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Tail Behaviour of the Random Products of Independent Regularly Varying Random Variable Glucocorticoids

S. Lakshmi[*] and S. Pamela[**]

* Reader and Head, Dept. of Mathematics, Kundavai Nachiaar Govt. Arts College for Women, Thanjavur, Tamilnadu,

E mail: lakshmi291082@yahoo.co.in

** Principal, Thiru. Vi. Ka Govt. Arts College, Thiruvarur, Tamilnadu, India.

Abstract

Human stress effects are cumulative. They are additive in nature. It affects all of us either directly or indirectly in day-today life. In this paper, stress effects are measured in terms of glucocorticoids(even though they are multivariate) particularly ACTH and Corticosterone. To find the long-term effect, the formula for finding the tail behaviour of the corresponding distributions with random products is developed.

***Keywords*:** Glucocorticoid, ACTH, Random product.

1. Introduction:

Generally human stress effects are cumulative [1]. It affects all of us either directly or indirectly. They are additive in nature [4]. Here we have measured the stress effects interms of glucocorticoids. It is tested for rats by giving severe emotional stressor. Current study about adaptation to repeated stress assume that it follows the rules of habituation[2], which predict a negative relationship between the degree of adaptation and both the intensity of the stressor and the interval between successive exposures. Here we have assumed that this successive exposures following a Poisson distribution with means υ as a random variable of time intervals.

2. Application:

However, we assumed that we are dealing with a special type of learning- like process linked to severe stressful situations [8]. In order to better characterize the phenomenon, we focused on the possible mechanisms involved in this induction. Taking into account that glucocorticoids are able to positively modulate memory about emotional events [1,8], the stress induced release of glucocorticoid may well be involved in the long term effects of a single exposure to IMO(severe emotional stressor immobilization) on the HPA axis[6,7].

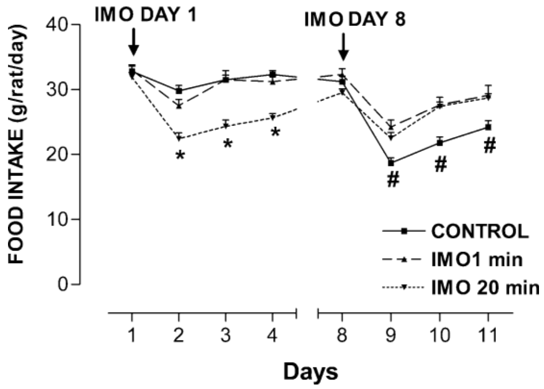

Figure 2.1

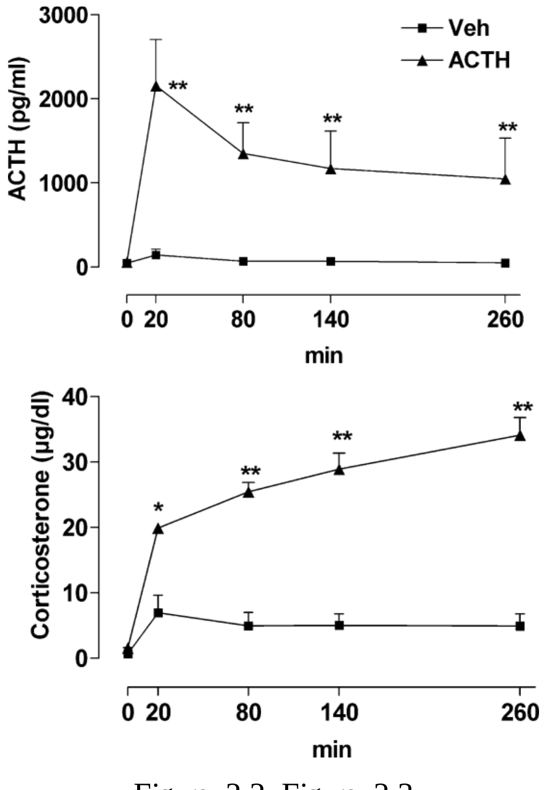

Figure 2.2, Figure 2.3

These data suggest that glucocorticoids (here ACTH and Corticosterone) are involved in the induction of the long-term effects of a single exposure to IMO [10]. To find the long-term effects, we have tested the tail behaviour of the corresponding distributions with random products of regularly varying random variables glucocorticoids $\{X_i\}$ in [3,9].

3. Mathematical Model:

Random products of independent regularly varying random variable. Here we study the tail behaviour of the random product

$$\Pi_\upsilon = \prod_{i=1}^{M} X_i \ ------- (3.1)$$

Where $\{X_i\}$ is an i.i.d sequence of non-negative regularly varying random variables independent of the Poisson random variable with mean υ.

Assume that X has the Pareto distribution with parameter $\alpha > 0$ and that M is Poisson distributed with mean $\upsilon > 0$.

$$P\left(\prod_{\upsilon} > x\right) = P\left(\sum_{i=1}^{M} logX_i > logx\right) ----- (3.2)$$

$$P(\propto^{-1} \Gamma_M > logx)$$

Where M is independent of the sequence (Γ_i). Thus we will study the tail of the compound Poisson sum $\frac{\Gamma_M}{\alpha}$ with i.i.d exponential sum and Y_i with mean α^{-1}.

$$\Phi_y(\theta) = (1 - \alpha^{-1}\theta)^{-1} ----------- (3.3)$$

Hence saddle point equation

$$\upsilon\, \Phi_y'(\theta) = y ------------- (3.4)$$

is given by

$$\upsilon \propto^{-1} \left(1 - \frac{\theta}{\alpha}\right)^{-2} = y$$

and has solution $\left(1 - \frac{\theta}{\alpha}\right)^{-2} = \frac{y\propto}{\upsilon}$

$$\frac{\theta}{\alpha} = \left[1 - \left(\frac{y \propto}{\upsilon}\right)^{-1/2}\right]$$

$$\theta = \theta(y) = \alpha\left[1 - \left(\frac{y \propto}{\upsilon}\right)^{-1/2}\right] ----- (3.5)$$

Consider the compound Poisson sum

$$S = \sum_{i=1}^{M} Y_i$$

Where $\{Y_i\}$ is an i.i.d. sequence of random variables independent of the Poisson random variable with mean $\upsilon > 0$.

$$\Phi_y(h) = E\ e^{hy},$$

$$\Phi_s(h) = E\ e^{hs} = e^{-\upsilon}(1 - \Phi_Y(hy)) - - - - (3.6)$$

$$\sigma_s^2(h) = \upsilon \Phi_Y''(h)$$

Here we assume that Y has a Gamma like density if Y has a Lebesque density f_Y with the property that there exist positive constants x_0, α, β and a slowly varying function l, s, t.

$$f_Y(x) = x^{\beta-1} l(x) e^{-\alpha x}, \qquad x \geq x_0$$

$$\Phi_s(\theta) = e^{-\upsilon}\left[1 - \left(\frac{y \propto}{\upsilon}\right)^{-1/2}\right]$$

$$\sigma_s^2(\theta) = 2\left(\frac{y^3}{\upsilon\alpha}\right)^{1/2}$$

$$P(\pi_\upsilon > x) \sim \frac{e^{-\propto y}\left[1 - \left(\frac{y \propto}{\upsilon}\right)^{-1/2}\right]\ e^{-\upsilon}\left[1 - \left(\frac{y \propto}{\upsilon}\right)^{1/2}\right]}{\propto\left[1 - \left(\frac{y \propto}{\upsilon}\right)^{-1/2}\right]\sqrt{2\left(\frac{y^3}{\upsilon \propto}\right)^{1/2}}\sqrt{2\pi}}$$

$$\sim e^{-\upsilon}\left(\frac{\upsilon}{\propto^3}\right)^{1/4}\frac{1}{2\sqrt{\pi}} e^{-\alpha y + 2\sqrt{\alpha \upsilon y}}\, y^{-3/4} - - - - - - - - - (3.7)$$

This is tail of the product Π_υ given in (3.1).

4. Results:

If Y has a Gamma like density and if we take the slowly varying function *l(x)* as food intake, from fig(2.1), fig(2.2) and fig(2.3).

Table : 1

	α	β
Vehicle	88.2	210
ACTH	2.2484	1.9723
Vehicle	19.0125	24.375

Corticosterone	4.4201	2.0091

$$f_Y(x) = x^{\beta-1} l(x) e^{-\alpha x}$$

5. Conclusion:

Human stress effects are cumulative. They are additive in nature. It affects all of us either directly or indirectly in day-today life. In this paper, stress effects are measured in terms of glucocorticoids, particularly ACTH and Corticosterone. Glucocorticoids are involved in the induction of the long-term effects of a single exposure to IMO. To find the long-term effects, the formula for finding the tail behaviour of the corresponding distribution with random products is developed. Food intake at the time of IMO stressor is taken as a slowly varying function and the corresponding results are obtained.

6. Open Problem:

Different mathematical models have been developed for the function of machines and corresponding man machine models in the engineering field. But corresponding ideas are not yet fully used in the development of medical situations. Every variable in medical science follows some rules of mathematics. So there is a dare need of developing mathematical models in medical science. The above model is one of the example in this direction.

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