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Multicriteria Optimization Approach of the Electricity Distribution Planning Network Problem

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Abstract

The management of electricity distribution networks raises many problems in Society, mainly due to network expansion, increased consumption of electricity and real-time management. This is due to the continued growth in demand for electricity, posing more challenges for Society. As the strengthening of electricity networks is difficult and expensive simultaneously, it is necessary to choose an optimal process management ensuring customer satisfaction, reducing costs and increasing margins profit. In this paper, we present a multicriteria decision support methods to make appropriate choices in the planning and the operating of such networks without omitting of any of the main objectives.

Keywords: *Planning, Distribution network, Electricity, Multicriteria optimization, Pareto optimality.*
MSC (2000): *90B50, 90B10.*

1 Introduction

For many years, the electric power development in the world has led to a vast system of production, transport and distribution of electricity. This system has been largely conditioned by a very strong constraint: the power is very difficult to stock, it must be sent in real-time from production centers to final consumers, industrial or domestic.

The system includes energy production facilities (nuclear, thermal, hydraulic, or distributed generation: wind, small hydro, cogeneration, etc ...) and consumption areas (municipalities, businesses, etc ...) connected by electrical networks for transportation and distribution. The distribution networks enable to deliver energy to consumption nodes, with steps of elevation and decline in the tension level at the transformer stations. The voltage at the output of large power plants, is transformed to reduce energy loss as heat in the cables. Then, the voltage is gradually reduced to closer to the consumer level, getting the different voltage levels to which consumers are connected.

The energy sector is experiencing unprecedented growth. Customers are increasingly demanding in terms of performance, and new competitors emerging daily. Track this frantic pace is, according to distributors, a genuine challenge and an immense opportunity. Distributors of electricity work to ensure the quality of electricity provision. The first efforts focused on continuity of service to make a permanent available access to energy by the user. Hence the need to focus on studying the problem of transportation and distribution of electric energy, to present methods of decision support enabling the company to make appropriate choices to meet critical needs at management of such networks.

The planning and operating process include all the problems related to managing a grid. Our work concerns this category. In fact, we focus our study to the problem of managing the electricity distribution. This requires great efforts to optimize decisions.

Increasingly, the decision to problems arising from life require consideration of multiple conflicting objectives and sometimes even contradictory. In such cases, there is no single optimum. Solving real world problems has led to the development of multi-criteria optimization (or multiobjective or multilevel), as it best reflects the various conflicting criteria that prohibit an "ideal" solution (optimal for each decision maker under each objective considered separately).

A multicriteria optimization problem is to choose an "optimal solution" among a set of "alternatives" which refers to certain optimality criteria by which solution quality is measured.

The methods of multicriteria analysis or, more accurately, the multicriteria decision techniques are relatively recent and rapidly expanding. By the way they integrate all types of criteria, these procedures seem better able to move toward a sensible compromise rather than optimum often outdated. The peculiarity of such methods leads to the use for the problem of managing the electricity distribution networks, as in this problem we are faced with conflicting objective functions (such as energy loss, sales cost, laborers number and energy production amount) for which determining the optimal configuration is set according to certain criteria.

2 Electric power distribution network model

2.1 Model of an electric line of a grid

In lines (or cables), the equivalent circuit diagram is defined by series connection of resistance and inductive reactance.

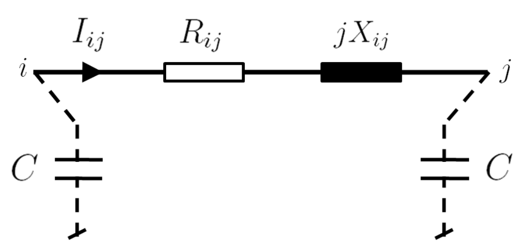

Figure 1: Electrical model of a line

where V_i is the complex voltage at node i, V_j denotes the complex voltage at node j, R_{ij} is the resistance, X_{ij} represents reactance, c is the capacity and I_{ij} is the current passing in class.

By knowing the voltage V_i at node i and current I_{ij}, we calculate the voltage V_j by the exact relationship

$$V_j = V_i - (R_{ij} + jX_{ij})I_{ij}$$

which is illustrated by the complex diagram of Figure 2, where α_{ij} is the phase shift between voltage V_i and voltage V_j, on one hand, and φ_{ij} is the phase shift between voltage V_i and current I_{ij}, on other hand.

For distribution networks, branches are generally short enough so that the phase shift, between the voltages at the extremities, is often negligible (($\alpha_{ij} \approx 0$). In such cases, the argument voltage V_j is almost identical to V_i and its module is written, according to the diagram of Figure 3, as follows

$$V_j = V_i - (R_{ij}cos(\varphi_{ij}) + X_{ij}sin(\varphi_{ij}))Iij$$

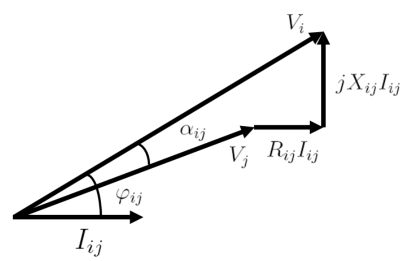

Figure 2: Exact diagram of current and voltage

where $tg(\varphi_{ij}) \approx \dfrac{X_{ij}}{R_{ij}}$ (this relationship can be used without risk of error too great).

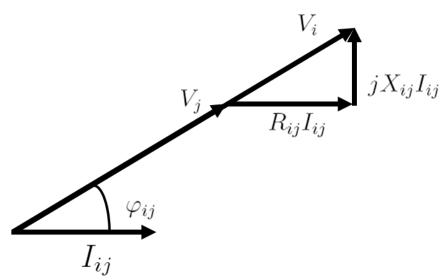

Figure 3: Simplified diagram of current and voltages

Remark 2.1 *In the rest of this paper, the reactance of each edge is omitted because it does not intervene in the current calculations, nor in the evaluation criteria and constraints.*

2.2 Electric power distribution problem formulation

Many formulations of the electricity distribution problem can be found in the literature. However, all these formulations were developed for specific situations that companies had requested for the planning or the operating process of their proper networks (see [2], [3], [7] and [8]). We propose to establish a general formulation of the electricity distribution network by a graph G where only few different characteristics of the network are taken into consideration (consumer demand, the capacity of edges, ...). Thus, the problem is to determine a maximal tree of G optimizing a certain objective function and respecting the capacity constraints of the edges.

Recall that an electrical distribution system can be exploited if the used edges form a maximum tree ([2] and [3]). It may even prove that if network has several sources, one can reduce all sources to one source.

In practice, a distribution network is often supplied by several sources. The operation of such a network requires that each item is supplied by exactly one source at a time. This leads to the exploitation pattern, by the non connectivity of the graph corresponding the distribution network. In such case, this graph is a forest corresponding to union of several arborescent graphs whose number corresponds to the number of source nodes.

The challenge is to determine, for each node, the source having the power to restore the network connectivity distribution. In most cases, this problem can be solved implicitly by reducing the number of sources to a single one by making the following transformation:

1. *Choose any source S from all sources of the network.*

2. *For each source s different from S, introduce the edge $[S, s]$, then transform the source s to a node application $d_s = 0$. (i.e. the source becomes a transition node).*

It may also proceed by adding an extra node that is connected to every source nodes by a branch of fictitious ideal characteristics.

This sources number reduction (as discussed later) can deal problems more effectively. Indeed, for any network having an unique source, operating configuration is a maximum tree (ie containing n nodes and $n-1$ edges, where n is the number of network nodes) which contains only one connected component.

However, there is a model which transforms this particular problem to an allocating resources problem without worrying about sources number.

The goal is to determine whether is a maximum tree to which capacity constraints edges are respected. One formulates this problem as follows.

Let's be given a graph $G(N, E)$, a particular vertex $S \in N$, a positive capacity c_e for every edge $e \in E$ and a positive request d_i for every vertex $i \in N - \{S\}$. N represents the nodes set and E corresponds to the edge set. Is there a maximum tree $A = (N^A, E^A)$ in graph G as for every edge $e \in E^A$, the relation $\sum\limits_{i \in N^{A_e}} d_i \leq c_e$ is satisfied?

Let

$$x_e = \begin{cases} 1, & \text{if } e \in E^A; \\ 0, & \text{else.} \end{cases}$$

To answer the question, it suffices to solve

$$(P)\begin{cases} \sum\limits_{e\in E} x_e = |N| - 1 \\ \\ \sum\limits_{e\in E^H} x_e \le |N^H| - 1,\ \forall H = (N^H, E^H) \subset G \\ \\ \sum\limits_{i\in N^{A_e}} d_i \le c_e, \forall e \in E^A. \end{cases}$$

The first two constraints guarantee that A is a maximum tree.

The constraints number to ensure that A is a maximum tree is equivalent to the number of possible vertices subsets from N. This number is set to $2^{|N|}$ and one finds that it increases exponentially with the network vertices number. Therefore, such a system of inequalities can be exploited to solve problem (P). This shows that the addition of the constraint maximum tree increases the complexity of problem (P) over a traditional waves problem.

In practice, different criteria for defining objective function are of potential interest in busines. Among these optimization criteria may be mentioned:

Thus, following a failure of a branch in a given network, the operators want, in general, restore continuity of service by using a scheme operating relief that is as close as possible to the original scheme. This desire translates into compliance with a limit on the number of laborers allowed.

$$\sum_{e\,\in\,E^A} |x_e - x_e^0| \le \eta_{max}$$

where u_e is the topological state branch e (0 or 1), u_e^0 is the initial topological state branch e and η_{max} is the maximum number of laborers allowed.

Because of security requirements, the state variables, namely the currents in the branches shall not exceed allowable limits, i.e.

$$I_e \le x_e I_{emax}$$

where x_e represents the topological status of the branch e (0 or 1), I_e is the complex current in the branch e and I_{emax} is the maximum allowable current in the branch.

For quality reasons, for example in case regarding the delivered energy, one must guarantee a voltage as close to the nominal voltage at each node of the network. In general, the absolute value of this difference varies between 2% and 7%

$$\frac{|V_{in} - V_i|}{V_{in}} \le \varepsilon_{imax}$$

where V_{in} is the nominal voltage at node x_i, V_i is the modulus of the complex voltage at node x_i and ε_{imax} is the gap maximum allowable voltage.

The goal is to determine an eligible configuration (maximum tree)

$$A = (N, E^A) \quad \left(\rightarrow I_e^A = \sum_{i \in N^{A_c}} d_i, \ \forall e \in E^A \right)$$

such that the objective function f is minimal among all eligible configurations.

2.2.1 Additional constraints

To account the extra constraints indicated in the previous subsection, there are two possible alternatives. The first alternative would be simply to ignore, in the choice of the substitution configuration, any configuration violating one or more neighboring inequality constraints. This means, firstly, it is imperative to start with a feasible configuration tree (except, if after the first stage, one determines a feasible alternative configuration) and secondly, that it must resign itself to eliminate any possibility of transiting from a common configuration violating inequality constraints to a common configuration in which this inequality constraints would be improved if it is not satisfied. It follows that the first alternative could, therefore, apply in case of a network managed under ordinary conditions.

To cope with the above disadvantages, the second alternative suggests treating the inequality constraints requested as additional optimization criteria with weights indicating the importance provided to each one. However, the value corresponding to each of additional criteria would not be different than 0 only if the associate inequality constraint is not respected.

Current:

$$P_C = \begin{cases} \dfrac{|I_e - I_{emax}|}{I_{emax}}, & \text{if } I_e \geq x_e I_{emax} \\ \\ 0, & \text{if } I_e \leq x_e I_{emax}. \end{cases}$$

Voltage:

$$P_V = \begin{cases} \dfrac{1}{\varepsilon_{imax}} \dfrac{|V_{in} - V_i|}{V_{in}}, & \text{if } \dfrac{|V_{in} - V_i|}{V_{in}} \geq \varepsilon_{imax} \\ \\ 0, & \text{if } \dfrac{|V_{in} - V_i|}{V_{in}} \leq \varepsilon_{imax}. \end{cases}$$

Maneuver:

$$P_M = \begin{cases} \dfrac{1}{\eta_{max}} \sum\limits_{e \in E^A} |x_e - x_e^0|, & \text{if } \sum\limits_{e \in E^A} |x_e - x_e^0| \geq \eta_{max} \\ 0, & \text{if } \sum\limits_{e \in E^A} |x_e - x_e^0| \leq \eta_{max}. \end{cases}$$

By defining, for each type of penalty, a weighting factor, the expression of total additional cost to add to the objective function of problem (P) can be expressed as follows

$$P = P^c \sum P_C + P^v \sum P_V + P^m P_M$$

where P^c, P^v and P^m means the weight penalties associated with current, voltage and laborers respectively. Thus, under the mentioned second alternative, methods of resolution for the problem (P) remain valid if one replaces the objective function f by $f + P$. Subsequently, we shall assume implicitly that the expression of f includes that of P.

Finally, we should point out that we could consider other alternatives replacing the second alternative. As an indication, we could multiply the objective function for P instead of adding it to P and or raise the penalties to a different wattage given.

2.2.2 Criteria function

In practice, different criteria for defining the function target are of interest in business. These objectives are vague and sometimes contradictory. Therefore, in order to remedy this, one has to impose the conditions required to cover the electricity production capacity from different sources and so some objectives are translated into constraints. In addition to minimize its profits, a company must take into account the specifications which it operates. These optimization criteria include:

$\rightharpoonup$ *Ohmic loss:*

It causes a heating network branch limiting its transfer ability.

$$f_1(A) = \sum_{e \in E^A} R_e \, (I_e^A)^2$$

where R_e is the resistance of the arc e, while I_e^A is the electric current flowing through the arc e.

$\rightharpoonup$ *Transactions number:*

This criterion has to be considered if, by following a disturbance of the initial network topology (failure of a branch, branch overload, ...), we desired to determine a relief topology, whose realization will require a certain number of operations to transform the initial configuration A_i to another configuration A. In this case, the function f is

$$f_2(A) = \sum_{e \in E^A} |x_e - x_e^0|$$

where x_e^0 is the state of branch e in the configuration initial A_i.

Also, one can be added to the latest formulation a cost α_e and our function will be rewritten as

$$f_2(A) = \sum_{e \in E^A} \alpha_e |x_e - x_e^0|$$

$\rightharpoonup$ *Configuration permissible operating :*

Determination of an eligible operating configuration can be achieved by minimizing an heuristic function

$$f_3(A) = \sum_{e \in E^A} \left(\frac{I_e^A}{c_e}\right)^k$$

where c_e is the maximum capacity of the arc e.

Recall that the addressed problem is to determine a maximum tree of G satisfying all capacity constraints. The first intuitive idea is to minimize the number of overloaded arcs. For every maximum tree A, the cost would equal the number of arcs for which there is a breach of the capacity constraint. In fact, one needs that for every arc e of E^A, the ratio $\frac{I_e^A}{c_e}$ is less than 1. Now, one knows that any number less than (resp. greater than) a high power k, is lower (resp. higher) than 1 and tends to 0 (resp. ∞) when k tends to ∞. So one uses this property to define the objective function f for any configuration tree A as follows:

$$f_3(A) = \sum_{e \in O^A} \frac{I_e^A}{c_e} + \sum_{e \in E^A} \left(\frac{I_e^A}{c_e}\right)^k$$

where $O^A = \{e \in E^A, I_e^A > c_e\}$ denotes the overloaded arcs set when $k \geq 1$ is a given parameter set.

In order for a solution A to be eligible for the problem (P), it is necessary and sufficient that $O^A \neq \emptyset$. The goal is to determine an eligible tree for problem (P) and the process may stop when a solution satisfying this criterion is reached.

The objective function can be decomposed as follows:

$$f_3(A) = g(A) + h(A)$$

where

$$g(A) = \sum_{e \in O^A} \frac{I_e^A}{c_e}$$

$$h(A) = \sum_{e \in E^A} \left(\frac{I_e^A}{c_e}\right)^k$$

Thus, A is an admissible solution of problem (P) if and only if $g(A) = 0$. This implies that the process of finding a feasible solution stops when $f_3(A) = h(A)$.

$\rightharpoonup$ *Distribution cost:*

The electrical energy distribution, from the source to consumers, demands increasingly financial resources important in order to meet the Secondary cost request:

$$f_4(A) = \sum_{e \in E^A} K_e x_e$$

where K_e is the estimated Secondary cost of the distribution from source to consumer.

3 Multicriteria optimization formalization of the electrical energy distribution problem

3.1 Mathematical model in multicriteria optimization

Recall that the general mathematical formulation of a multicriteria optimization problem can be specified by

$$(Pmc) \qquad \begin{cases} min\ F(x) = \{f_1(x), .., f_k(x)\} \\ \text{s.t} \qquad x \in X \end{cases}$$

where $F(x)$ is the objective functions set and X denotes the constraints set given by $X = \{x \in I\!R^n / g_i(x) \leq 0, i = 1, .., m\}$. If constraints and objectives functions are linear, it corresponds to a multicriteria linear programming problem which is simply represented by

$$\begin{cases} min\ Cx \\ \text{s.t} \qquad x \in X \end{cases}$$

where C is a matrix $n \times k$ and $X = \{x \in I\!R^n / Ax \leq b, x \geq 0\}$ with $A = (a_{ij})_{i=1,n,\, j=1,k}$ a matrix $n \times m$ and $b = (b_i)_{i=1,m}$ column vector of $I\!R_+^m$ while n, k and m are integers. To define the meaning of min, one needs to define how the objective functions $F(x)$ must be compared to other alternatives for $x \in X$ because, for $k \geq 2$, there is no canonical order from $I\!R^k$ to $I\!R$. In general, there is no solution which simultaneously optimizes all objective functions. For $x \in X$, it seems natural to impose that there is no other solution $y \in X$ that provides values as good as any x. The solutions x are called effective solutions.

3.2 Multicriteria optimization formalization of the electrical energy distribution problem

As a consequence, the objective function of the multicriteria planning problem for electric energy distribution is given by

$$min\, F(A) = \{f_1(A), f_2(A), f_3(A), f_4(A)\}$$

where

$$f_1(A) = \sum_{e \in E^A} R_e\, (I_e^A)^2,$$

$$f_2(A) = \sum_{e \in E^A} |x_e - x_e^0|,$$

$$f_3(A) = \sum_{e \in E^A} \left(\frac{I_e^A}{c_e}\right)^k,$$

$$f_4(A) = \sum_{e \in E^A} K_e x_e.$$

On other hand, the associated constraints with the problem can be grouped as follows:

Maximum tree

$$\sum_{e \in E} x_e = |N| - 1$$

$$\sum_{e \in E^H} x_e \leq |N^H| - 1,\ \forall H = (N^H, E^H) \subset G$$

Capacity

$$\sum_{i \in V^{T_e}} d_i \leq c_e, \forall e \in E^T$$

Maneuver

$$\sum_{e \in E^A} |x_e - x_e^0| \leq \eta_{max}$$

Current

$$I_e \leq x_e I_{emax}$$

Voltage

$$\frac{|V_{in} - V_i|}{V_{in}} \leq \varepsilon_{imax}$$

3.3 Method for solving multicriteria optimization problem

Under the criteria optimization, most often the decision maker rather argues in terms of evaluating a solution on each criterion in the criteria space (or objective space) and $y = (y_1, .., y_k)$ where $y_i = f_i(x)$ is a point in criteria space. It imposes a partial order relation on this set of points, called *dominance relation*.

Definition 3.1 *A solution $y = (y_1, .., y_k)$ dominates a solution $z = (z_1, .., z_k)$ if and only if $\forall i \in [1..k]$ one has $y_i \leq z_i$ and $\exists i \in [1..k]$ such that $y_i < z_i$.*

One rarely has a vector x^* which is optimum for all objectives

$$\forall x \in X, \ f_i(x^*) \leq f_i(x), \ i \in [1..k].$$

Since a such situation rarely happens for real world problems where the criteria are in conflict, other concepts have been established to consider an optimal solution. Mostly, one uses the notion of *Pareto optimality*.

Definition 3.2 *A solution $x^* \in X$ is Pareto optimal if and only if there is not a solution $x \in X$ such that $F(x)$ dominates $F(x^*)$.*

The Pareto optimal solution definition follows directly from dominance concept. It means that it is impossible to find a solution improving performance on one criterion without causing a performance degradation for at least one other criterion ([1] and [4]).

Pareto optimal solutions are also known as *admissible solutions*, *effective*, *non-dominated* or *not inferior*.

Some Pareto optimal solutions can be obtained by solving the following mathematical program

$$(Pmc_\lambda) \qquad \begin{cases} min\ F(x) = \sum_{i=1}^{k} \lambda_i f_i(x) \\ \text{s.t} \qquad x \in X \end{cases}$$

where $\lambda_i \geq 0$ for $i = 1, ..., k$ and $\sum_{i=1}^{k} \lambda_i = 1$. All these solutions, also called supported solutions, can be generated by resolution of (Pmc_λ) for different values of weight vector λ.

Definition 3.3 *The ideal vector $y^* = (y_1^*, .., y_k^*)$ is the vector optimizing each objective functions f_i, i.e. $y^* = min(f_i(x))$ for $x \in X$.*

To generate Pareto optimal solutions, one uses the aggregation method (considered as one of the best methods used to solve this class of problem). It consists to transform problem (Pmc) into another problem (Pmc_λ) which combines the objective functions f_i of the problem in a single function F which, generally, can be written as follows

$$F(x) = \sum_{i=1}^{k} \lambda_i f_i(x)$$

where $\lambda_i \geq 0$ for $i = 1, ..., k$ and $\sum_{i=1}^{k} \lambda_i = 1$. Different weights provide different supported solutions .

The obtained results by solving problem (Pmc_λ) depend strongly on parameters chosen for the vector of weight λ. The weight λ_i must also be selected based on preferences associated with objectives. This is a delicate task. Thus an approach, commonly used, is to solve problem (Pmc_λ) with different values of λ.

If the different criteria are not commensurable, as is the case for electricity distribution planning problem, where there are several objectives treated without the same nature (which is observed in our model in the subsection 3.2), one can transform the above equation as

$$F(x) = \sum_{i=1}^{k} c_i \lambda_i f_i(x)$$

where c_i represent constants that are the same across the different objectives. The constants c_i are usually initialized to $\frac{1}{f_i(x^*)}$ where $f_i(x^*)$ corresponds to the solution optimal objective function associated with f_i

$$(P_i) \qquad \begin{cases} min\ f_i(x) \\ \text{s.t} \qquad x \in X \end{cases}$$

In this case, the vector is normalized with respect to the ideal vector.

Remark *Distribution models of electrical energy, used in ([2], [3], [7] and [8]), can be improved by applying multicriteria optimization and by transforming the multicriteria function into a single criterion, instead of stacking the different objective functions because there are not similar. In practice, the weights are not identical since policymakers have, generally, several needs and multiple choices to consider.*

4 Conclusion

In this paper, we present a method for decision support enabling the company to make appropriate choices in managing the electricity distribution networks. This Management raises many challenges to society, mainly caused by the combinatorial nature of the problem and the explosive size of the associated potential solutions (or configurations). A multicriteria formulation is proposed to solve electricity distribution planning problem, by considering objectives considered by the company (other objectives can be treated). To study this multicriteria problem, one uses a transforming method of multicriteria problem into a problem with a single criterion since in such a case there are several methods guarantying the existence of an optimal solution ([2], [3], [6], [7], [8] and [11]) (which is, at least, a local solution). This transformation requires a priori knowledge of problem. The optimization problem to a single criterion can not guarantee Pareto optimality for the solution based and hence the need for several alternatives. Indeed, one must know the entire electricity grid and all its features. If certain targets are noisy or uncertain data, such methods are not effective. Another disadvantage of such methods is their sensitivity to weight constraints and/ or target levels for each goal.

5 Some Open Problems

It would be interesting to suggest a sequel to this work. Indeed, several elements could also become subject of extensive research. In this regard,

- We have studied the power distribution problem in general. Other cases (according to needs and goals of distribution company and countries) can be considered and studied in a similar manner.

- It can be treated, equivalently, many transportation problems encountered or distribution in society namely the transport and distribution of gas, water and oil or even road networks problem.

- In case of the existence of a configuration, it is interesting to find a configuration that minimizes some criteria in order to increase system performance. For example, when it is not possible to satisfy all customers, declines in demand for certain customers should be operated in order to exploit the network and to maximize the overall satisfaction of customers.

References

[1] M. Basseur, E. Talbi, A. Nebro, E. Àlba, Metaheuristics for multiobjetctive combinatorial optimization problems : Review and recent issues, *INRIA, Rapport de recherche N. 5978, September 2006.*

[2] A. Benchakroun, Une approche constructive pour la planification des réseaux de distribution d'énergie électrique, *Investigacion Operativa, Vol. 2, No. 1, pp. 5-22, 1990.*

[3] S.R. Cherkaoui, C. Rossier, M. Stalder, Renforcement et reconfiguration optimaux d'un réseau électrique de distribution, *NEFF 405, rapport 405/1, EPFL, 1989.*

[4] Y. Collette, P. Siarry, Multiobjective optimization : Principles and case studies, *Springer-Verlag, Berlin*, 2004.

[5] N. Dubois, Méthodes d'optimisation combinatoire pour des problèmes de graphes, *Ph.D Thesis, EPFL, Switzerland*, 1994.

[6] K. El Yassini, R. Zine, M. Raïssouli, Outils d'aide à la décision pour la planification des réseaux de distribution de l'énergie électrique, *ARIMA, Vol. 13, pp. 105 - 118*, 2010.

[7] E. Kallel, Planification des réseaux de distribution d'énergie électrique avec demande incertaine, *Ph.D Thesis, University of Sherbrooke, Canada*, 2002.

[8] Y. Meynard, Heuristique pour la planification des réseaux de distribution d'énergie électrique, *Ph.D Thesis, University of Montreal, Canada*, 1995.

[9] C.H. Papadimitriou, K. Steiglitz, Combinatorial optimization: algorithms and complexity, *Englewoods Cliffs, Prentice Hall*, 1982.

[10] Roseaux, Exercices et problèmes résolus de recherche opérationnelle, 1. Graphes : leurs usages, leurs algorithmes, *Dunod, Paris*, 1998.

[11] D.L. Wall, G.L. Thompson, J.E.D. NorthcoteGreen, An optimization model for planning radial distribution networks, *IEEE Transactions on Power Apparatus and Systems PAS-98, pp. 1061-1067*, 1979.