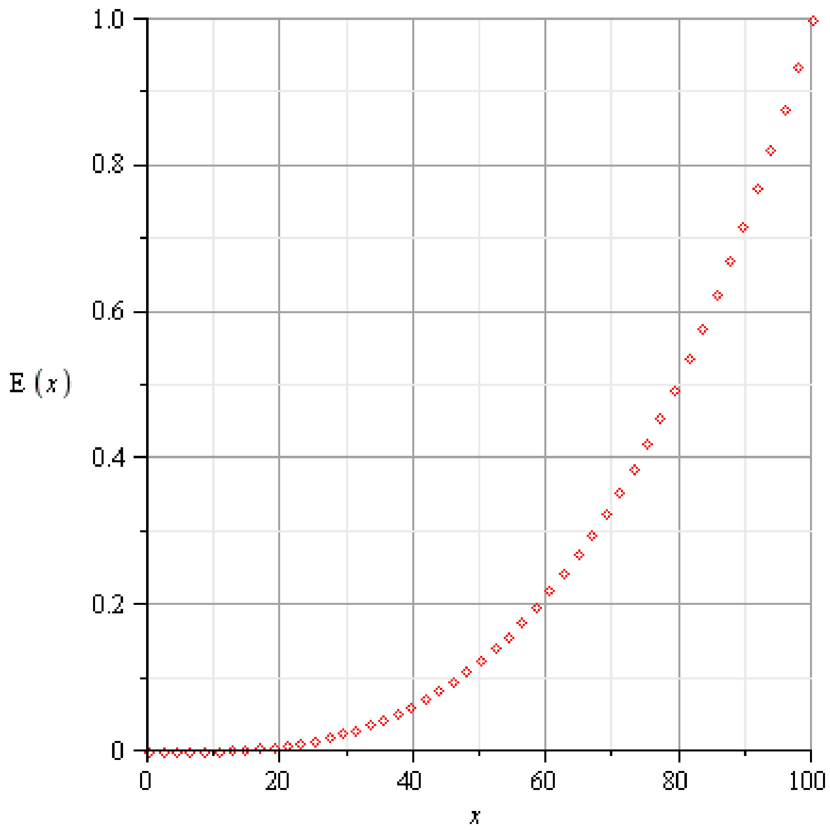

Figure 2: Membership function E of English

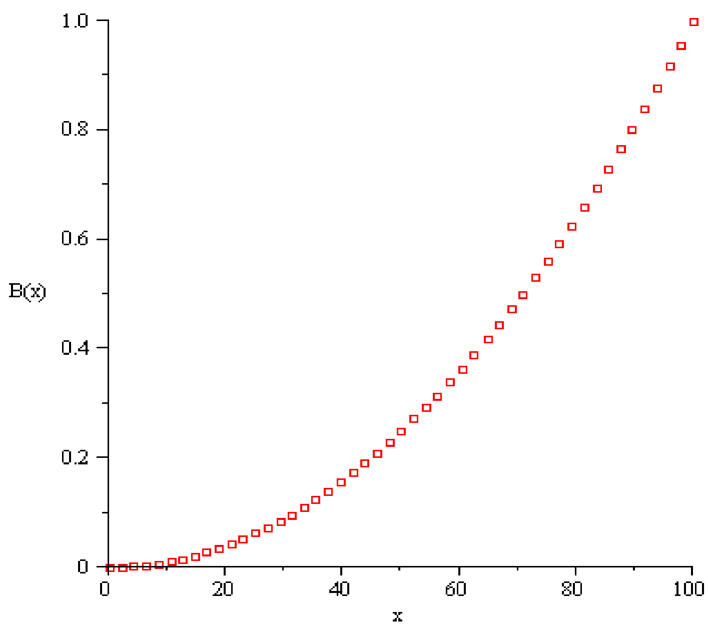

Figure 3: Membership function B of Biology

Now using equations (1)-(3) we convert the numbers of the students given in Table 1 into grades which are given in Table 2.

Table 2: Conversion table

Sub./students	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	μ
Mathematics(m)	0.80	0.65	0.79	0.45	0.87	0.98	0.87	0.36	0.54	0.32	0.9
English(e)	0.15	0.19	0.25	0.12	0.18	0.05	0.40	0.11	0.17	0.26	0.7
Biology(b)	0.42	0.49	0.64	0.36	0.49	0.42	0.64	0.2	0.29	0.44	0.8

Thus we have the GFSS $F_\mu : E \to I^U \times I$, describing the situation, as follows:

$F_\mu(m) = \{\{\frac{x_1}{0.80}, \frac{x_2}{0.65}, ..., \frac{x_{10}}{0.32}\}, 0.9\}$, $F_\mu(e) = \{\{\frac{x_1}{0.22}, \frac{x_2}{0.27}, ..., \frac{x_{10}}{0.37}\}, 0.7\}$

and $F_\mu(b) = \{\{\frac{x_1}{0.42}, \frac{x_2}{0.49}, ..., \frac{x_{10}}{0.44}\}, 0.8\}$.

Now we calculate resultant grades of each student in each subject by taking multiplication of grade of a subject with the corresponding value of μ. Next we find the Choice Values of each student by adding the three resultant grades in three subjects, which has been shown in the following Table 3. Finally we rank the students depending on their choice values.

Table 3: Resultant table

Sub./students	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Mathematics (m)	0.72	0.585	0.711	0.405	0.783	0.882	0.783	0.324	0.486	0.28
English(e)	0.154	0.189	0.252	0.119	0.175	0.049	0.399	0.112	0.168	0.259
Biology(b)	0.336	0.392	0.512	0.288	0.392	0.336	0.512	0.16	0.232	0.352
Choice Value (c_i)	1.21	1.166	1.587	0.868	1.441	1.241	1.694	0.645	0.977	1.003

Analysis: This new method of ranking is not based on actual marks obtained by a student. Rather it converts the number obtained by a student in each subject into grades. Therefore the credit can be assigned according to the difficulty of the subject and not uniformly. Also social aspects can also be incorporated (by proper choice of μ) while ranking a student. In this new method of ranking the problems described earlier has been solved. For example the ranking of x_{10} has improved compared to x_9. Also the rankings of x_1 & x_6 have been changed. And in this method there is a very little chance of having equal choice values and hence equal ranks.

The algorithm of the method is described as follows:

Algorithm [1]

1. Input the marks of the students.
2. Input the fuzzy sets M, E, B & μ.
3. Convert the marks into grades.
4. Calculate resultant grades.
5. Compute Choice value c_i of each x_i.
6. Arrange the students according to the descending values of c_i's.
7. Stop

4 Conclusion

In this paper we have proposed a new student ranking system based on generalized fuzzy soft set theory. We have studied some actual cases and have shown how class ranks can be calculated using this method. The author is hopeful that this new system will be helpful in assigning student grades more efficiently and realistically. One can further study the effect of this new gradation system on students of different age groups and from different socio-economic backgrounds.

5 Future Work

Process of grading students involves many factors where fuzzy set theory and soft set theory may be useful. One can study student grading systems based on the techniques of similarity measurement of two fuzzy soft sets or two generalised fuzzy soft sets as discussed in [4, 5].

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