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On Subclass Of Harmonic Univalent Functions Associated With The Poisson Distribution Series

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Abstract

The purpose of the present paper is to introduce a new subclass of harmonic functions in the unit disk U associated with the Poisson distribution series. Coefficient conditions, extreme points, distortion bounds, convex combination are studied. Furthermore, several corollaries of the main theorems are presented.

Keywords: *Hadamard product, Harmonic functions, Poisson distribution series, Univalent functions.*
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1 Introduction

In a complex domain G, let u and v be harmonic real-valued functions. Then the continuous function $f = u + iv$ defined on G is said to be harmonic. In any simply connected domain D, we can write f as $h + \bar{g}$, where h and g are analytic on D. A necessary and sufficient condition for f to be locally univalent and sense preserving on D is that $|h^{'}(z)| > |g^{'}(z)|$ for all z in D(see [1]).
Clunie and Sheil-Small [2] introduced a class $\mathcal{S}_{\mathcal{H}}$ of harmonic complex-valued functions f which are univalent and sense-preserving on the open unit disk

$U = \{z \in C : |z| < 1\}$ with standard normalization $f(0) = f_z(0) - 1 = 0$. Then for $f = h + \bar{g} \in \mathcal{S_H}$ we may express the analytic functions h and g as

$$h(z) = z + \sum_{n=2}^{\infty} a_n z^n, \ \ g(z) = \sum_{n=1}^{\infty} b_n z^n, |b_1| < 1. \tag{1}$$

Let $\overline{\mathcal{S_H}}$ be the subclass of $\mathcal{S_H}$ consisting of functions of the form $f(z) = h(z) + \overline{g(z)}$, where

$$h(z) = z - \sum_{n=2}^{\infty} |a_n| z^n, \ \ g(z) = \sum_{n=1}^{\infty} |b_n| z^n, |b_1| < 1. \tag{2}$$

Also, Sheil-Small [3] investigated the class $\mathcal{S_H}$ as well as its geometric subclasses and obtained some coefficient bounds. Since then, there has been several related papers on $\mathcal{S_H}$ and its subclasses studied by Avci and Zlotkiewicz [4], Silverman [5], and Jahangiri [6]. Furthermore, Some several researcher such as (e.g see [7], [8], [9]) have recently studied the harmonic univalent functions and many others researchers.

A variable x is said to be Poisson distributed if it takes the values $0, 1, 2, 3, ...$ with probabilities $e^m, m\frac{e^m}{1!}, m\frac{e^m}{3!}, ...$ respectively, where m is called the parameter. Thus

$$P(x = r) = \frac{m^n e^{-m}}{r!}, r = 0, 1, 2, ... \ .$$

Recently, Porwal [10] introduce a power series whose coefficients are probabilities of Poisson distribution

$$K(m, z) = z + \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} z^n, \quad (m > 0, \ z \in U).$$

We note that, by ratio test, the radius of convergence of the above series is infinity. They also obtained some interesting results on certain classes of analytic univalent functions.

In 2016, Porwal and Kumar [11] introduced a new linear operator $I(m, n) : A \to A$ by using the convolution (or Hadamard product), and defined as follows

$$I(m, z)f = K(m, z) * f(z) = z + \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} a_n z^n, \quad (m > 0, \ z \in U).$$

where $*$ denote the convolution (or Hadamard product) of two series.

Corresponding to $I(m, z)f(z)$, we define the following class of functions.

Definition 1.1 *Let $f(z) = h(z) + \overline{g(z)}$, be the harmonic univalent function given by (1), then $f \in \mathcal{S}_{\mathcal{H}}(m, \lambda, \alpha)$ if and only if*

$$\Re\left((1-\lambda)\frac{I(m,z)f(z)}{z} + \lambda\frac{[I(m,z)f(z)]'}{z'}\right) \geq \alpha, \tag{3}$$

where $0 \leq \alpha < 1, 0 \leq \lambda \leq 1$ and $z = re^{i\theta} \in U$.

We also let $\overline{\mathcal{S}_{\mathcal{H}}}(m, \lambda, \alpha) = \mathcal{S}_{\mathcal{H}}(m, \lambda, \alpha) \cap \overline{\mathcal{S}_{\mathcal{H}}}$. In this paper, a new subclass of harmonic univalent functions associated with Poisson distribution $I(m, z)$ examined to be in the function class $\mathcal{S}_{\mathcal{H}}(m, \lambda, \alpha)$. The coefficient condition for the function class $\mathcal{S}_{\mathcal{H}}(m, \lambda, \alpha)$ is given. Furthermore, we determine distortion theorem, convex combinations , and extreme points for the functions f in $\overline{\mathcal{S}_{\mathcal{H}}}(m, \lambda, \alpha)$.

2 Coefficient bound

We begin with a sufficient coefficient condition for functions f in $\mathcal{S}_{\mathcal{H}}(m, \lambda, \alpha)$.

Theorem 2.1 *Let $f = h + \overline{g}$ be given by (1). If*

$$\sum_{n=2}^{\infty}[\lambda(n-1)+1]\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n| + \sum_{n=1}^{\infty}[\lambda(n+1)-1]\frac{m^{n-1}}{(n-1)!}e^{-m}|b_n| \leq 1-\alpha. \tag{4}$$

Then $f \in \mathcal{S}_{\mathcal{H}}(m, \lambda, \alpha)$.

Proof. Using the fact that $\Re(w) > \alpha$ if and only if $|w - \alpha + 1| \geq |w - \alpha - 1|$, where

$$w = (1-\lambda)\frac{I(m,z)f(z)}{z} + \lambda\frac{[I(m,z)f(z)]'}{z'}.$$

It is enough to show that $|w - \alpha + 1| - |w - \alpha - 1| \geq 0$.

Now, we have

$$|w-\alpha+1| = |(1-\lambda)\left(1 + \sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}a_n z^{n-1} + \sum_{n=1}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}b_n\overline{z}^{n-1}\right)$$

$$+\lambda\left(1 + \sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}na_n z^{n-1} - \sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}nb_n\overline{z}^{n-1}\right) + 1 - \alpha|$$

$$\geq 2 - \alpha - \sum_{n=2}^{\infty}|1+\lambda(n-1)|\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n||z^{n-1}|$$

$$-\sum_{n=1}^{\infty}|1-\lambda(n+1)|\frac{m^{n-1}}{(n-1)!}e^{-m}|b_n||z^{n-1}|$$

and

$$|w-\alpha-1|=|(1-\lambda)\left(1+\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}a_nz^{n-1}+\sum_{n=1}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}b_n\overline{z}^{n-1}\right)$$

$$+\lambda\left(1+\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}na_nz^{n-1}-\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}nb_n\overline{z}^{n-1}\right)-1-\alpha|$$

$$\leq\alpha+\sum_{n=2}^{\infty}|1+\lambda(n-1)|\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n||z^{n-1}|$$

$$+\sum_{n=1}^{\infty}|1-\lambda(n+1)|\frac{m^{n-1}}{(n-1)!}e^{-m}|b_n||z^{n-1}|.$$

So by using (4) we have

$$|w-\alpha+1|-|w-\alpha-1|\geq 2[1-\alpha-\sum_{n=2}^{\infty}|\lambda(n-1)+1|\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n|$$

$$-\sum_{n=1}^{\infty}|\lambda(n+1)-1|\frac{m^{n-1}}{(n-1)!}e^{-m}|b_n|]\geq 0$$

and so the proof is completed.

Remark 2.2 *The coefficient bound (4) in previous theorem is sharp for the function*

$$f(z)=z+\sum_{n=2}^{\infty}\frac{x_n}{|\lambda(n-1)+1|}\frac{m^{n-1}}{(n-1)!}e^{-m}z^n+\sum_{n=1}^{\infty}\frac{\overline{y_n}}{|\lambda(n-1)+1|}\frac{m^{n-1}}{(n-1)!}e^{-m}\overline{z}^n, \tag{5}$$

where $\sum_{n=2}^{\infty}|x_n|+\sum_{n=1}^{\infty}|y_n|=1-\alpha$.

Theorem 2.3 *Let* $f=h+\overline{g}\in\overline{\mathcal{S}_{\mathcal{H}}}$. *Then* $f\in\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$ *If and only if*

$$\sum_{n=2}^{\infty}[\lambda(n-1)+1]\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n|+\sum_{n=1}^{\infty}[\lambda(n+1)-1]\frac{m^{n-1}}{(n-1)!}e^{-m}|b_n|\leq 1-\alpha. \tag{6}$$

Proof. Since $\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha) \subset \mathcal{S}_{\mathcal{H}}(m,\lambda,\alpha)$, we only need to prove the "only if " part, assume that $f \in \overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$. Therefore, for assume that $z = re^{i\theta} \in U$, we have

$$\Re\left\{(1-\lambda)\frac{I(m,z)f(z)}{z} + \lambda\frac{[I(m,z)f(z)]'}{z'}\right\}$$

$$= \Re\{(1-\lambda)\left(1+\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}a_n z^{n-1} + \sum_{n=1}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}b_n\overline{z}^{n-1}\right)$$

$$+\lambda\left(1-\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}na_n z^{n-1} - \sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}nb_n\overline{z}^{n-1}\right)\}$$

$$= \Re\left\{1-\sum_{n=2}^{\infty}(1-\lambda+n\lambda)\frac{m^{n-1}}{(n-1)!}e^{-m}a_n z^{n-1} + \sum_{n=1}^{\infty}(1-\lambda-n\lambda)\frac{m^{n-1}}{(n-1)!}e^{-m}b_n\overline{z}^{n-1}\right\}$$

$$\geq 1-\sum_{n=2}^{\infty}(\lambda(n-1)+1)\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n|\mu^{n-1}-\sum_{n=1}^{\infty}(\lambda(n+1)-1)\frac{m^{n-1}}{(n-1)!}e^{-m}|b_n|\mu^{n-1} \geq \alpha.$$

The last inequality hold for all values of $z \in U$ on the positive real axes. So if $z = \mu \to 1$, we obtain the required result given by (6). So the proof of the Theorem 2.3 is completed.

If $\lambda = 0$ in Theorem 2.3, we obtained the following Corollary

Corollary 2.4 *Let* $f = h + \overline{g} \in \overline{\mathcal{S}_{\mathcal{H}}}(m,\alpha)$ *if and only if*

$$\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n| - \sum_{n=1}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}|b_n| \leq 1-\alpha.$$

If $\lambda = 1$ in Theorem 2.3, we obtained the following Corollary

Corollary 2.5 *Let* $f = h + \overline{g} \in \overline{\mathcal{S}_{\mathcal{H}}}(m,1,\alpha)$ *if and only if*

$$\sum_{n=2}^{\infty}n\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n| + \sum_{n=1}^{\infty}n\frac{m^{n-1}}{(n-1)!}e^{-m}|b_n| \leq 1-\alpha.$$

3 Distortion bounds

In this section, we obtain distortion bounds for functions f in $\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$.

Theorem 3.1 *Let* $f \in \overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$*, then for* $|z| < 1$*, we have*

$$|f(z)| \geq (1-|b_1|)\mu - \frac{1}{me^{-m}}\left(\frac{(1-\alpha)}{1+\lambda} - \frac{2\lambda-1}{1+\lambda}|b_1|\right)\mu^2 \tag{7}$$

and

$$|f(z)| \leq (1+|b_1|)\mu + \frac{1}{me^{-m}}\left(\frac{(1-\alpha)}{1+\lambda} - \frac{2\lambda-1}{1+\lambda}|b_1|\right)\mu^2. \tag{8}$$

Proof. Let $f \in \overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$. Taking the absolute value of f, we have

$$|f(z)| = |z - \textstyle\sum_{n=2}^{\infty}|a_n|z^n + \sum_{n=1}^{\infty}|b_n|\overline{z}^n|$$

$$\geq (1-|b_1|)\mu - \textstyle\sum_{n=2}^{\infty}(|a_n|+|b_n|)\mu^n$$

$$\geq (1-|b_1|)\mu - \textstyle\sum_{n=2}^{\infty}(|a_n|+|b_n|)\mu^2$$

$$\geq (1-|b_1)\mu - \tfrac{1-\alpha}{1+\lambda}\left(\tfrac{1}{me^{-m}}\right)\left(\textstyle\sum_{n=2}^{\infty}\tfrac{1+\lambda}{1-\alpha}me^{-m}(|a_n|+|b_n|)\right)\mu^2$$

$$\geq (1-|b_1)\mu -$$

$$\tfrac{1-\alpha}{1+\lambda}\left(\tfrac{1}{me^{-m}}\right)\textstyle\sum_{n=2}^{\infty}\left(\tfrac{\lambda(n-1)+1}{1-\alpha}\left(\tfrac{m^{n-1}}{(n-1)!}e^{-m}\right)|a_n| + \tfrac{\lambda(n+1)-1}{1-\alpha}\left(\tfrac{m^{n-1}}{(n-1)!}e^{-m}\right)|b_n|\right)\mu^2$$

$$\geq (1-|b_1)\mu - \tfrac{1-\alpha}{1+\lambda}\left(\tfrac{1}{me^{-m}}\right)\left(1-\tfrac{2\lambda-1}{1-\alpha}|b_1|\right)\mu^2$$

$$= (1-|b_1)\mu - \left(\tfrac{1}{me^{-m}}\right)\left(\tfrac{1-\alpha}{1+\lambda} - \tfrac{2\lambda-1}{1+\lambda}|b_1|\right)\mu^2.$$

Relation (8) can be proved by using the similar statements. Therefore, it is omitted. So the proof is completed. The following covering result follows from the left hand inequality in Theorem 3.1.

If $\lambda = 0$ in Theorem 3.1, we obtained the following Corollary

Corollary 3.2 *Let* $f \in \overline{\mathcal{S}_{\mathcal{H}}}(m,\alpha)$*, then for* $|z| < 1$*, we have*

$$|f(z)| \geq (1-|b_1|)\mu - \frac{1}{me^{-m}}(1-\alpha+|b_1|)\mu^2 \tag{9}$$

and

$$|f(z)| \leq (1+|b_1|)\mu + \frac{1}{me^{-m}}(1-\alpha+|b_1|)\mu^2. \tag{10}$$

If $\lambda = 1$ in Theorem 3.1, we obtained the following Corollary

Corollary 3.3 *Let* $f \in \overline{\mathcal{S}_{\mathcal{H}}}(m,1,\alpha)$*, then for* $|z| < 1$*, we have*

$$|f(z)| \geq (1-|b_1|)\mu - \frac{1}{me^{-m}}\left(\frac{(1-\alpha)}{2} - \frac{1}{2}|b_1|\right)\mu^2 \tag{11}$$

and

$$|f(z)| \leq (1+|b_1|)\mu + \frac{1}{me^{-m}}\left(\frac{(1-\alpha)}{2} - \frac{1}{2}|b_1|\right)\mu^2. \tag{12}$$

4 Convex combinations and Extreme points

Now we introduce $\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$ is closed under convex combination.

Theorem 4.1 *If $f_{n,i}$ $(i=1,2,...)$ belongs to $\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$, then the function $\Phi(z)=\sum_{i=1}^{\infty}t_i f_{n,i}(z)$, is also in $\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$, where $f_{n,i}(z)$ is defined by*

$$f_{n,i}(z)=z-\sum_{n=2}^{\infty}a_{n,i}z^n+\sum_{n=1}^{\infty}b_{n,i}\bar{z}^n,\ (i=1,2,3,...,\ 0\le t_i\le 1,\ \sum_{i=1}^{\infty}t_i=1). \tag{13}$$

Proof. Since $f\in\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$. Then by (4) we have

$$\sum_{n=2}^{\infty}(|\lambda(n-1)+1|)\left(\frac{m^{n-1}}{(n-1)!}e^{-m}\right)|a_{n,i}|$$

$$+\sum_{n=1}^{\infty}(|\lambda(n+1)-1|)\left(\frac{m^{n-1}}{(n-1)!}e^{-m}\right)|b_{n,i}|\le 1-\alpha,\ (i=1,2,3,...).$$

Also, we have

$$\Phi(z)=\sum_{i=1}^{\infty}t_i f_{n,i}(z)=z-\sum_{n=2}^{\infty}\left(\sum_{i=1}^{\infty}t_i a_{n,i}\right)z^n+\sum_{n=1}^{\infty}\left(\sum_{i=1}^{\infty}t_i b_{n,i}\right)\overline{z}^n.$$

Now according to Theorem 2.3 we have

$$\sum_{n=2}^{\infty}(|\lambda(n-1)+1|)\left(\frac{m^{n-1}}{(n-1)!}e^{-m}\right)\left|\sum_{i=1}^{\infty}t_i a_{n,i}\right|$$

$$+\sum_{n=1}^{\infty}(|\lambda(n+1)-1|)\left(\frac{m^{n-1}}{(n-1)!}e^{-m}\right)\left|\sum_{i=1}^{\infty}t_i b_{n,i}\right|$$

$$=\sum_{i=1}^{\infty}t_i\{\sum_{n=2}^{\infty}(|\lambda(n-1)+1|)\left(\frac{m^{n-1}}{(n-1)!}e^{-m}\right)|a_{n,i}|$$

$$+\sum_{n=1}^{\infty}(|\lambda(n+1)-1|)\left(\frac{m^{n-1}}{(n-1)!}e^{-m}\right)|b_{n,i}|\}$$

$$\leq (1-\alpha)\sum_{i=1}^{\infty} t_i = 1-\alpha.$$

Thus

$$\Phi(z) \in \overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha).$$

So, we note that $\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$ is a convex set.

Next we determine the extreme points of closed convex hulls of $\overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$, denoted by clco $\mathcal{S}_{\mathcal{H}}(m,\lambda,\alpha)$.

Theorem 4.2 *Let $f = h+\overline{g} \in \overline{\mathcal{S}_{\mathcal{H}}}(m,\lambda,\alpha)$ if and only if it can be expressed as*

$$f(z) = X_1 z + \sum_{n=2}^{\infty} X_n h_n(z) + \sum_{n=1}^{\infty} Y_n g_n(z), z \in U, \tag{14}$$

where

$$h_n(z) = z - \tfrac{1-\alpha}{(|\lambda(n-1)+1|)}\left(\tfrac{(n-1)!}{m^{n-1}e^{-m}}\right) z^n, (n = 2,3,...),$$

$$g_n(z) = z + \tfrac{1-\alpha}{(|\lambda(n+1)-1|)}\left(\tfrac{(n-1)!}{m^{n-1}e^{-m}}\right) \overline{z}^n, (n = 1,2,...),$$

$$X_1 z + \textstyle\sum_{n=2}^{\infty} X_n + \sum_{n=1}^{\infty} Y_n = 1,\ X_n \geq 0,\ Y_n \geq 0.$$

Proof. For functions f of the form (14) we have

$$f(z) = z - \textstyle\sum_{n=2}^{\infty} \tfrac{1-\alpha}{(|\lambda(n-1)+1|)}\left(\tfrac{(n-1)!}{m^{n-1}e^{-m}}\right) X_n z^n$$

$$+\textstyle\sum_{n=1}^{\infty} \tfrac{1-\alpha}{(|\lambda(n+1)-1|)}\left(\tfrac{(n-1)!}{m^{n-1}e^{-m}}\right) Y_n \overline{z}^n.$$

Since by (6), we have

$$\textstyle\sum_{n=2}^{\infty}(|\lambda(n-1)+1|)\tfrac{1-\alpha}{(|\lambda(n-1)+1|)}\left(\tfrac{(n-1)!}{m^{n-1}e^{-m}}\right)|X_n|$$

$$+\textstyle\sum_{n=1}^{\infty}(|\lambda(n+1)-1|)\tfrac{1-\alpha}{(|\lambda(n+1)-1|)}\left(\tfrac{(n-1)!}{m^{n-1}e^{-m}}\right)|Y_n|.$$

$$= (1-\alpha)\left(\sum_{n=2}^{\infty}|X_n| + \sum_{n=1}^{\infty}|Y_n|\right) = (1-\alpha)(1+X_1) \leq 1-\alpha, \tag{15}$$

and so $f \in$ *clco* $\mathcal{S}_{\mathcal{H}}(m,\lambda,\alpha)$.

Conversely, suppose that $f \in$ *clco* $\mathcal{S}_{\mathcal{H}}(m,\lambda,\alpha)$. Setting

$$X_n = \frac{(|\lambda(n-1)+1|)}{1-\alpha}\left(\frac{m^{n-1}}{(n-1)!}e^{-m}\right)|a_n|,\ 0 \le X_n \le 1, n = 2, 3, \dots ,$$

$$Y_n = \frac{(|\lambda(n+1)-1|)}{1-\alpha}\left(\frac{m^{n-1}}{(n-1)!}e^{-m}\right)|b_n|,\ 0 \le Y_n \le 1, n = 1, 2, \dots ,$$

and $X_1 = 1 - \sum_{n=2}^{\infty} X_n - \sum_{n=1}^{\infty} Y_n$, and note that, by Theorem 2.1, $X_1 \ge 0$. Consequently, we obtain

$$\begin{aligned} f(z) &= z - \textstyle\sum_{n=2}^{\infty} |a_n| z^n + \sum_{n=}^{\infty} |b_n| \overline{z}^n, \\ &= \sum_{n=2}^{\infty} \frac{(1-\alpha)X_n}{(|\lambda(n-1)+1|)}\left(\frac{(n-1)!}{m^{n-1}e^{-m}}\right) z^n \\ &+ \sum_{n=1}^{\infty} \frac{(1-\alpha)Y_n}{(|\lambda(n+1)-1|)}\left(\frac{(n-1)!}{m^{n-1}e^{-m}}\right) \overline{z}^n \\ &= \left(1 - \sum_{n=2}^{\infty} X_n - \sum_{n=1}^{\infty} Y_n\right) z + \sum_{n=1}^{\infty} g_n(z) Y_n + \sum_{n=2}^{\infty} h_n(z) X_n \\ &= X_1 z + \sum_{n=1}^{\infty} g_n(z) Y_n + \sum_{n=2}^{\infty} h_n(z) X_n, \end{aligned}$$

that is the required representation.

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