

Some Inequalities for Meromorphic Multivalent Functions Associated with Mittag-Leffler Function

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Abstract

The purpose of this paper is to prove some differential inequalities for meromorphic multivalent functions by using a new operator associated with Mittag-Leffler function.

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1 Introduction

Denote by Σ_p the class of meromorphic multivalent functions of the form:

$$f(z) = \frac{1}{z^p} + \sum_{n=1-p}^{\infty} a_n z^n, \quad (1)$$

which are analytic in $\mathbb{U}^* = \{z : z \in \mathbb{C} \text{ and } 0 < |z| < 1\} = \mathbb{U} \setminus \{0\}$.

The Mittag-Leffler function $E_\alpha(z)$ ($z \in \mathbb{C}$) ([7], [8] and see also [3-5]) is defined by

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(n\alpha + 1)} z^n, \alpha \in \mathbb{C}, \operatorname{Re}(\alpha) > 0.$$

For $\alpha, \beta, \gamma \in \mathbb{C}$, $Re(\alpha) > \max\{0, Re(k)-1\}$ and $Re(k) > 0$, Srivastava and Tomovski [9] generalized Mittag-Leffler function and introduced the function

$$E_{\alpha, \beta}^{\gamma, k}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nk}}{\Gamma(n\alpha + \beta)n!} z^n \quad (2)$$

and proved that it is an entire function in the complex z -plane, where

$$(\gamma)_{\theta} = \frac{\Gamma(\gamma + \theta)}{\Gamma(\gamma)} \begin{cases} 1, & \theta = 0 \\ \gamma(\gamma + 1)\dots(\gamma + \theta - 1), & \theta \neq 0 \end{cases} .$$

Using the function $E_{\alpha, \beta}^{\gamma, k}(z)$, let

$$\begin{aligned} \mathcal{M}_{p, \alpha, \beta}^{\gamma, k}(z) &= z^{-p} \Gamma(\beta) E_{\alpha, \beta}^{\gamma, k}(z) \\ &= z^{-p} + \sum_{n=1-p}^{\infty} \frac{\Gamma(\beta) \Gamma[\gamma + (n+p)k]}{\Gamma(\gamma) \Gamma[\beta + (n+p)\alpha] \Gamma(n+p)} z^n, \\ &Re\alpha = 0 \text{ when } Re k = 1 \text{ with } \beta \neq 0. \end{aligned} \quad (3)$$

For $f(z) \in \Sigma_p$, we define the operator

$$\begin{aligned} \mathcal{H}_{p, \alpha, \beta}^{\gamma, k} f(z) &= \mathcal{M}_{p, \alpha, \beta}^{\gamma, k}(z) * f(z) \\ &= z^{-p} + \sum_{n=1-p}^{\infty} \frac{\Gamma(\beta) \Gamma[\gamma + (n+p)k]}{\Gamma(\gamma) \Gamma[\beta + (n+p)\alpha] \Gamma(n+p)} a_n z^n. \end{aligned} \quad (4)$$

From (4) it is easy to have

$$\mathcal{H}_{p, 0, \beta}^{1, 1} f(z) = p f(z) + z f'(z) + z^{-p},$$

$$kz(\mathcal{H}_{p, \alpha, \beta}^{\gamma, k} f(z))' = \gamma \mathcal{H}_{p, \alpha, \beta}^{\gamma+1, k} f(z) - (\gamma + pk) \mathcal{H}_{p, \alpha, \beta}^{\gamma, k} f(z) \quad (5)$$

and

$$\alpha z \left(\mathcal{H}_{p, \alpha, \beta+1}^{\gamma, k} f(z) \right)' = \beta \mathcal{H}_{p, \alpha, \beta}^{\gamma, k} f(z) - (p\alpha + \beta) \mathcal{H}_{p, \alpha, \beta+1}^{\gamma, k} f(z), \alpha \neq 0. \quad (6)$$

In the present paper using the operator $\mathcal{H}_{p, \alpha, \beta}^{\gamma, k} f(z)$, we investigate some inequalities for meromorphic univalent functions.

Definition 1. Let H be the set of complex valued functions $h(r, s, t) : \mathbb{C}^3 \rightarrow \mathbb{C}$ such that:

- i) is continuous in a domain $D \subset \mathbb{C}^3$
- ii) $(1, 1, 1) \in D$, $|h(1, 1, 1)| < 1$

$$\text{iii)} \quad \left| h \left(e^{i\theta}, \left(1 + \frac{k}{\gamma}\zeta\right)e^{i\theta}, k^2 \frac{\frac{\gamma(\gamma+1)}{k^2}e^{ii\theta} + \frac{2\gamma+k+1}{k}\zeta e^{i\theta} + L}{\gamma(\gamma+1)} \right) \right| > 1, \quad (7)$$

whenever

$$\left(e^{i\theta}, \left(1 + \frac{k}{\gamma}\zeta\right)e^{i\theta}, k^2 \frac{\frac{\gamma(\gamma+1)}{k^2}e^{ii\theta} + \frac{2\gamma+k+1}{k}\zeta e^{i\theta} + L}{\gamma(\gamma+1)} \right) \in D$$

with $Re(e^{-i\theta})L \geq \zeta(\zeta - 1)$, $\zeta \geq 1$ and θ real.

Definition 2. Let K be the set of complex valued functions $k(r, s, t) : \mathbb{C}^3 \rightarrow \mathbb{C}$ such that:

- i) is continuous in a domain $D \subset \mathbb{C}^3$
- ii) $(1, 1, 1) \in D$, $|h(1, 1, 1)| < 1$
- iii)

$$\left| h \left(e^{i\theta}, \left(1 + \frac{\alpha}{\beta-1}\zeta\right)e^{i\theta}, \alpha^2 \frac{\frac{(\beta-1)(\beta-2)}{\alpha^2}e^{ii\theta} + \frac{\alpha+2\beta-3}{\alpha}\zeta e^{i\theta} + L}{(\beta-1)(\beta-2)} \right) \right| > 1, \quad (8)$$

whenever

$$\left(e^{i\theta}, \left(1 + \frac{\alpha}{\beta-1}\zeta\right)e^{i\theta}, \alpha^2 \frac{\frac{(\beta-1)(\beta-2)}{\alpha^2}e^{ii\theta} + \frac{\alpha+2\beta-3}{\alpha}\zeta e^{i\theta} + L}{(\beta-1)(\beta-2)} \right) \in D$$

with $Re(e^{-i\theta})L \geq \zeta(\zeta - 1)$, $\zeta \geq 1$ and θ real.

Definition 3. Let M be the set of complex valued functions $k(r, s, t) : \mathbb{C}^3 \rightarrow \mathbb{C}$ such that:

- i) is continuous in a domain $D \subset \mathbb{C}^3$
- ii) $(1, 1, 1) \in D$, $|h(1, 1, 1)| < 1$
- iii)

$$\left| h \left(e^{i\theta}, \frac{\gamma e^{i\theta} + k\zeta + 1}{\gamma + 1}, \frac{1}{(\gamma + 2)} \left\{ \gamma e^{i\theta} + k\zeta + 2 + k^2 \frac{\frac{\gamma}{k}\zeta e^{i\theta} + \zeta - \zeta^2 + L}{\gamma e^{i\theta} + k\zeta + 1} \right\} \right) \right| > 1, \quad (9)$$

whenever

$$\left(e^{i\theta}, \frac{\gamma e^{i\theta} + k\zeta + 1}{\gamma + 1}, \frac{1}{(\gamma + 2)} \left\{ \gamma e^{i\theta} + k\zeta + 2 + k^2 \frac{\frac{\gamma}{k}\zeta e^{i\theta} + \zeta - \zeta^2 + L}{\gamma e^{i\theta} + k\zeta + 1} \right\} \right) \in D$$

with $ReL \geq \zeta(\zeta - 1)$, $\zeta \geq 1$ and θ real.

Definition 4. Let N be the set of complex valued functions $k(r, s, t) : \mathbb{C}^3 \rightarrow \mathbb{C}$ such that:

- i) is continuous in a domain $D \subset \mathbb{C}^3$
- ii) $(1, 1, 1) \in D$, $|h(1, 1, 1)| < 1$

iii)

$$\left| h \left(e^{i\theta}, \frac{\beta e^{i\theta} + \alpha\zeta - 1}{\beta - 1}, \frac{1}{(\beta - 2)} \left\{ \beta e^{i\theta} + \alpha\zeta - 2 + \alpha^2 \frac{\frac{\beta}{\alpha} \zeta e^{i\theta} + \zeta - \zeta^2 + L}{\beta e^{i\theta} + \alpha\zeta + 1} \right\} \right) \right| > 1, \quad (10)$$

whenever

$$\left(e^{i\theta}, \frac{\beta e^{i\theta} + \alpha\zeta - 1}{\beta - 1}, \frac{1}{(\beta - 2)} \left\{ \beta e^{i\theta} + \alpha\zeta - 2 + \alpha^2 \frac{\frac{\beta}{\alpha} \zeta e^{i\theta} + \zeta - \zeta^2 + L}{\beta e^{i\theta} + \alpha\zeta + 1} \right\} \right) \in D$$

with $ReL \geq \zeta(\zeta - 1)$, $\zeta \geq 1$ and θ real.

2 Main Results

Unless otherwise maintained we assume that $L \geq \zeta(\zeta - 1)$, $\zeta \geq 1$, $\alpha, k \neq 0$, $\beta \neq 0, 1, 2$, $\gamma \neq -1, -2$ and θ real.

To prove our main results, we need the following lemma.

Lemma 1[6]. *Let $w(z) = a + a_\tau z^\tau + a_{\tau+1} z^{\tau+1} \dots$ be analytic in \mathbb{U} with $w(z) \neq a$ and $\tau \geq 1$. If $z_0 = r_0 e^{i\theta}$ ($0 < r_0 < 1$) and $|w(z_0)| = \max_{|z| \leq |z_0|} |w(z)|$.*

Then

$$z_0 w'(z_0) = \zeta w(z_0) \quad (11)$$

and

$$Re \left(1 + \frac{z_0 w''(z_0)}{w'(z_0)} \right) \geq \zeta, \quad (12)$$

where ζ is a real number and

$$\zeta \geq n \frac{|w(z_0) - a|^2}{|w(z_0)|^2 - |a|^2} \geq \tau \frac{|w(z_0)| - |a|}{|w(z_0)| + |a|}.$$

Theorem 1. *Let $h(r, s, t) \in H$ and $f(z) \in \Sigma_p$ satisfy:*

$$\left(z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z) \right) \in D \subset \mathbb{C}^3 \quad (13)$$

and

$$\left| h \left(z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z) \right) \right| < 1. \quad (14)$$

Then

$$\left| z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z) \right| < 1.$$

Proof. For $f \in \Sigma_p$, let

$$w(z) = z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z),$$

then $w(z)$ is either analytic or meromorphic in \mathbb{U} , $w(0) = 1$ and $w(z) \neq 1$. Using (5) we have

$$z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z) = w(z) + \frac{k}{\gamma} z w'(z)$$

and

$$z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z) = \frac{k^2}{\gamma(\gamma+1)} \left\{ \frac{\gamma(\gamma+1)}{k^2} w(z) + \frac{k+2\gamma+1}{k} z w'(z) + z^2 w''(z) \right\}.$$

Claim that $|w(z)| < 1, z \in \mathbb{U}$. If it is not, then there exists a point $z_0 \in \mathbb{U}$, $\max_{|z| < |z_0|} |w(z)| = |w(z_0)| = 1$. Taking $w(z_0) = e^{i\theta}$ and by Lemma 1 with $a = \tau = 1$, we see that

$$\begin{aligned} z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z_0) &= w(z_0) = e^{i\theta}, \\ z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z_0) &= e^{i\theta} + \frac{k}{\gamma} \zeta e^{i\theta} \end{aligned}$$

and

$$z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z_0) = \frac{k^2}{\gamma(\gamma+1)} \left\{ \frac{\gamma(\gamma+1)}{k^2} e^{i\theta} + \frac{k+2\gamma+1}{k} \zeta e^{i\theta} + L \right\}, L = z_0^2 w''(z_0).$$

Using (12), we have

$$Re \left\{ \frac{z_0 w''(z_0)}{w'(z_0)} \right\} = Re \left\{ \frac{z_0 w''(z_0)}{\zeta e^{i\theta}} \right\} \geq \zeta - 1,$$

that is $Re(e^{-i\theta} L) \geq \zeta(\zeta - 1)$. Since $h(r, s, t) \in H$, we have

$$\begin{aligned} & \left| h \left(z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z_0), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z_0), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z_0) \right) \right| \\ &= \left| h \left(e^{i\theta}, e^{i\theta} + \frac{k}{\gamma} \zeta e^{i\theta}, \frac{k^2}{\gamma(\gamma+1)} \left\{ \frac{\gamma(\gamma+1)}{k^2} e^{i\theta} + \frac{k+2\gamma+1}{k} \zeta e^{i\theta} + L \right\} \right) \right| \geq 1. \end{aligned}$$

This contradicts (14). Therefor, we conclude that $\left| z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z) \right| < 1$.

Using (6) instead of (5), we can prove the following

Theorem 2. Let $h(r, s, t) \in K$ and $f(z) \in \Sigma_p$ satisfy:

$$\left(z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta-1}^{\gamma,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta-2}^{\gamma,k} f(z) \right) \in D \subset \mathbb{C}^3 \quad (15)$$

and

$$\left| h \left(z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta-1}^{\gamma,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta-2}^{\gamma,k} f(z) \right) \right| < 1. \quad (16)$$

Then

$$\left| z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z) \right| < 1.$$

Theorem 3. Let $h(r, s, t) \in M$ and $f(z) \in \Sigma_p$ satisfy:

$$\left(\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)} \right) \in D \subset \mathbb{C}^3 \quad (17)$$

and

$$\left| h \left(\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)} \right) \right| < 1. \quad (18)$$

Then

$$\left| \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)} \right| < 1.$$

Proof. For $f \in \Sigma_p$, let

$$w(z) = \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)},$$

then $w(z)$ is either analytic or meromorphic in \mathbb{U} , $w(0) = 1$ and $w(z) \neq 1$. Using (5) we have

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)} = \frac{1}{\gamma+1} \left\{ \gamma w(z) + k \frac{zw'(z)}{w(z)} + 1 \right\}$$

and

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)} = \frac{1}{(\gamma+2)} \left\{ \gamma w(z) + k \frac{zw'(z)}{w(z)} + 2 + k^2 \frac{\frac{\gamma}{k} zw'(z) + \frac{zw'(z)}{w(z)} + z^2 \frac{w''(z)}{w(z)} - \left(\frac{zw'(z)}{w(z)} \right)^2}{\gamma w(z) + k \frac{zw'(z)}{w(z)} + 1} \right\}.$$

Claim that $|w(z)| < 1, z \in \mathbb{U}$. If it is not, then there exists a point $z_0 \in \mathbb{U}$, $\max_{|z| < |z_0|} |w(z)| = |w(z_0)| = 1$. Taking $w(z_0) = e^{i\theta}$ and by Lemma 1 with $a = \tau = 1$, we see that

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z_0)} = w(z_0) = e^{i\theta},$$

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z_0)} = \frac{1}{\gamma+1} (\gamma e^{i\theta} + k\zeta + 1)$$

and

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z_0)} = \frac{1}{\gamma+2} \left\{ \gamma e^{i\theta} + k\zeta + 2 + k^2 \frac{\frac{\gamma}{k} e^{i\theta} + \zeta - \zeta^2 + L}{\gamma e^{i\theta} + k\zeta + 1} \right\}, L = z_0^2 \frac{w''(z_0)}{w(z_0)}, \zeta > 1.$$

Using (12), we have

$$\Re \left\{ \frac{z_0 w''(z_0)}{w'(z_0)} \right\} = \operatorname{Re} \left\{ \frac{z_0 w''(z_0)}{\zeta e^{i\theta}} \right\} \geq \zeta - 1,$$

that is $\Re L \geq \zeta(\zeta - 1)$. Since $h(r, s, t) \in M$, we have

$$\begin{aligned} & \left| h \left(\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z_0)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z_0)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z_0)} \right) \right| \\ &= \left| h \left(e^{i\theta}, \frac{\gamma e^{i\theta} + k\zeta + 1}{\gamma + 1}, \frac{1}{\gamma + 2} \left\{ \gamma e^{i\theta} + k\zeta + 2 + k^2 \frac{\gamma e^{i\theta} + \zeta - \zeta^2 + L}{\gamma e^{i\theta} + k\zeta + 1} \right\} \right) \right| \geq 1. \end{aligned}$$

This contradicts (18). Therefore, we conclude that $\left| \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)} \right| < 1$.

Using (6) instead of (5), we can prove the following

Theorem 4. Let $h(r, s, t) \in N$ and $f(z) \in \Sigma_p$ satisfy:

$$\left(\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}{\mathcal{H}_{p,\alpha,\beta+1}^{\gamma,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta-1}^{\gamma,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta-2}^{\gamma,k} f(z)}{\mathcal{H}_{p,\alpha,\beta-1}^{\gamma,k} f(z)} \right) \in D \subset \mathbb{C}^3 \tag{19}$$

and

$$\left| h \left(\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}{\mathcal{H}_{p,\alpha,\beta+1}^{\gamma,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta-1}^{\gamma,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta-2}^{\gamma,k} f(z)}{\mathcal{H}_{p,\alpha,\beta-1}^{\gamma,k} f(z)} \right) \right| < 1. \tag{20}$$

Then

$$\left| \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}{\mathcal{H}_{p,\alpha,\beta+1}^{\gamma,k} f(z)} \right| < 1.$$

Remark. Putting $p = 1$ in the above results, we obtain the results in [1].

3 Open Problem

The authors suggest investigating the classes in this article with the recent differential operators in [1, 2].

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