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Certain Sufficient Conditions for Starlike Functions

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Abstract

We here obtain certain sufficient conditions for normalized analytic functions to be starlike. We also find some sandwich-type results ensuring the starlikeness of the normalized analytic functions.

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1 Introduction

Let $\mathcal{H}$ be the class of functions analytic in $\mathbb{E} = \{z : |z| < 1\}$ and $\mathcal{H}[a, n]$ be the subclass of $\mathcal{H}$ consisting functions of the form

$$f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots .$$

Let $\mathcal{A}$ be the subclass of $\mathcal{H}$ consisting functions f, analytic in the open unit disk $\mathbb{E} = \{z : |z| < 1\}$ and normalized by the conditions $f(0) = f'(0) - 1 = 0$. A function $f \in \mathcal{A}$ is said to be starlike of order β, $0 \leq \beta < 1$,if and only if

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \beta,\ z \in \mathbb{E}.$$

The class of such functions is denoted by $\mathcal{S}^*(\beta)$. Note that $\mathcal{S}^*(0) = \mathcal{S}^*$ the class of univalent starlike functions.

Let $\Phi : \mathbb{C}^2 \times \mathbb{E} \to \mathbb{C}$ be an analytic function, p be an analytic function in $\mathbb{E}$ with $(p(z), zp'(z); z) \in \mathbb{C}^2 \times \mathbb{E}$ for all $z \in \mathbb{E}$ and h be univalent in $\mathbb{E}$. Then the function p is said to satisfy first order differential subordination if

$$\Phi(p(z), zp'(z); z) \prec h(z),\quad \Phi(p(0), 0; 0) = h(0). \tag{1}$$

A univalent function q is called a dominant of the differential subordination (1) if $p(0) = q(0)$ and $p \prec q$ for all p satisfying (1). A dominant $\tilde{q}$ that satisfies $\tilde{q} \prec q$ for all dominants q of (1), is said to be the best dominant of (1).

Let $\Psi : \mathbb{C}^2 \times \mathbb{E} \to \mathbb{C}$ be analytic and univalent in domain $\mathbb{C}^2 \times \mathbb{E}$, h be analytic in $\mathbb{E}$, p be analytic and univalent in $\mathbb{E}$, with $(p(z), zp'(z); z) \in \mathbb{C}^2 \times \mathbb{E}$ for all $z \in \mathbb{E}$. Then p is called a solution of the first order differential superordination if

$$h(z) \prec \Psi(p(z), zp'(z); z),\ h(0) = \Psi(p(0), 0; 0). \tag{2}$$

An analytic function q is called a subordinant of the differential superordination (2), if $q \prec p$ for all p satisfying (2). A univalent subordinant $\tilde{q}$ that satisfies $q \prec \tilde{q}$ for all subordinant q of (2), is said to be the best subordinant of (2).

A number of sufficient conditions for $f \in \mathcal{A}$ to be starlike are available in literature on univalent functions. In 1973, Miller, Mocanu and Reade [4] studied the class of $\alpha-$convex functions $f \in \mathcal{A}$ satisfying the differential inequality

$$\Re\left[(1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right)\right] > 0,\ z \in \mathbb{E},$$

where α is any real number and proved that members of this class are starlike in $\mathbb{E}$.

In 1976, Lewandowski et al. [2] proved that the functions $f \in \mathcal{A}$ which satisfy

$$\Re\left[\frac{zf'(z)}{f(z)}\left(1 + \frac{zf''(z)}{f'(z)}\right)\right] > 0,\ z \in \mathbb{E},$$

are starlike in $\mathbb{E}$. In 2001, Padmanabhan [7] proved that for a function $f \in \mathcal{A}$, the differential inequality

$$\alpha\frac{z^2f''(z)}{f'(z)} + \frac{zf'(z)}{f(z)} \prec \frac{1+z}{1-z},\ \alpha \geq 0,\ z \in \mathbb{E},$$

ensures the membership of f in class $\mathcal{S}^*$. For more results we refer to [5, 6, 8, 9, 10, 11, 12].

The main objective of the present paper is to derive certain sufficient conditions for members of the class $\mathcal{A}$ to be starlike. For this purpose, we establish a subordination theorem to get some criteria for starlikeness of $f \in \mathcal{A}$. We also obtain a superordination theorem and consequently get certain sandwich-type results for starlikeness of $f \in \mathcal{A}$.

2 Preliminaries

We shall need the following definition and Lemmas to prove our main results.

Definition 2.1. *[3, Def. 2.2b, p.21]. We denote by Q the set of functions p that are analytic and injective on $\bar{\mathbb{E}} \setminus \mathbb{B}(p)$, where*

$$\mathbb{B}(p) = \left\{\zeta \in \partial\mathbb{E} : \lim_{z\to\zeta} p(z) = \infty\right\},$$

are such that $p'(\zeta) \neq 0$ for $\zeta \in \partial\mathbb{E} \setminus \mathbb{B}(p)$.

Lemma 2.2. *[3, Theorem 3.4h, p.132]. Let q be univalent in $\mathbb{E}$ and let θ and ϕ be analytic in a domain D containing $q(\mathbb{E})$, with $\phi(w) \neq 0$, when $w \in q(\mathbb{E})$. Set $Q_1(z) = zq'(z)\phi[q(z)],\ h(z) = \theta[q(z)] + Q_1(z)$ and suppose that either*
(i) h is convex, or
(ii) Q_1 is starlike.
in addition, assume that
(iii) $\Re\left(\dfrac{zh'(z)}{Q_1(z)}\right) > 0$.
If p is analytic in $\mathbb{E}$, with $p(0)= q(0)$, $p(\mathbb{E}) \subset D$ and

$$\theta[p(z)] + zp'(z)\phi[p(z)] \prec \theta[q(z)] + zq'(z)\phi[q(z)],$$

then $p \prec q$ and q is the best dominant.

Lemma 2.3. *[1]. Let q be univalent in $\mathbb{E}$ and let θ and ϕ be analytic in a domain $\mathbb{D}$ containing $q(\mathbb{E})$. Set $Q_1(z) = zq'(z)\phi[q(z)],\ h(z) = \theta[q(z)] + Q_1(z)$ and suppose that*
(i) Q_1 is starlike in $\mathbb{E}$ and
(ii) $\Re\left[\dfrac{\theta'(q(z))}{\phi(q(z))}\right] > 0,\ z \in \mathbb{E}$.

If $p \in \mathcal{H}[q(0), 1] \cap Q$, with $p(\mathbb{E}) \subset \mathbb{D}$ and $\theta[p(z)] + zp'(z)\phi[p(z)]$ is univalent in $\mathbb{E}$ and

$$\theta[q(z)] + zq'(z)\phi[q(z)] \prec \theta[p(z)] + zp'(z)\phi[p(z)],\ z \in \mathbb{E},$$

then $q(z) \prec p(z)$ and q is the best subordinant.

3 Main Results

In what follows, all the powers taken are the principal ones.

Theorem 3.1. *Let q, $q(z) \neq 0$, be a univalent function in $\mathbb{E}$, satisfying the conditions*

$$\Re\left(1+\frac{zq''(z)}{q'(z)}+(\lambda-1)\frac{zq'(z)}{q(z)}\right)>0$$

and

$$\Re\left(\frac{1-2\alpha}{\alpha}\lambda+(\lambda+1)q(z)\right)>0$$

where α and λ are complex numbers and $\alpha \neq 0$. If $f \in \mathcal{A}$, $\dfrac{zf'(z)}{f(z)} \neq 0$, satisfies the differential subordination

$$\left(\frac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right)\prec(1-2\alpha+\alpha q(z))q^{\lambda}(z)+\alpha zq'(z)q^{\lambda-1}(z) \quad (3)$$

then

$$\frac{zf'(z)}{f(z)}\prec q(z)$$

and q(z) is the best dominant.

Proof: Define the function $p(z)$ by

$$p(z)=\frac{zf'(z)}{f(z)}.$$

Therefore

$$\frac{zp'(z)}{p(z)}=1+\frac{zf''(z)}{f'(z)}-\frac{zf'(z)}{f(z)}$$

and (3) reduces to

$$p^{\lambda}(z)(1-2\alpha+\alpha p(z))+\alpha zp'(z)p^{\lambda-1}(z)\prec q^{\lambda}(z)(1-2\alpha+\alpha q(z))+\alpha zq'(z)q^{\lambda-1}(z) \quad (4)$$

Define θ and ϕ as under:

$\theta(w)=(1-2\alpha+\alpha w)w^{\lambda}$ & $\phi(w)=\alpha w^{\lambda-1}$ where θ is analytic function in $\mathbb{C}$ and ϕ is analytic in $\mathbb{C}\setminus\{0\}$ and $\phi(w)\neq 0,\ w\in\mathbb{C}\setminus\{0\}$. Therefore $Q_1(z)=zq'(z)\phi(q(z))=\alpha zq'(z)q^{\lambda-1}(z)$ and

$$h(z)=\theta(q(z))+Q_1(z)=q^{\lambda}(z)(1-2\alpha+\alpha q(z))+\alpha zq'(z)q^{\lambda-1}(z).$$

A little calculation yields

$$\frac{zQ_1(z)}{Q_1(z)}=1+\frac{zq''(z)}{q'(z)}+(\lambda-1)\frac{zq'(z)}{q(z)}$$

and

$$\frac{zh'(z)}{Q_1(z)} = \frac{(1-2\alpha)}{\alpha}\lambda + (\lambda+1)q(z) + 1 + \frac{zq''(z)}{q'(z)} + (\lambda-1)\frac{zq'(z)}{q(z)}.$$

In view of the given conditions, we have $Q_1(z)$ is starlike in $\mathbb{E}$ and $\Re\left(\frac{zh'(z)}{Q_1(z)}\right) > 0$. The proof now follows from the Lemma 2.2.

Remark 3.2. *It is easy to verify that dominant* $q(z) = \frac{1+(1-2\beta)z}{1-z}$, $0 \leq \beta < 1$, *satisfies the conditions of Theorem 3.1 for* $\lambda = 1$ *and for real number* α, $0 < \alpha \leq 1/2$. *Consequently, we get the following result.*

Corollary 3.3. *If* $f \in \mathcal{A}$, $\frac{zf'(z)}{f(z)} \neq 0$, $z \in \mathbb{E}$ *and for real number* α, $0 < \alpha \leq 1/2$, *satisfies*

$$\frac{zf'(z)}{f(z)}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right) \prec \left(1-2\alpha+\alpha\frac{1+(1-2\beta)z}{1-z}\right)\left(\frac{1+(1-2\beta)z}{1-z}\right)+\frac{2\alpha(1-\beta)z}{(1-z)^2},$$

then

$$\frac{zf'(z)}{f(z)} \prec \frac{1+(1-2\beta)z}{1-z}$$

and hence $f(z) \in \mathcal{S}^*(\beta)$, $0 \leq \beta < 1$.

Remark 3.4. *When we select the dominant* $q(z) = e^z$ *in Theorem 3.1, it satisfies the conditions of Theorem 3.1 for real numbers* α *and* λ *be such that* $0 \leq \lambda < 1$ and $0 < \alpha \leq 1/2$, *we obtain the following result:*

Corollary 3.5. *Let* $f \in \mathcal{A}$, $\frac{zf'(z)}{f(z)} \neq 0$, $z \in \mathbb{E}$, *satisfies*

$$\left(\frac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right) \prec (1-2\alpha+\alpha(e^z+z))\,e^{\lambda z},$$

where α *and* λ *are real numbers be such that* $0 \leq \lambda < 1$ and $0 < \alpha \leq 1/2$, *then*

$$\frac{zf'(z)}{f(z)} \prec e^z,\ z \in \mathbb{E} \ \ \textit{i.e. } f \in \mathcal{S}^*.$$

Remark 3.6. *On selecting the dominant* $q(z) = 1 + az$, $0 < a < 1$, *it is easy to check that this dominant satisfies the conditions given in Theorem 3.1 for* $\lambda = 1$ *and for real number* α, $0 < \alpha \leq 1/2$, *we have the following corollary:*

Corollary 3.7. *For $\alpha \in \mathbb{C}$, $0 < \alpha \leq 1/2$, if $f \in \mathcal{A}$, $\dfrac{zf'(z)}{f(z)} \neq 0$, $z \in \mathbb{E}$, satisfies*

$$\frac{zf'(z)}{f(z)}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right) \prec (1-\alpha)+az(1+\alpha)+\alpha a^2z^2,$$

then

$$\frac{zf'(z)}{f(z)} \prec 1+az,\ 0<a<1,$$

and therefore $f(z)$ is starlike.

Remark 3.8. *For $q(z) = \dfrac{\beta(1-z)}{\beta-z}$, as the dominant in Theorem 3.1, the given conditions are satisfied by this dominant for $\lambda = 1$, α, and β are real numbers such that $0 < \alpha \leq 1/2$ and $\beta > 1$. In view of this remark, we obtain the following result:*

Corollary 3.9. *If $f \in \mathcal{A}$, $\dfrac{zf'(z)}{f(z)} \neq 0$, $z \in \mathbb{E}$, for real numbers α, and β be such that $0 < \alpha \leq 1/2$ and $\beta > 1$, satisfies*

$$\frac{zf'(z)}{f(z)}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right) \prec \left(1-2\alpha+\alpha\frac{\beta(1-z)}{\beta-z}+\frac{\alpha(1-\beta)z}{(\beta-z)(1-z)}\right)\frac{\beta(1-z)}{(\beta-z)},$$

then

$$\frac{zf'(z)}{f(z)} \prec \frac{\beta(1-z)}{\beta-z}$$

and hence $f(z) \in \mathcal{S}^*$.

Theorem 3.10. *Let α, λ are complex numbers with $\alpha \neq 0$, and let q, $q(z) \neq 0$ be univalent function in the unit disc $\mathbb{E}$ and be such that*

$$\Re\left(1+\frac{zq''(z)}{q'(z)}+(\lambda-1)\frac{zq'(z)}{q(z)}\right) > 0$$

and

$$\Re\left(\frac{1-2\alpha}{\alpha}\lambda+(\lambda+1)q(z)\right) > 0$$

If $f \in \mathcal{A}$, $\dfrac{zf'(z)}{f(z)} \in \mathcal{H}[q(0),1] \cap \mathcal{Q}$ with $\left(\dfrac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\dfrac{zf''(z)}{f'(z)}\right)$ is univalent in $\mathbb{E}$, then

$$(1-2\alpha+\alpha q(z))q^{\lambda}(z)+\alpha zq'(z)q^{\lambda-1}(z) \prec \left(\frac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right), \quad (5)$$

then

$$q(z) \prec \frac{zf'(z)}{f(z)}, \; z \in \mathbb{E}.$$

And $q(z)$ is the best subordinant.

Proof: Write $p(z) = \dfrac{zf'(z)}{f(z)}$, then (5) becomes

$$(1-2\alpha+\alpha q(z))q^{\lambda}(z)+\alpha zq'(z)q^{\lambda-1}(z) \prec p^{\lambda}(z)(1-2\alpha+\alpha p(z))+\alpha zp'(z)p^{\lambda-1}(z) \tag{6}$$

By defining θ and ϕ as under:

$\theta(w) = (1-2\alpha+\alpha w)w^{\lambda}$ & $\phi(w) = \alpha w^{\lambda-1}$, where θ is analytic function in $\mathbb{C}$ and ϕ is analytic in $\mathbb{C}\setminus\{0\}$ and $\phi(w) \neq 0, \; w \in \mathbb{C}\setminus\{0\}$. Therefore,

$$Q_1(z) = zq'(z)\phi(q(z)) = \alpha zq'(z)q^{\lambda-1}(z)$$

and observing that

$$\frac{zQ_1(z)}{Q_1(z)} = 1 + \frac{zq''(z)}{q'(z)} + (\lambda-1)\frac{zq'(z)}{q(z)}$$

and

$$\frac{\theta'(q(z))}{\phi(q(z))} = \frac{1-2\alpha}{\alpha}\lambda + (\lambda+1)q(z).$$

In view of the given conditions, $Q_1(z)$ is starlike and $\Re\left[\dfrac{\theta'(q(z))}{\phi(q(z))}\right] > 0$, for $z \in \mathbb{E}$. Therefore, the proof now follows from Lemma (2.3).

4 Sandwich-Type Results

Theorem 4.1. *Let $q_i(z) \neq 0$ $(i = 1,2)$ be univalent in $\mathbb{E}$ and $\lambda, \; \alpha$ are complex numbers where $\alpha \neq 0$. Further assume that*

$$(i)\; \Re\left(1 + \frac{zq_i''(z)}{q_i'(z)} + (\lambda-1)\frac{zq_i'(z)}{q_i(z)}\right) > 0$$

and

$$(ii)\; \Re\left(\frac{1-2\alpha}{\alpha}\lambda + (\lambda+1)q_i(z)\right) > 0, \quad for\ (i = 1,2)$$

If $f \in \mathcal{A}, 0 \neq \dfrac{zf'(z)}{f(z)} \in \mathcal{H}[q(0),1] \cap Q$, and $\left(\dfrac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\dfrac{zf''(z)}{f'(z)}\right)$ is univalent in $\mathbb{E}$, then

$$\left(1-2\alpha+\alpha q_1(z)+\alpha\frac{zq_1'(z)}{q_1(z)}\right)q_1^{\lambda}(z) \prec \left(\frac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right)$$

$$\prec \left(1 - 2\alpha + \alpha q_2(z)) + \alpha \frac{zq_2'(z)}{q_2(z)}\right) q_2^{\lambda}(z), \tag{7}$$

then

$$q_1(z) \prec \frac{zf'(z)}{f(z)} \prec q_2(z).$$

Moreover q_1 and q_2 are the best subordinant and the best dominant respectively.

Taking $q_1(z) = 1 + az$ and $q_2(z) = 1 + bz$, $0 < a < b < 1$. Also for α, $0 < \alpha \leq 1/2$ and $\lambda = 1$ in Theorem 4.1, we conclude the following result:

Corollary 4.2. *For α, $0 < \alpha \leq 1/2$, if $f \in \mathcal{A}$ be such that $\frac{zf'(z)}{f(z)} \in \mathcal{H}[1,1] \cap Q$ with $\frac{zf'(z)}{f(z)}\left(1 - \alpha + \alpha\frac{zf''(z)}{f'(z)}\right)$ is univalent in $\mathbb{E}$ and satisfies*

$$(1 - \alpha + \alpha az)(1 + az) \prec \frac{zf'(z)}{f(z)}\left(1 - \alpha + \alpha\frac{zf''(z)}{f'(z)}\right) \prec (1 - \alpha + \alpha bz)(1 + bz)$$

then

$$1 + az \prec \frac{zf'(z)}{f(z)} \prec 1 + bz,\ z \in \mathbb{E}.$$

where a and b are real numbers such that $0 < a < b < 1$.

Example 4.3. *Taking $b = 1/2$, $a = 1/4$, $\alpha = 1/2$ and f to be same as in Corollary 4.2, we obtain:*

$$\left(1 + \frac{z}{4}\right)^2 \prec \frac{zf'(z)}{f(z)}\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec \left(1 + \frac{z}{2}\right)^2 \tag{8}$$

which implies

$$1 + \frac{z}{4} \prec \frac{zf'(z)}{f(z)} \prec 1 + \frac{z}{2},\ z \in \mathbb{E}. \tag{9}$$

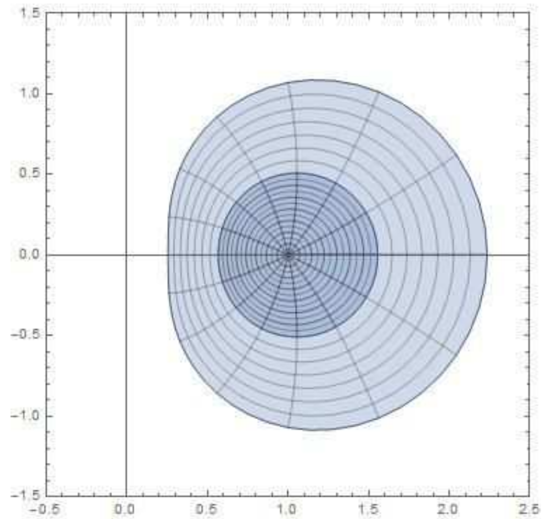

Fig.1

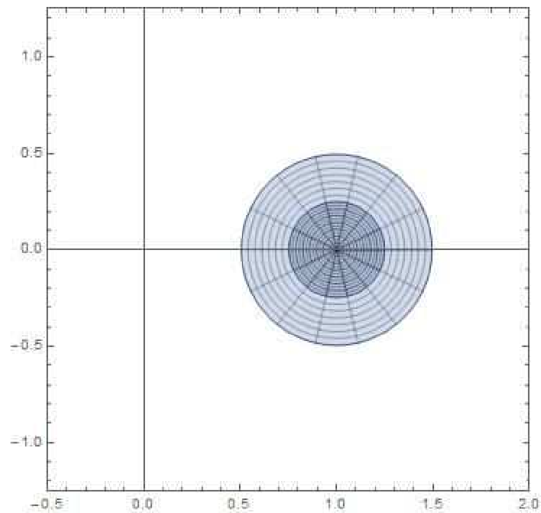

Fig.2

Using Mathematica 10.0, we plot the images of the unit disk under the functions $\left(1+\dfrac{z}{4}\right)^2$ and $\left(1+\dfrac{z}{2}\right)^2$ of (8) in Fig.1 and $\left(1+\dfrac{z}{4}\right)$ and $\left(1+\dfrac{z}{2}\right)$ of (9) in Fig. 2. It follows that if $\dfrac{zf'(z)}{f(z)}\left(1+\dfrac{zf''(z)}{f'(z)}\right)$ takes values in the light shaded portion of Fig. 1, then $\dfrac{zf'(z)}{f(z)}$ will take values in the light shaded portion of Fig. 2. Hence f is starlike in $\mathbb{E}$.

By selecting $q_1(z) = e^{z/2}$ and $q_2(z) = e^z$. And for α and λ, $0 < \alpha \leq 1/2$ *and* $0 \leq \lambda < 1$ in Theorem 4.1, we obtain:

Corollary 4.4. *For real numbers α and λ be such that $0 < \alpha \leq 1/2$ and $0 \leq \lambda < 1$, if $f \in \mathcal{A}$, $\dfrac{zf'(z)}{f(z)} \in \mathcal{H}[1,1] \cap Q$, with $\left(\dfrac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\dfrac{zf''(z)}{f'(z)}\right)$ is univalent in $\mathbb{E}$, and satisfies*

$$\left(1-2\alpha+\alpha e^{z/2}+\frac{\alpha z}{2}\right)e^{\lambda z/2} \prec \left(\frac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right) \prec (1-2\alpha+\alpha e^{z}+\alpha z)e^{\lambda z}$$

then

$$e^{z/2} \prec \frac{zf'(z)}{f(z)} \prec e^{z}.$$

Example 4.5. *By selecting $\alpha = 1/2$, $\lambda = 1/2$ and f as same in the above corollary, we get :*

$$(e^{z/2}+z/2)e^{z/4} \prec \left(\frac{zf'(z)}{f(z)}\right)^{1/2}\left(1+\frac{zf''(z)}{f'(z)}\right) \prec (e^{z}+z)e^{z/2} \tag{10}$$

which implies that

$$e^{z/2} \prec \frac{zf'(z)}{f(z)} \prec e^{z}. \tag{11}$$

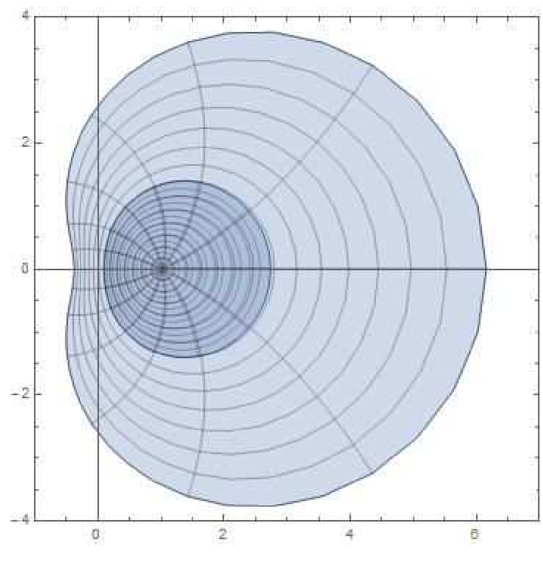

Fig.3

Here we plot, using mathematica 10.0, the functions $(e^{z/2}+z/2)e^{z/4}$ and $(e^{z}+z)e^{z/2}$ of (10) in Fig.3 and functions $e^{z/2}$ and e^{z} of (11) in Fig.4. We observe that when $\left(\dfrac{zf'(z)}{f(z)}\right)^{1/2}\left(1+\dfrac{zf''(z)}{f'(z)}\right)$ takes values in light shaded portion of Fig.3 then $\dfrac{zf'(z)}{f(z)}$ takes values in the light shaded portion of Fig.4. Hence f is starlike in $\mathbb{E}$.

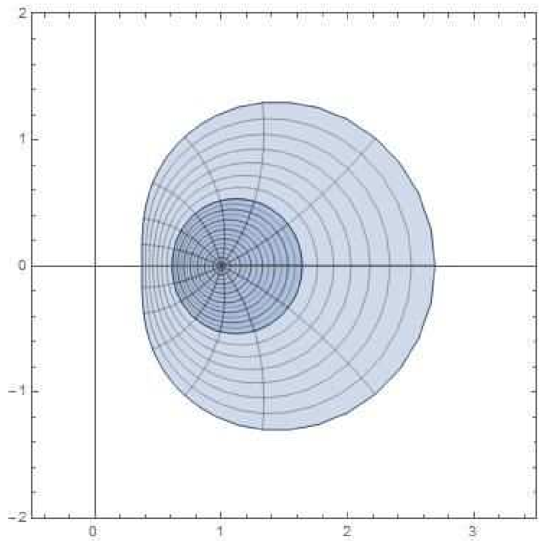

Fig.4

5 Open Problem

In the present paper, we here prove the starlikeness of $f \in \mathcal{A}$ satisfying a differential subordination involving the operator $\left(\frac{zf'(z)}{f(z)}\right)^{\lambda}\left(1-\alpha+\alpha\frac{zf''(z)}{f'(z)}\right)$. The problem of finding the order of starlikeness is yet open for $\lambda \neq 1$.

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