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Some Rational Contractions for Coupled Coincidence and Common Coupled fixed point theorems in Complex-valued metric spaces

A. Gupta[1], N. Kaur[2], B. Sood[3], S. Manro[4] and R. Rani[5]
[1]H.No. 93/654, Ward No. 2, Gandhi Chowk Pachmarhi - 461881
, Dist. Hoshangabad (M.P.), India
[2]Department of Mathematics, Desh Bhagat University,
Mandi Gobindgarh, Punjab, India
[3]Department of Mathematics, Desh Bhagat University,
Mandi Gobindgarh, Punjab, India
[4,*] Department of Mathematics,
Thapar University, Patiala, Punjab, India
[5] Department of Mathematics,
A.S. College for Women, Khanna, Punjab, India
email:saurabh.manro@thapar.edu
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Abstract

The aim of this paper is to obtain a coupled coincidence point theorem and a common coupled fixed point theorem of contractive type mappings involving rational expressions in the framework of a complex-valued metric spaces. We also improve the result obtain by Jhade and Khan "Some Coupled Coincidence and Common Fixed Point Theorems in Complex-valued Metric spaces, Ser. Math. Inform. 29, (4) (2014), 385-395". The results of this paper generalize and extend the results of Kang etal. "Coupled Fixed Point Theorems in Complex Valued Metric Spaces, Int. J. of Math. Analysis, 7(46) (2013) , 2269 - 2277", in complex-valued metric spaces..

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1 Introduction and Preliminaries

In 2011, Azam et al. [2] introduced the notion of complex valued metric space which is a generalization of the classical metric space. They established some fixed point results for mappings satisfying a rational inequality. The idea of complex valued metric spaces can be exploited to define complex valued normed spaces and complex valued Hilbert spaces; additionally, it offers numerous research activities in mathematical analysis.

Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$, we define a partial order $\preceq$ on $\mathbb{C}$ as follows:

$$z_1 \preceq z_2 \quad \text{if and only if} \quad Re(z_1) \le Re(z_2) \quad \text{and} \quad Im(z_1) \le Im(z_2).$$

It follows that $z_1 \preceq z_2$ if one of the following conditions is satisfied:

(C1) $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$;

(C2) $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$;

(C3) $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$;

(C4) $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.

In particular, we will write $z_1 \prec z_2$ if $z_1 \neq z_2$ and one of (C2), (C3) and (C4) is satisfied and we will write $z_1 \prec z_2$ if only (C4) is satisfied.

Remark 1.1 *We obtained that the following statements hold:*

1. *If $a, b \in \mathbb{R}$ with $a \le b$, then $az \prec bz$ for all $z \in \mathbb{C}$.*
2. *If $0 \preceq z_1 \prec z_2$, then $|z_1| < |z_2|$.*
3. *If $z_1 \preceq z_2$ and $z_2 \prec z_3$, then $z_1 \prec z_3$.*

Consistent with Azam et al. [2], we state some definitions and results about the complex-valued metric space to prove our main results.

Definition 1.2 *Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathbb{C}$ satisfies the following conditions:*

(d1) $0 \preceq d(x, y)$ *for all* $x, y \in X$*;*

(d2) $d(x, y) = 0$ *if and only if* $x = y$ *for all* $x, y \in X$*;*

(d3) $d(x, y) = d(y, x)$ *for all* $x, y \in X$*;*

(d4) $d(x, y) \preceq d(x, z) + d(z, y)$ *for all* $x, y, z \in X$*.*

Then d is called a complex-valued metric on X and (X, d) is called a complex-valued metric space.

Example 1.3 *Example 1.3. Let $X = \mathbb{C}$. Define the mapping $d : X \times X \to \mathbb{C}$ by $d(z_1, z_2) = 2i|z_1 - z_2|$ for all $z_1, z_2 \in X$. Then (X, d) is a complex valued metric space.*

Definition 1.4 *Let (X, d) be a complex-valued metric space.*

I. *Apoint $x \in X$ is called interior point of a set $B \subseteq X$ whenever there exists $0 < r \in \mathbb{C}$ such that*

$$N(x, r) := \{y \in X : d(x, y) < r\} \subseteq B.$$

II. *A point $x \in X$ is called limit point of a set $B \subseteq X$ whenever there exists $0 < r \in \mathbb{C}$ such that$N(x, r) \cap (B\{x\}) \neq \phi$.*

III. *A subset $B \subseteq X$ is called open whenever each element of B is an interior point of B.*

IV. *A subset $B \subseteq X$ is called closed whenever each limit point of B belongs to B.*

V. *The family $F = \{N(x, r) : x \in X, 0 \prec r\}$ is a sub-basis for a topology on X. We denote this complex topology by τ_c. Indeed, the topology τ_c is Hausdorff.*

Definition 1.5 *Let (X, d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X and $x \in X$.*

I. *If for every $c \in \mathbb{C}$ with $0 < c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x) < c$ then $\{x_n\}$ is said to be convergent, if $\{x_n\}$ converges to x and x is the limit point of $\{x_n\}$. We denote this by $x_n \to x$ as $n \to \infty$ or $\lim_{n\to\infty} x_n = x$.*

II. *If for every $c \in \mathbb{C}$ with $0 < c$ there is $N \in \mathbb{N}$ such that for all $n, m > N$, $d(x_n, x_m) < c$ then $\{x_n\}$ is said is said to be Cauchy sequence.*

III. *If every Cauchy sequence in X is convergent, then (X, d) is said to be a complete complex-valued metric space.*

Lemma 1.6 *Let (X, d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.*

Lemma 1.7 *Let (X,d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$.*

In 2006, Bhaskar et al. [1] introduced the notion of coupled fixed point and proved some fixed point results in this context. Similarly, Kang etal. [3] introduce the notion of coupled fixed point for a mapping in complex valued metric spaces as follows.

Definition 1.8 *Let (X,d) be a complex-valued metric space, an element $(x,y) \in X \times X$ is said to be a coupled fixed point of the mapping $F : X \times X \to X$ if $F(x,y) = x$ and $F(y,x) = y$.*

Definition 1.9 *Let (X,d) be a complex valued metric space. An element $(x,y) \in X \times X$ is said to be*

I. *A coupled coincidence point of mappings $F : X \times X \to X$ and $g : X \to X$ if $g(x) = F(x,y)$ and $g(y) = F(y,x)$, and (gx, gy) is called a coupled point of coincidence if there exists $(u,v) \in X \times X$ such that $x = gu = F(u,v)$ and $y = gv = F(v,u)$.*

II. *A common coupled fixed point of mappings $F : X \times X \to X$ and $g : X \to X$ if $x = gx = F(x,y)$ and $y = gy = F(y,x)$.*

Definition 1.10 *Let (X,d) be a complex-valued metric space. The mappings $F : X \times X \to X$ and $g : X \to X$ are called w-compatible if $g(F(x,y)) = F(gx, gy)$, whenever $gx = F(x,y)$ and $gy = F(y,x)$.*

Kang et al [3] prove following result,

Theorem 1.11 *([3], Theorem-2.1) Let (X,d) be a complex valued metric space. Suppose that the mapping $F : X \times X \to X$ satisfies*

$$d(F(x,y), F(u,v)) \precsim hd(x,u) + kd(y,v) \tag{1}$$

for all $x, y, u, v \in X$, where h and k are non-negative constants with $h+k < 1$. Then F has a unique coupled fixed point.

In [4], Jhade and Khan prove following result,

Theorem 1.12 *([4], Theorem 3.1) Let (X,d) be a complex-valued metric space. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that* *there exist nonnegative constants* $a_i \in [0,1), i = 1,2,\ldots,6$ *such that* $\Sigma_{i=1}^{6} a_i < 1$ *and for all* $x, y, u, v \in X$

$$\begin{aligned} d(F(x,y),F(u,v)) \preceq\; & a_1 d(gx,gu) + a_2(gy,gv) \\ & + a_3 \frac{d(gx,F(x,y))d(gu,F(u,v))}{d(gx,gu)} \\ & + a_4 \frac{d(gx,F(u,v))d(gu,F(x,y))}{d(gx,gu)} \\ & + a_5 \frac{d(gy,F(y,x))d(gv,F(v,u))}{d(gy,gv)} \\ & + a_6 \frac{d(gy,F(v,u))d(gv,F(y,x))}{d(gy,gv)}. \end{aligned} \tag{2}$$

Suppose $F(X \times X) \subseteq g(X)$ *and* $g(X)$ *is a complete subspace of* X*. Then* F *and* g *have a coupled coincidence point* $(x^*, y^*) \in X \times X$*.*

Remark 1.13 *It should be noted that Theorem 1.12 is not true for* $x = u$ *and* $y = v$*, i.e., 2 is not valid for* $x = u$ *and* $y = v$ *and we can not obtain coupled fixed point.*

2 Main Results

First we improve the Theorem 1.12 and prove a coupled coincidence point theorem which state is as follows,

Theorem 2.1 *Let* (X, d) *be a complex-valued metric space. Let* $F : X \times X \to X$ *and* $g : X \to X$ *be two mappings. Suppose that there exist nonnegative constants* $a_i \in [0,1), i = 1,2,\ldots,6$ *such that* $\Sigma_{i=1}^{6} a_i < 1$ *and for all* $x, y, u, v \in X$

$$\begin{aligned} d(F(x,y),F(u,v)) \preceq\; & a_1 d(gx,gu) + a_2 d(gy,gv) \\ & + a_3 \frac{[1 + d(gx,F(x,y))]d(gu,F(u,v))}{d(gx,gu)+1} \\ & + a_4 \frac{[1 + d(gx,F(u,v))]d(gu,F(x,y))}{d(gx,gu)+1} \\ & + a_5 \frac{[1 + d(gy,F(y,x))]d(gv,F(v,u))}{d(gy,gv)+1} \\ & + a_6 \frac{[1 + d(gy,F(v,u))]d(gv,F(y,x))}{d(gy,gv)+1}. \end{aligned} \tag{3}$$

Suppose $F(X \times X) \subseteq g(X)$ *and* $g(X)$ *is a complete subspace of* X*. Then* F *and* g *have a coupled coincidence point* $(x^*, y^*) \in X \times X$*.*

Proof 2.2 *Let $x_0, y_0 \in X$ are arbitrary. Set $gx_1 = F(x_0, y_0)$ and $gy_1 = F(y_0, x_0)$, this can be done because $F(X \times X) \subseteq g(X)$. Continuing the process, we obtain two sequences $\{x_n\}$ and $\{y_n\}$ such that $gx_{n+1} = F(x_n, y_n)$ and $gy_{n+1} = F(y_n, x_n)$ for all $n \geq 0$. Then we have*

$$\begin{aligned} d(gx_n, gx_{n+1}) &= d(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\ &\preceq a_1 d(gx_{n-1}, gx_n) + a_2 d(gy_{n-1}, gy_n) \\ &\quad + a_3 \frac{[1 + d(gx_{n-1}, F(x_{n-1}, y_{n-1}))] d(gx_n, F(x_n, y_n))}{d(gx_{n-1}, gx_n) + 1} \\ &\quad + a_4 \frac{[1 + d(gx_{n-1}, F(x_n, y_n))] d(gx_n, F(x_{n-1}, y_{n-1}))}{d(gx_{n-1}, gx_n) + 1} \\ &\quad + a_5 \frac{[1 + d(gy_{n-1}, F(y_{n-1}, x_{n-1}))] d(gy_n, F(y_n, x_n))}{d(gy_{n-1}, gy_n) + 1} \\ &\quad + a_6 \frac{[1 + d(gy_{n-1}, F(y_n, x_n))] d(gy_n, F(y_{n-1}, x_{n-1}))}{d(gy_{n-1}, gy_n) + 1} \end{aligned}$$

$$\begin{aligned} d(gx_n, gx_{n+1}) &\preceq a_1 d(gx_{n-1}, gx_n) + a_2 d(gy_{n-1}, gy_n) \\ &\quad + a_3 \frac{[1 + d(gx_{n-1}, gx_n)] d(gx_n, gx_{n+1})}{d(gx_{n-1}, gx_n) + 1} \\ &\quad + a_4 \frac{[1 + d(gx_{n-1}, gx_{n+1})] d(gx_n, gx_n)}{d(gx_{n-1}, gx_n) + 1} \\ &\quad + a_5 \frac{[1 + d(gy_{n-1}, gy_n)] d(gy_n, gy_{n+1})}{d(gy_{n-1}, gy_n) + 1} \\ &\quad + a_6 \frac{[1 + d(gy_{n-1}, d(gy_{n-1}, gy_n))] d(gy_n, gy_n)}{d(gy_{n-1}, gy_n) + 1} \end{aligned}$$

which implies

$$\begin{aligned} |d(gx_n, gx_{n+1})| &\preceq a_1 |d(gx_{n-1}, gx_n)| + a_2 |d(gy_{n-1}, gy_n)| \\ &\quad + a_3 |d(gx_n, gx_{n+1})| + a_5 |d(gy_n, gy_{n+1})| \end{aligned} \tag{4}$$

Similarly we have

$$\begin{aligned} |d(gy_n, gy_{n+1})| &\preceq a_1 |d(gy_{n-1}, gy_n)| + a_2 |d(gx_{n-1}, gx_n)| \\ &\quad + a_3 |d(gy_n, gy_{n+1})| + a_5 |d(gx_n, gx_{n+1})|. \end{aligned} \tag{5}$$

Suppose that

$$d_n = \|d(gx_n, gx_{n+1}\| + \|d(gy_n, gy_{n+1}\|.$$

Adding inequalities 4 and 5, we obtain

$$d_n \leq (a_1 + a_2)d_{n-1} + (a_3 + a_5)d_n \tag{6}$$

that is

$$d_n \leq hd_{n-1}$$

where

$$h = \frac{a_1 + a_2}{1 - (a_3 + a_5)} < 1.$$

Thus, we have

$$d_n \leq hd_{n-1} \leq h^2 d_{n-2} \leq h^3 d_{n-3} \leq h^4 d_{n-4} \leq \cdots \leq h^n d_0. \tag{7}$$

We shall show that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. If $m > n$, then we have

$$\begin{aligned}
|d(gx_n, gx_m| + |d(gy_n, gy_m| &\leq |d(gx_n, gx_{n+1}| + |d(gy_n, gy_{n+1}| \\
&\quad +|d(gx_{n+1}, gx_{n+2}| + |d(gy_{n+1}, gy_{n+2}| \\
&\quad +|d(gx_{n+2}, gx_{n+3}| + |d(gy_{n+2}, gy_{n+3}| \\
&\quad + \cdots + |d(gx_{m-1}, gx_m| + |d(gy_{m-1}, gy_m| \\
&\leq h^n d_0 + h^{n+1} d_0 + h^{n+2} d_0 + h^{n+3} d_0 + \cdots + h^{m-1} d_0 \\
&\leq \frac{h^n}{1-h} d_0 \to 0 \quad as \quad n \to \infty.
\end{aligned}$$

Hence $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in $g(X)$. Since $g(X)$ is complete, there exists x^* and y^* such that $gx_n \to x^*$ and $gy_n \to y^*$ as $n \to \infty$.

On the other hand, we have from 3,

$$\begin{aligned}
d(F(x^*, y^*), gx^*) &\preceq d(F(x^*, y^*), gx_{n+1}) + d(gx_{n+1}, gx^*) \\
&= d(F(x^*, y^*), F(x_n, y_n)) + d(gx_{n+1}, gx^*)
\end{aligned}$$

$$\begin{aligned} d(F(x^*, y^*), gx^*) \preceq\ & a_1 d(gx^*, gx_n) + a_2 d(gy^*, gy_n) \\ & + a_3 \frac{[1 + d(gx^*, F(x^*, y^*))] d(gx_n, F(x_n, y_n))}{d(gx^*, gx_n) + 1} \\ & + a_4 \frac{[1 + d(gx^*, F(x_n, y_n))] d(gx_n, F(x^*, y^*))}{d(gx^*, gx_n) + 1} \\ & + a_5 \frac{[1 + d(gy^*, F(y^*, x^*))] d(gy_n, F(y_n, x_n))}{d(gy^*, gy_n) + 1} \\ & + a_6 \frac{[1 + d(gy^*, F(y_n, x_n))] d(gy_n, F(y^*, x^*))}{d(gy^*, gy_n) + 1} \\ & + |d(gx_{n+1}, gx^*)| \end{aligned}$$

$$\begin{aligned} d(F(x^*, y^*), gx^*) \preceq\ & a_1 d(gx^*, gx_n) + a_2 d(gy^*, gy_n) \\ & + a_3 \frac{[1 + d(gx^*, F(x^*, y^*))] d(gx_n, gx_{n+1})}{d(gx^*, gx_n) + 1} \\ & + a_4 \frac{[1 + d(gx^*, gx_{n+1})] d(gx_n, F(x^*, y^*))}{d(gx^*, gx_n) + 1} \\ & + a_5 \frac{[1 + d(gy^*, F(y^*, x^*))] d(gy_n, gy_{n+1})}{d(gy^*, gy_n) + 1} \\ & + a_6 \frac{[1 + d(gy^*, gy_{n+1})] d(gy_n, F(y^*, x^*))}{d(gy^*, gy_n) + 1} \\ & + |d(gx_{n+1}, gx^*)| \end{aligned}$$

$$\begin{aligned} |d(F(x^*, y^*), gx^*)| \preceq\ & a_1 |d(gx^*, gx_n)| + a_2 |d(gy^*, gy_n)| \\ & + a_3 \frac{[1 + |d(gx^*, F(x^*, y^*))|](|d(gx_n, gx^*)| + |d(gx^*, gx_{n+1})|)}{|d(gx^*, gx_n)| + 1} \\ & + a_4 \frac{[1 + |d(gx^*, gx_{n+1})|] |d(gx_n, F(x^*, y^*))|}{|d(gx^*, gx_n)| + 1} \\ & + a_5 \frac{[1 + |d(gy^*, F(y^*, x^*))|](|d(gy_n, gy^*)| + |d(gy^*, gy_{n+1})|)}{|d(gy^*, gy_n)| + 1} \\ & + a_6 \frac{[1 + |d(gy^*, gy_{n+1})|] |d(gy_n, F(y^*, x^*))|}{|d(gy^*, gy_n)| + 1} \\ & + |d(gx_{n+1}, gx^*)|. \end{aligned}$$

Since $gx_n \to gx^*$ and $gy_n \to gy^*$ as $n \to \infty$, we have $|d(F(x^*, y^*), gx^*)| \leq 0$. That is, $F(x^*, y^*) = gx^*$.

Similarly one can show that $F(y^*, x^*) = gy^*$.

Hence (x^*, y^*) is a coupled coincidence point of F and g.

For common coupled fixed point for the mappings F and g, the condition of Theorem 2.1 are not enough. So by applying the condition of w-compatibility on F and g, we obtain the following common coupled fixed point theorem.

Theorem 2.3 *In addition to the hypotheses of Theorem 2.1 are not enough to prove the existence of a common coupled fixed point for the mappings F and g. By applying the condition of w-compatibility on F and g, we obtain the following common coupled fixed point theorem, if F and g are w-compatible, then F and g have a unique common coupled fixed point. Moreover, a common coupled fixed point of F and g is of the form (u, v) for some $u, v \in X$.*

Proof 2.4 *The existence of coupled coincidence point (x^*, y^*) of F and g follows from Theorem 2.1. Then (gx^*, gy^*) is a coupled point of coincidence of F, g and so $gx^* = F(x^*, y^*)$ and $gy^* = F(y^*, x^*)$.*

First we will show that this coupled point of coincidence is unique.

For this, suppose that F and g have another coupled point of coincidence (gu, gv), that is, $gu = F(u, v)$ and $gv = F(v, u)$ where $(u, v) \in X \times X$. Then we have

$$\begin{aligned} d(F(x^*, y^*), F(u, v)) \preceq\ & a_1 d(gx^*, gu) + a_2 d(gy^*, gv) \\ & + a_3 \frac{[1 + d(gx^*, F(x^*, y^*))] d(gu, F(u, v))}{d(gx^*, gu) + 1} \\ & + a_4 \frac{[1 + d(gx^*, F(u, v))] d(gu, F(x^*, y^*))}{d(gx^*, gu) + 1} \\ & + a_5 \frac{[1 + d(gy^*, F(y^*, x^*))] d(gv, F(v, u))}{d(gy^*, gv) + 1} \\ & + a_6 \frac{[1 + d(gy^*, F(v, u))] d(gv, F(y^*, x^*))}{d(gy^*, gv) + 1}. \end{aligned}$$

Hence

$$\begin{aligned} |d(gx^*, gu)| \preceq\ & a_1 |d(gx^*, gu)| + a_2 |d(gy^*, gv)| \\ & + a_4 |d(gx^*, gu)| + a_6 |d(gy^*, gv)|. \end{aligned} \tag{8}$$

Similarly we obtain

$$\begin{aligned} |d(gy^*, gv)| \preceq\ & a_1 |d(gy^*, gv)| + a_2 |d(gx^*, gu)| \\ & + a_4 |d(gy^*, gv)| + a_6 |d(gx^*, gu)|. \end{aligned} \tag{9}$$

Adding 8 and 9 we obtain

$$|d(gx^*, gu)| + |d(gy^*, gv)| \leq (a_1 + a_2 + a_4 + a_6)[|d(gx^*, gu)| + |d(gy^*, gv)|].$$

Since $(a_1 + a_2 + a_4 + a_6) < 1$. Therefore,

$$|d(gx^*, gu)| + |d(gy^*, gv)| \leq 0$$

which contradiction. Hence $d(gx^*, gu) = 0$ *and* $d(gy^*, gv) = 0$*, i.e.,* $gx^* = gu$ *and* $gy^* = gv$*.*

Thus $(gx^*, gy^*) = (u, v)$ *is the unique coupled point of coincidence of* F *and* g*. Now if* F *and* g *are w-compatible, then* $gu = g(F(x^*, y^*)) = F(gx^*, gy^*) = F(u, v) = w$*(say). Similarly, we obtain* $gv = g(F(y^*, x^*)) = F(gy^*, gx^*) = F(v, u) = z$*(say). So,* (w, z) *is another coupled point of coincidence of* F *and* g*. By uniqueness, we have* $(u, v) = (w, z)$*, that is,* $gu = F(u, v) = u$ *and* $gv = F(v, u) = v$*. Thus* (u, v) *is the unique common coupled fixed point of* F *and* g*.*

Example 2.5 *Let* $X = \{ix : x \in [0, 1]\}$ *and consider a complex valued metric* $d : X \times X \to X$ *defined by*

$$d(x, y) = i|x - y|$$

for all $x, y \in X$*. Then* (X, d) *is a complex valued metric space.*

Define the mappings $F : X \times X \to X$ *and* $g : X \to X$ *by* $F(x, y) = i\left(\frac{x}{10} + \frac{y}{15}\right)$ *and* $g(x) = \frac{x}{5}i$ *for all* $x, y \in [0, 1]$*. Then we have*

$$\begin{aligned} d(F(x,y), F(u,v)) &= i|i\left(\frac{x}{10} + \frac{y}{15}\right) - i\left(\frac{u}{10} + \frac{v}{15}\right)| \\ &= i|i\left(\frac{x}{10} - \frac{u}{10}\right) - i\left(\frac{y}{15} - \frac{v}{15}\right)| \\ &\le \frac{5}{10} i|i\left(\frac{x}{5} - \frac{u}{5}\right)| + \frac{5}{15} i|i\left(\frac{y}{5} - \frac{v}{5}\right)| \\ &\le \frac{1}{2} d(gx, gu) + \frac{1}{3} d(gy, gv) \end{aligned}$$

where $a_1 = \frac{1}{2}$*,* $a_2 = \frac{1}{3}$*,* $a_i = 0, i = 3, 4, 5, 6$*. Note that* $a_1 + a_2 = \frac{5}{6} + \frac{5}{6} < 1$*,* $F(X \times X) \subseteq g(X)$ *and* $g(X)$ *is a complete subspace of* X*. Hence the condition of Theorem 2.1 are satisfied, that is,* F *and* g *have a coupled coincidence point (0,0). Furthermore, since* F *and* g *are w-compatible, hence, Theorem 2.3 shows that (0,0) is the unique common coupled fixed point of* F *and* g*.*

Remark 2.6 *It should be noted that Example 2.5 is valid for Theorem 2.1 as well as for Theorem 1.12 . In fact Theorem 2.1 is more general the Theorem 1.12 .*

Remark 2.7 *If we take* $a_i = 0 \quad for \quad i = 3, 4, 5, 6$ *and* $g = I_X$ *(identity mapping over X) in Theorem 2.1 then we get result of Kang et al [3] .*

Corollary 2.8 *([3], Corollary-2.2) Let* (X, d) *be a complete complex valued metric space. Suppose that the mapping* $F : X \times X \to X$ *satisfies*

$$d(F(x, y), F(u, v)) \le h[(d(x, u) + d(y, v))]$$

for all $x, y, u, v \in X$*, where* h *is a non-negative constant with* $h < \frac{1}{2}$*. Then* F *has a unique coupled fixed point.*

Proof 2.9 *If we take* $a_1 = a_2 = h$, $a_i = 0 \quad for \quad i = 3, 4, 5, 6$ *and* $g = I_X$ *(identity mapping over X) in Theorem 2.1, then we get required result.*

Example 2.10 *Let* $X = \{ix : x \in [0, 1]\}$ *and consider a complex valued metric* $d : X \times X \to X$ *defined by*

$$d(x, y) = i|x - y|$$

for all $x, y \in X$. *Then* (X, d) *is a complex valued metric space.*

Define the mappings $F : X \times X \to X$ *by* $F(x, y) = i\left(\frac{x+y}{3}\right)$ *for all* $x, y \in [0, 1]$. *Then we have* $h = \frac{1}{3} < \frac{1}{2}$. *So all condition of Corollary 2.8 are satisfied and we get (0,0) is a coupled fixed of* F.

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4 Conflict of Interests

The authors declare that they have no competing interests.

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