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Certain class of analytic functions with varying arguments associated with the convolution of q-analogue of Salagean and Ruscheweyh differential operator

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Abstract

In the present paper, a new class $\mathcal{V}_q(\lambda, \beta, A, B)$ of analytic functions with varying arguments in the open unit disk $\Delta := \{z \in \mathbb{C} : |z| < 1\}$ is introduced. The class is defined by the convolution of q-analogue of the well-known Salagean and Ruscheweyh differential operator. We derive coefficient estimates, distortion theorem and extreme points for the function belongs to the above mentioned class.

Keywords: *Analytic function, q-derivative, Salagean operator, Ruscheweyh differential operator, varying argument, distortion theorem, extreme point.*

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1 Introduction and Motivation

Let $\mathbb{N}$ and $\mathbb{C}$ be denote the set of natural numbers and complex numbers respectively. In view of Riemann mapping theorem, the unit disk $\Delta := \{z \in \mathbb{C} : |z| < 1\}$ can be taken as a standard domain in the theory of analytic function theory. Let $\mathcal{H}(\Delta)$ be represent the set of all analytic (or holomorphic) functions in Δ. The class of functions $f \in \mathcal{H}(\Delta)$ with normalization condition $f(0) = f'(0) - 1 = 0$ be denoted by $\mathcal{A}$. Thus, the function $f \in \mathcal{A}$ have the following Taylor-Maclaurin series representation:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in \Delta). \tag{1}$$

We denote $\mathcal{S}$, the class of all functions $f \in \mathcal{A}$ which are univalent in Δ.
The quantum calculus, so called q-calculus (q-analysis) is the usual calculus without using the notion of limit. It provides important tools that has been used in order to investigate several subclasses of $\mathcal{A}$. It has attracted the attention of various researchers due to its numerous applications in mathematics and physics. Researchers all over the globe have applied it to construct and investigated several classes of analytic and bi-univalent functions. It was Jackson (see [6, 7]) who first developed q integral and q derivative in a systematic way and later geometrical interpretation of q-analysis has been recognized through studies on quantum groups. The q-analogue of differential operators in some subclasses of analytic functions in compact disk have been introduced by various authors (see [1, 8, 9]). These q-operators are defined by using convolution of normalized analytic functions and q-hypergeometric functions.
Here we mention some notations and concept of q-calculus that is used in this paper. The notations and terminology can be found in [3, 4, 5]. We recall the definition of fractional q-calculus operator of a complex-valued function $f(z)$.
Definition 1: For $0 < q < 1$, define the q-number $[\alpha]_q$ by

$$[\alpha]_q = \begin{cases} \frac{1-q^\alpha}{1-q} & (\alpha \in \mathbb{C}) \\ \sum_{i=0}^{\alpha-1} q = 1 + q + q^2 + \cdots + q^{n-1} & (\alpha = n \in \mathbb{N}). \end{cases} \tag{2}$$

Note that as $q \longrightarrow 1^-$, $[n]_q \longrightarrow n$.
Definition 2: For $0 < q < 1$, define the q-factorial $[n]_q!$ by

$$[n]_q! = \begin{cases} 1 & (n = 0) \\ \prod_{k=1}^{n} [k]_q & (n \in \mathbb{N}). \end{cases} \tag{3}$$

Definition 3 (see [5, 6]) : The q-derivative $D_q f$ of a function f is defined in a given subset of $\mathbb{C}$ by

$$(D_q f)(z) = \begin{cases} \frac{f(z)-f(qz)}{(1-q)z} & (z \neq 0) \\ f'(0) & (z = 0) \end{cases} \tag{4}$$

provided that $f'(0)$ exists.

It follows from (4) that

$$lim_{q\longrightarrow 1^-} D_q f(z) = lim_{q\longrightarrow 1^-} \frac{f(z)-f(qz)}{(1-q)z} = f'(z)$$

for a function f which is differentiable in a given subset of $\mathbb{C}$. Thus for a function $f(z)$ of the form (1), we have

$$D_q f(z) = 1 + \sum_{k=2}^{\infty} [k]_q a_k z^{k-1}. \tag{5}$$

Making use of q-operator, we generalize Salagean and Ruscheweyh differential operators as follows:

Definition 4: For $f \in \mathcal{A}$, $\lambda \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$, the q-analogue of Salagean differential operator $\mathfrak{J}_q^\lambda : \mathcal{A} \longrightarrow \mathcal{A}$ is defined by

$$\begin{aligned} \mathfrak{J}_q^0 f(z) &= f(z) \\ \mathfrak{J}_q^1 f(z) &= z(D_q f(z)) \\ &\cdots \\ &\cdots \\ \mathfrak{J}_q^\lambda f(z) = \mathfrak{J}_q^1(\mathfrak{J}_q^{\lambda-1} f(z)) &= z(D_q \mathfrak{J}_q^{\lambda-1} f(z)). \end{aligned}$$

Thus, for a function $f(z)$ of the form (1), we have

$$\mathfrak{J}_q^\lambda f(z) = z + \sum_{k=2}^{\infty} [k]_q^\lambda a_k z^k \quad (z \in \Delta). \tag{6}$$

Definition 5 (see [1]): Let $f \in \mathcal{A}$. Denote by $\mathcal{R}_q^\lambda$, the q-analogue of Ruscheweyh differential operator defined by

$$\mathcal{R}_q^\lambda f(z) = z + \sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q![k-1]_q!} a_k z^k, \tag{7}$$

where $[\alpha]_q!$ is defined as (3). It may be noted that when $q \longrightarrow 1^-$ we have

$$\begin{aligned} lim_{q\longrightarrow 1^-} \mathcal{R}_q^\lambda f(z) &= z + lim_{q\longrightarrow 1^-} \sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q![k-1]_q!} a_k z^k \\ & z + \sum_{k=2}^{\infty} \frac{(k+\lambda-1)!}{\lambda!(k-1)!} a_k z^k = \mathcal{R}^\lambda f(z), \end{aligned} \tag{8}$$

where $\mathcal{R}^\lambda$ is Ruscheweyh differential operator which was defined in [13] and has been studied by various researchers (for details, see [10, 12, 14]).

Definition 6: For $f \in \mathcal{A}$ given by (1), we define a linear operator $\mathfrak{J}\mathcal{R}_q^\lambda$: $\mathcal{A} \longrightarrow \mathcal{A}$ define by the Hadamard product (or convolution) of q-analogue of Salagean operator $\mathfrak{J}_q^\lambda$ and Ruscheweyh operator $\mathcal{R}_q^\lambda$ as:

$$\begin{aligned}\mathfrak{J}\mathcal{R}_q^\lambda f(z) &= (\mathfrak{J}_q^\lambda \star \mathcal{R}_q^\lambda) f(z)\\ &= z + \sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q![k-1]_q!}[k]_q^\lambda a_k^2 z^k \quad (z \in \Delta).\end{aligned} \tag{9}$$

Motivated from the work studied in [2, 11], we now introduce a new subclass of $\mathcal{A}$ by using the operator $\mathfrak{J}\mathcal{R}_q^\lambda$ as follows:
Definition 7: A function $f \in \mathcal{A}$ given by (1) is in the class $\mathcal{T}_q(\lambda, \beta, A, B)$ $(0 < q < 1,\ \lambda \in \mathbb{N}_0, \beta \geq 0, -1 \leq A < B \leq 1,\ 0 < B \leq 1)$ if it satisfies the following subordination condition:

$$(1-\beta)\frac{\mathfrak{J}\mathcal{R}_q^\lambda f(z)}{z} + \beta(\mathfrak{J}\mathcal{R}_q^\lambda f(z))' \prec \frac{1+Az}{1+Bz} \quad (z \in \Delta). \tag{10}$$

Silverman (see [15]) introduced and studied the univalent function with varying arguments of coefficients as follows:
Definition 8: (see [15]) A function $f(z)$ of the form (1) is in the class $\mathcal{V}(\theta_k)$ if $f(z) \in \mathcal{S}$ (the class of analytic and univalent function in Δ) and $arg(a_k) = \theta_k$ for all k $(k \geq 2)$. Further, if there exists a real number η such that

$$\theta_k + (k-1)\eta \equiv \pi (mod\ 2\pi), \tag{11}$$

then $f(z)$ is said to be in the class $\mathcal{V}(\theta_k, \eta)$. The union of $\mathcal{V}(\theta_k, \eta)$ taken over all possible sequence $\{\theta_k\}$ and all possible real numbers η is denoted by $\mathcal{V}$.

Let $\mathcal{V}_q(\lambda, \beta, A, B)$ denote the subclass of $\mathcal{V}$ consisting of functions $f(z) \in \mathcal{T}_q(\lambda, \beta, A, B)$.
In this paper, the authors obtain coefficient estimates, distortion theorem and extreme point for the function $f \in \mathcal{A}$ belongs to the class $\mathcal{V}_q(\lambda, \beta, A, B)$.

2 Coefficient Estimates

Unless otherwise stated, we assume throughout the sequel that $-1 \leq A < B \leq 1,\ 0 < B \leq 1,\ \lambda,\ \beta \in \mathbb{N}_0,\ 0 < q < 1;\ z \in \Delta$.
The sufficient condition for a function $f(z)$ of the form (1) to be in the class $\mathcal{T}_q(\lambda, \beta, A, B)$ is given by the following theorem.

Theorem 1: Let the function $f(z)$ be of the form (1). If

$$\sum_{k=2}^{\infty}[1+\beta(k-1)](1+\beta)D_{k,q}^\lambda |a_k|^2 \leq (B-A) \tag{12}$$

then $f(z) \in \mathcal{T}_q(\lambda, \beta, A, B)$ where $D_{k,q}^{\lambda} = \frac{[k]_q^{\lambda}[k+\lambda-1]_q!}{[\lambda]_q![k-1]_q!}$.
Proof:
A function $f(z)$ of the form (1) belongs to the class $\mathcal{T}_q(\lambda, \beta, A, B)$ if and only if there exists an analytic function $w(z)$, satisfying the condition of Schwarz lemma such that

$$(1-\beta)\frac{\Im\mathcal{R}_q^{\lambda} f(z)}{z} + \beta(\Im\mathcal{R}_q^{\lambda} f(z))' = \frac{1+Aw(z)}{1+Bw(z)}.$$

Or equivalently,

$$\left| \frac{(1-\beta)\frac{\Im\mathcal{R}_q^{\lambda} f(z)}{z} + \beta(\Im\mathcal{R}_q^{\lambda} f(z))' - 1}{\beta[(1-\beta)\frac{\Im\mathcal{R}_q^{\lambda} f(z)}{z} + \beta(\Im\mathcal{R}_q^{\lambda} f(z))'] - A} \right| < 1. \tag{13}$$

Thus, it is sufficient to show that

$$\left|(1-\beta)\frac{\Im\mathcal{R}_q^{\lambda} f(z)}{z} + \beta(\Im\mathcal{R}_q^{\lambda} f(z))' - 1\right| - \left|B\big[(1-\beta)\frac{\Im\mathcal{R}_q^{\lambda} f(z)}{z} + \beta(\Im\mathcal{R}_q^{\lambda} f(z))'\big] - A\right| \leq 0.$$

Letting $|z| = r \quad (0 \leq r < 1)$, we have

$$\begin{aligned}
&\left|(1-\beta)\frac{\Im\mathcal{R}_q^{\lambda} f(z)}{z} + \beta(\Im\mathcal{R}_q^{\lambda} f(z))' - 1\right| - \left|B\big[(1-\beta)\frac{\Im\mathcal{R}_q^{\lambda} f(z)}{z} + \beta(\Im\mathcal{R}_q^{\lambda} f(z))'\big] - A\right| \\
&= \left|\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda} a_k^2 z^{k-1}\right| - \left|(B-A) + B\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda} a_k^2 z^{k-1}\right| \\
&\leq \sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}|a_k|^2 r^{k-1} - (B-A) + B\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}|a_k|^2 r^{k-1} \\
&\leq \sum_{k=2}^{\infty}[1+\beta(k-1)](1+B)D_{k,q}^{\lambda}|a_k|^2 - (B-A).
\end{aligned}$$

In view of (12), the last inequality is less than zero. Hence $f(z) \in \mathcal{T}_q(\lambda, \beta, A, B)$. This completes the proof of Theorem 2.

Theorem 2: Let the function $f(z) \in \mathcal{A}$ be of the form (1). Then $f(z) \in \mathcal{V}_q(\lambda, \beta, A, B)$ if and only if

$$\sum_{k=2}^{\infty}[1+\beta(k-1)](1+B)D_{k,q}^{\lambda}|a_k|^2 \leq (B-A). \tag{14}$$

Proof. In view of Theorem 1, we need only to show that function $f(z) \in \mathcal{V}_q(\lambda, \beta, A, B)$ satisfies the coefficient inequalities (14). Let $f(z) \in \mathcal{V}_q(\lambda, \beta, A, B)$.

Then from (1) and (13), we have

$$\left| \frac{\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}a_k^2 z^{k-1}}{(B-A)+\sum_{k=2}^{\infty}B[1+\beta(k-1)]D_{k,q}^{\lambda}a_k^2 z^{k-1}} \right| < 1. \tag{15}$$

Since $f(z) \in \mathcal{V}$, $f(z)$ lies in the class $\mathcal{V}(\theta_k, \eta)$ for some sequence $\{\theta_k\}$ and real number η such that $\theta_k + (k-1)\eta \equiv \pi(mod 2\pi)$ for all $k \geq 2$.
Set $z = re^{i\eta}$ in (15), we have

$$\frac{\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}|a_k|^2 e^{i(\theta_k+(k-1)\eta)} r^{k-1}}{(B-A)+B\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}|a_k|^2 e^{i(\theta_k+(k-1)\eta)} r^{k-1}} < 1,$$

which implies

$$\left| \frac{-\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}|a_k|^2 r^{k-1}}{(B-A)-B\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}|a_k|^2 r^{k-1}} \right| < 1.$$

Since $\Re(w(z)) < |w(z)| < 1$ implies

$$\Re\left[\frac{\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}|a_k|^2 r^{k-1}}{(B-A)-B\sum_{k=2}^{\infty}[1+\beta(k-1)]D_{k,q}^{\lambda}|a_k|^2 r^{k-1}} \right] < 1. \tag{16}$$

It has been observed that the denominator of the left hand side of (16) cannot vanish for $[0,1)$. Furthermore, it is positive for $r = 0$ and therefore for $r \in [0,1)$. Thus, we have

$$\sum_{k=2}^{\infty}[1+\beta(k-1)](1+B)D_{k,q}^{\lambda}|a_k|^2 r^{k-1} < (B-A)$$

which, upon letting $r \longrightarrow 1^-$ gives the require assertion of Theorem 2. The proof of Theorem 2 is thus completed.

Corollary 3: Let the function $f(z) \in \mathcal{A}$ defined by (1) be in the class $\mathcal{V}_q(\lambda, \beta, A, B)$. Then

$$|a_k| \leq \sqrt{\frac{(B-A)}{[1+\beta(k-1)](1+B)D_{k,q}^{\lambda}}} \quad (k \geq 2).$$

The result is sharp for the function

$$f(z) = z + \sqrt{\frac{(B-A)}{[1+\beta(k-1)](1+B)D_{k,q}^{\lambda}}} e^{i\theta_k} z^k \quad (k \geq 2).$$

3 Distortion Theorem

Theorem 4: Let the function $f(z)$ defined by (1) be in the class $\mathcal{V}_q(\lambda, \beta, A, B)$. Then

$$|z| - \sqrt{\frac{(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}|z|^2 \le |f(z)| \le |z| + \sqrt{\frac{(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}|z|^2 \quad (17)$$

The result is sharp.

Proof: Corollary 3 and elementary inequality

$$(1+\beta)D_{2,q}^{\lambda} \le [1+\beta(k-1)]D_{k,q}^{\lambda} \quad (k \ge 2)$$

yield

$$\sum_{k=2}^{\infty} |a_k| \le \sqrt{\frac{(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}. \quad (18)$$

Thus,

$$\begin{aligned} |f(z)| &= |z + \sum_{k=2}^{\infty} a_k z^k| \\ &\le |z| + \sum_{k=2}^{\infty} |a_k||z|^k \\ &\le |z| + |z|^2 \sum_{k=2}^{\infty} |a_k| \\ &\le |z| + \sqrt{\frac{(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}|z|^2. \end{aligned} \quad (19)$$

Similarly, we have

$$\begin{aligned} |f(z)| &= |z + \sum_{k=2}^{\infty} a_k z^k| \\ &\ge |z| - \sum_{k=2}^{\infty} |a_k||z|^k \\ &\ge |z| - |z|^2 \sum_{k=2}^{\infty} |a_k| \\ &\ge |z| - \sqrt{\frac{(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}|z|^2. \end{aligned} \quad (20)$$

Combining (19) and (20) we obtain the desire result. The result is sharp for the function

$$f(z) = z + \sqrt{\frac{(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}e^{i\theta_2}z^2 \tag{21}$$

at $z = \pm|z|e^{-i\theta_2}$, This completes the proof of Theorem 4.

Corollary 5: Under the hypothesis of Theorem 4, $f(z)$ included in a disk with center at origin and radius r_1 given by

$$r_1 = 1 + \sqrt{\frac{(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}.$$

Theorem 6: Let the function $f(z)$ defined by (1) belong to the class $\mathcal{V}_q(\lambda, \beta, A, B)$. Then

$$1 - \sqrt{\frac{4(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}|z| \le |f'(z)| \le 1 + \sqrt{\frac{4(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}|z|. \tag{22}$$

The result is sharp for the function $f(z)$ given by (21) at $z = \pm|z|e^{-i\theta_2}$.

Proof. In view of the inequality

$$k(1+\beta)D_{2,q}^{\lambda} \le 2[1+\beta(k-1)]D_{k,q}^{\lambda} \quad (k \ge 2),$$

it follows that

$$\sum_{k=2}^{\infty} k|a_k| \le 2\sqrt{\frac{(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}} = \sqrt{\frac{4(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}.$$

Thus, we have

$$\begin{aligned} |f'(z)| &= |1 + \sum_{k=2}^{\infty} k a_k z^{k-1}| \\ &\le 1 + |z|\sum_{k=2}^{\infty} k|a_k| \\ &\le 1 + \sqrt{\frac{4(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}|z|. \end{aligned} \tag{23}$$

Similarly, we obtain

$$\begin{aligned} |f'(z)| &= |1+\sum_{k=2}^{\infty} k a_k z^{k-1}| \\ &\geq 1-|z|\sum_{k=2}^{\infty} k|a_k| \\ &\geq 1-\sqrt{\frac{4(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}|z|. \end{aligned} \tag{24}$$

The assertion (22) of Theorem 6 follows from (23) and (24). The result is sharp for the function $f(z)$ given by (21).

Corollary 7: Let the function $f(z) \in \mathcal{A}$ defined by (1) be in the class $\mathcal{V}_q(\lambda, \beta, A, B)$. Then $f(z)$ is included in a disk with center at origin and radius r_2 given by

$$r_2 = 1+\sqrt{\frac{4(B-A)}{(1+\beta)(1+B)D_{2,q}^{\lambda}}}.$$

4 Extreme Points

Theorem 8: Let the function $f(z)$ defined by (1) be in the class $\mathcal{V}_q(\lambda, \beta, A, B)$ with $arg(a_k) = \theta_k$ where $[\theta_k + (k-1)\eta] \equiv \pi \ (mod 2\pi)$. Define $f_1(z) = z$ and

$$f_k(z) = z+\sqrt{\frac{(B-A)}{[1+\beta(k-1)](1+B)D_{k,q}^{\lambda}}}e^{i\theta_k}z^k \quad (k \geq 2; z \in \Delta).$$

Then $f(z)$ is in the class $\mathcal{V}_q(\lambda, \beta, A, B)$ if and only if it can be expressed in the form

$$f(z) = \sum_{k=1}^{\infty} \mu_k f_k(z),$$

where $\mu_k \geq 0 \ \ (k \geq 1)$ and $\sum_{k=1}^{\infty} \mu_k = 1$.

Proof: If $f(z) = \sum_{k=1}^{\infty} \mu_k f_k(z)$ with $\sum_{k=1}^{\infty} \mu_k = 1$ and $\mu_k \geq 0$, then

$$\begin{aligned} &\sum_{k=2}^{\infty}[1+\beta(k-1)](1+B)D_{k,q}^{\lambda}\frac{(B-A)}{[1+\beta(k-1)](1+B)D_{k,q}^{\lambda}}\mu_k \\ &= \sum_{k=2}^{\infty}(B-A)\mu_k = (B-A)(1-\mu_1) \leq (B-A). \end{aligned}$$

So, by Theorem 6, we have $f(z) \in \mathcal{V}_q(\lambda, \beta, A, B)$. Conversely, let the function $f(z)$ defined by (1) be in the class $\mathcal{V}_q(\lambda, \beta, A, B)$. Define

$$\mu_k = \sqrt{\frac{[1+\beta(k-1)](1+B)D^{\lambda}_{k,q}}{(B-A)}|a_k|} \quad (k \geq 2)$$

and

$$\mu_1 = 1 - \sum_{k=2}^{\infty} \mu_k.$$

From Theorem 6, $\sum_{k=2}^{\infty} \mu_k \leq 1$ which implies $\mu_1 \geq 0$. Since $\mu_k f_k(z) = \mu_k z + a_k z^k$, we have

$$\sum_{k=1}^{\infty} \mu_k f_k(z) = z + \sum_{k=2}^{\infty} a_k z^k = f(z).$$

This completes the proof of Theorem 8.

5 Open Problem

Using post quantum analysis or (p,q)-differential operator, Salagean and Ruscheweyh differential operator can be further generalized. The class defined in Definition 7 can be redefined by help of generalized operator. Accordingly, coefficient estimates, distortion theorem and extreme points for the function belongs to the generalized class can be found.

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