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A General Class of Polynomials Involving a Generalized Mittag-Leffler Function

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Abstract

This paper is devoted for the study of a general class of polynomials defined as $A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)=\frac{x^{-\sigma-an}}{n!}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}\theta^n\left[x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}\right]$***, where*** $\theta\equiv x^a\left(s+xD\right)$ ***is the differential operator and*** $E_{\alpha,\beta,p}^{\gamma,\delta,q}(z)$ ***is the generalized Mittag-Leffler type function defined as***

$$E_{\alpha,\beta,p}^{\gamma,\delta,q}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)}\frac{z^n}{(\delta)_{pn}}.$$

Several families of generating relations and finite summation formulae for the general class of polynomials have been stated and proved. Also several special cases have been discussed.

Keywords: Mittag-Leffler function, general class of polynomials, generating relations, finite summation formulae.

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1. Introduction

In 1903, the Swidish mathematician Mittag–Leffler [2] introduced the function $E_{\alpha}(z)$ defined as

$$E_{\alpha}(z)=\sum_{n=0}^{\infty}\frac{z^{n}}{\Gamma(\alpha n+1)} \tag{1.1}$$

where z is a complex variable and $\Gamma(s)$ is the Gamma function; $\alpha \geq 0$.

Generalizations of Mittag-Leffler type function occurred in the last century and recently like $E_{\alpha,\beta}(z)$ defined by Wiman [12], $E^{\gamma}_{\alpha,\beta}(z)$ defined by Prabhakar [3], $E^{\gamma,q}_{\alpha,\beta}(z)$ defined by Shukla and Prajapati [7] and $E^{\gamma,\delta}_{\alpha,\beta}(z)$ defined by Salim [4] , where

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^{n}}{\Gamma(\alpha n+\beta)} \tag{1.2}$$

$$E^{\gamma}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)}\frac{z^{n}}{n!} \tag{1.3}$$

$$E^{\gamma,q}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)}\frac{z^{n}}{n!} \tag{1.4}$$

and

$$E^{\gamma,\delta}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)}\frac{z^{n}}{(\delta)_n} \tag{1.5}$$

Recently a new generalization of Mittag-Leffler function investigated by Salim and Faraj [5] which is defined as

$$E^{\gamma,\delta,q}_{\alpha,\beta,p}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)}\frac{z^{n}}{(\delta)_{pn}} \quad , \tag{1.6}$$

where

$$z,\alpha,\beta,\gamma,\delta\in C\ ;\ \min\left\{\mathrm{Re}\,(\alpha),\mathrm{Re}\,(\beta),\mathrm{Re}\,(\gamma),\mathrm{Re}\,(\delta)\right\}>0\ \mathit{and}\ \ p,q>0 \tag{1.7}$$

In this paper, a general class of polynomials associated with generalized Mittag – Leffler function defined in (1.6) is introduced as

$$A^{(\alpha,\beta,\gamma,\delta,\sigma)}_{qn,pn}(x;a,k,s)=\frac{x^{-\sigma-an}}{n!}E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{P_k(x)\right\}\theta^{n}\left[x^{\sigma}E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{-P_k(x)\right\}\right] \quad , \tag{1.8}$$

where $\theta\equiv x^{a}\left(s+xD\right)$ and $\theta^{n}\equiv x^{an}\prod_{j=0}^{n-1}\left(xD+s+ja\right)$.

We have also derived several families of generating relations and finite summation formulae by employing operational techniques. At the end, several special cases have been obtained.

Throughout this paper , we need the following well-known facts and rules.

- $\theta_1^{n} \equiv x^{an} \prod_{j=0}^{n-1}\left(xD+1+ja\right)$ (1.9)

- $\theta^{n}(x\,u\,v) = x \sum_{m=0}^{\infty}\binom{n}{m}\theta^{n-m}(v)\,\theta_1^{m}(u)$ (1.10)

- $e^{t\theta}\left(x^{\sigma}f(x)\right) = \dfrac{x^{\sigma}}{\left(1-tax^{a}\right)^{\left(\frac{\sigma+s}{a}\right)}} f\left(\dfrac{x}{\left(1-tax^{a}\right)^{\frac{1}{a}}}\right)$ (1.11)

- $e^{t\theta}\left(x^{\sigma}\right) = x^{\sigma}\left(1-tax^{a}\right)^{-\left(\frac{\sigma+s}{a}\right)}$ (1.12)

- $e^{t\theta}\left(x^{\sigma-n}f(x)\right) = x^{\sigma}\left(1+at\right)^{-1+\left(\frac{\sigma+s}{a}\right)} f\left(x\left(1+at\right)^{\frac{1}{a}}\right)$ (1.13)

- $\left(1-at\right)^{\frac{-\sigma}{a}} = \left(1-at\right)^{\frac{-\beta}{a}} \sum_{m=0}^{\infty}\left(\dfrac{\sigma-\beta}{a}\right)_m \dfrac{(at)^m}{m!}$ (1.14)

- The Stirling number of second kind [9]

$$S(n,k) = \frac{1}{k!}\sum_{j=1}^{\infty}(-1)^{k-j}\binom{k}{j} j^{n} \tag{1.15}$$

so that $$S(n,0) = \begin{cases} 1 & n=0 \\ 0 & n\in N \end{cases} \tag{1.16}$$

and $$S(n,1) = S(n,n) = 1,\quad S(n,n-1) = \binom{n}{2} \tag{1.17}$$

- Srivastava [9] proved that:

Let the sequence $\left\{\varepsilon_{n+k}(x)\right\}_{n=0}^{\infty}$ be generated by

$$\sum_{k=0}^{\infty}\binom{n+k}{k}\varepsilon_{n+k}(x)t^{k} = f(x,t)\left\{g(x,t)\right\}^{-n}\varepsilon_n\left\{h(x,t)\right\}\ , \tag{1.18}$$

where f , g and h are functions of x and t, then in terms of Stirling number $S(n,k)$ defined on (1.15) the following family of generating function

$$\sum_{k=0}^{\infty}k^{n}\varepsilon_{n+k}\left(h(x,-z)\right)\left(\frac{z}{g(x,-z)}\right)^{k} = \left\{f(x,-z)\right\}^{-1}\sum_{k=0}^{\infty}k!\,S(n,k)\varepsilon_k(x)\,.\,z^{k} \tag{1.19}$$

holds provided that each number of equation (1.26) exists.

2. Some Special Cases

By using general class of polynomials defined by (1.8), we can deduce special cases as follows:

- Putting $\delta=1$, $p=1$ in (1.8), then it reduces to

$$A_{qn}^{(\alpha,\beta,\gamma,\sigma)}(x;a,k,s)=\frac{x^{-\sigma-an}}{n!}E_{\alpha,\beta}^{\gamma,q}\{P_k(x)\}\,\theta^n\left[x^{\sigma}E_{\alpha,\beta}^{\gamma,q}\{-P_k(x)\}\right], \tag{2.1}$$

which is the general class of polynomial defined by Shukla & Prajapati [6]

- Setting $\alpha=\beta=\gamma=q=1$, $s=0$ and replacing σ by α, a by k, k by r and $p_k(x)=px^r$ in (2.1), we get

$$A_n^{(1,1,1,\alpha)}(x;k,r,0)=G_n^{(\alpha)}(x;r,p,k) \tag{2.2}$$

where $G_n^{(\alpha)}(x;r,p,k)$ is the general class of polynomials studied by Srivastava and Singhal [11]

- Setting $\alpha=\beta=\gamma=q=1$ and replacing σ by α in (2.1) gives

$$A_n^{(1,1,1,\alpha)}(x;a,k,s)=x^{-an}V_n^{(\alpha)}(x;a,k,s) \tag{2.3}$$

where $V_n^{(\alpha)}(x;a,k,s)$ is the general sequence of function introduced by Srivastava and Singh [10]

- Also by putting $s=0$, $a=\alpha=\beta=\gamma=q=1$, replacing σ by a and $p_k(x)=px^r$ in (2.1), we get

$$A_n^{(1,1,1,\alpha)}(x;-1,r,0)=\frac{(-x)^n}{n!}J_n^{(-1)}(x;r,p,0)\ , \tag{2.4}$$

where $J_n^{(-1)}(x;r,p,0)$ is a particular case of the general class of polynomials studied by Singhal and Joshi [8]

- Setting $\alpha=\beta=\gamma=q=1$, replacing σ by α, $a=k$, $s=\eta$, $p_k(x)=\beta x^r$ in (2.1) gives

$$\begin{aligned}A_n^{(1,1,1,\alpha)}(x;k,r,\eta)&=\frac{x^{-\alpha-kn}}{n!}\exp\left(\beta x^r\right)\left[x^k\left(\eta+xD\right)\right]^n\left[x^{\alpha}\exp\left(-\beta x^r\right)\right]\\&=\tau_n^{(\alpha)}(x;r,k,\eta) \qquad\qquad ,n\in N_0\end{aligned} \tag{2.5}$$

where $\tau_n^{(\alpha)}(x;r,k,\eta)$ is the class of polynomials studied by Chen et al. [1].

3. Generating Relations

A considerably large number of special functions are known to possess generating relations, we use $\theta \equiv x^{a}\left(s+xD\right)$ and $\theta_1 \equiv x^{a}\left(1+xD\right)$, $D=\dfrac{d}{dx}$ to obtain such generating relations in the following formulae.

(I)

$$\sum_{n=0}^{\infty} A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)t^{n} = \left(1-at\right)^{-\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\left(1-at\right)\right)^{\frac{-1}{a}}\right\} \quad (3.1)$$

Proof: From (1.8), we have

$$\sum_{n=0}^{\infty} x^{an} A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)t^{n} = \sum_{n=0}^{\infty} x^{an} \frac{x^{-\sigma-an}}{n!} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}\theta^{n}\left[x^{\sigma} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}\right]t^{n}$$

$$= x^{-\sigma} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\} e^{t\theta}\left[x^{\sigma} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}\right]$$

$$= x^{-\sigma} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\} x^{\sigma}\left(1-ax^{a}t\right)^{-\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\left(1-tax^{a}\right)^{\frac{-1}{a}}\right)\right\}$$

and replacing t by tx^{-a} yields ,

$$\sum_{n=0}^{\infty} x^{an} A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)(tx^{-a})^{n}$$

$$= x^{-\sigma} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\} x^{\sigma}\left(1-at\right)^{-\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\left(1-at\right)^{\frac{-1}{a}}\right)\right\}.$$

Hence,

$$\sum_{n=0}^{\infty} A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)\,t^{n} = E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}\left(1-at\right)^{-\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\left(1-at\right)^{\frac{-1}{a}}\right)\right\}.$$

(II)

$$\sum_{n=0}^{\infty}\left(\frac{u}{a}\right)^{n} \sum_{m=0}^{\infty} A_{n,n}^{(1,1,1,1,\sigma+km)}(x;a,k,s)\frac{t^{m}}{m!} = e^{t}\sum_{n=0}^{\infty} A_{n,n}^{(1,1,1,1,\sigma)}\left[\left(x^{k}-\frac{t}{p_k}\right)^{\frac{1}{k}};a,k,s\right]\left(\frac{u}{a}\right)^{n} \quad (3.2)$$

Proof:

Setting $\alpha=\beta=q=p=\gamma=\delta=1$ in (3.1), we get

$$\sum_{n=0}^{\infty} A_{n,n}^{(1,1,1,1,\sigma)}(x,a,k,s)t^n = (1-at)^{-\left(\frac{\sigma+s}{a}\right)} E_{1,1,1}^{1,1,1}\{p_k(x)\} E_{1,1,1}^{1,1,1}\left\{-P_k\left(x(1-at)^{\frac{-1}{a}}\right)\right\}$$
$$= (1-at)^{-\left(\frac{\sigma+s}{a}\right)} \exp\left[\{p_k(x)\} - \left\{P_k x(1-at)^{\frac{-1}{a}}\right\}\right].$$

Taking $P_k(x) = p_k x^k$ and $t = \dfrac{u}{a}$, then we get

$$\sum_{n=0}^{\infty}\left(\frac{u}{a}\right)^n A_{n,n}^{(1,1,1,1,\sigma)}(x;a,k,s) = = (1-u)^{-\left(\frac{\sigma+s}{a}\right)} \exp\left[p_k x^k \left\{1-(1-u)^{\frac{-k}{a}}\right\}\right] . \tag{3.3}$$

Replacing σ by $\sigma + km$ and multiplying both side of (3.3) by $\dfrac{t^m}{m!}$ yields

$$\sum_{n=0}^{\infty}\left(\frac{u}{a}\right)^n A_{n,n}^{(1,1,1,1,\sigma+km)}(x;a,k,s)\frac{t^m}{m!} = (1-u)^{-\left(\frac{\sigma+km+s}{a}\right)} \exp\left[p_k x^k \left\{1-(1-u)^{\frac{-k}{a}}\right\}\right]\frac{t^m}{m!}$$

and now summing up over $m = 0$ to ∞, yields

$$\sum_{n=0}^{\infty}\left(\frac{u}{a}\right)^n \sum_{m=0}^{\infty} A_{n,n}^{(1,1,1,1,\sigma+km)}(x;a,k,s)\frac{t^m}{m!} = e^t (1-u)^{-\left(\frac{\sigma+s}{a}\right)} \exp\left[(p_k x^k - t)\left\{1-(1-u)^{\frac{-k}{a}}\right\}\right] .$$

Hence

$$\sum_{n=0}^{\infty}\left(\frac{u}{a}\right)^n \sum_{m=0}^{\infty} A_{n,n}^{(1,1,1,1,\sigma+km)}(x;a,k,s)\frac{t^m}{m!}$$
$$= e^t (1-u)^{-\left(\frac{\sigma+s}{a}\right)} \exp\left[p_k\left(x^k - \frac{t}{p_k}\right)\left\{1-(1-u)^{\frac{-k}{a}}\right\}\right] \tag{3.4}$$

and from (3.3) and (3.4), we get

$$\sum_{n=0}^{\infty}\left(\frac{u}{a}\right)^n \sum_{m=0}^{\infty} A_{n,n}^{(1,1,1,1,\sigma+km)}(x;a,k,s)\frac{t^m}{m!} = e^t \sum_{n=0}^{\infty} A_{n,n}^{(1,1,1,1,\sigma)}\left[\left(x^k - \frac{t}{p_k}\right)^{\frac{1}{k}};a,k,s\right]\left(\frac{u}{a}\right)^n .$$

(III)

$$\sum_{n=0}^{\infty} A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)t^n = (1+at)^{\left(\frac{\sigma+s}{a}\right)-1} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x(1+at)^{\frac{1}{a}}\right)\right\} \tag{3.5}$$

Proof:

$$\sum_{n=0}^{\infty} A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)t^n = x^{-(\sigma-an)-an} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\} e^{t\theta}\left[x^{\sigma-an} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{-P_k(x)\}\right].$$

$$= x^{-\sigma} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{P_k\,(x)\right\} x^{\sigma}\,(1+at)^{\left(\frac{\sigma+s}{a}\right)-1} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{-P_k\;x\,(1+at)^{\frac{1}{a}}\right\}$$

$$= (1+at)^{\left(\frac{\sigma+s}{a}\right)-1} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{P_k\,(x)\right\} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{-P_k\;x\,(1+at)^{\frac{1}{a}}\right\}$$

which ends the proof.

(IV)

$$\sum_{m=0}^{\infty}\binom{n+m}{m} A^{(\alpha,\beta,\gamma,\delta,\sigma)}_{q(m+n),p(m+n)}(x;a,k,s)\,t^{m}$$
$$= \frac{(1-at)^{-n-\left(\frac{\sigma+s}{a}\right)} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{P_k\,(x)\right\}}{E^{\gamma,\delta,q}_{\alpha,\beta,p}\left(P_k\left\{x\,(1-at)^{-\frac{1}{a}}\right\}\right)}\; A^{(\alpha,\beta,\gamma,\delta,\sigma)}_{qn,pn}\left[x\,(1-at)^{-\frac{1}{a}};a,k,s\right] \tag{3.6}$$

Proof:

Writing equation (1.8) as

$$\theta^{n}\left[x^{\sigma} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{-P_k\,(x)\right\}\right] = \frac{n!\;x^{\sigma+an}}{E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{P_k\,(x)\right\}}\; A^{(\alpha,\beta,\gamma,\delta,\sigma)}_{qn,pn}(x;a,k,s)\,,$$

and applying the operator $e^{t\theta}$ to both sides, then we get

$$e^{t\theta}\left[\theta^{n}\left(x^{\sigma} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{-P_k\,(x)\right\}\right)\right] = n!\,e^{t\theta}\left[\frac{x^{\sigma+an}\;A^{(\alpha,\beta,\gamma,\delta,\sigma)}_{qn,pn}(x;a,k,s)}{E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{P_k\,(x)\right\}}\right],$$

$$\sum_{m=0}^{\infty}\frac{t^{m}\;\theta^{m+n}}{m!\;n!}\left[x^{\sigma} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{-P_k\,(x)\right\}\right]$$
$$= \frac{x^{\sigma+an}\left(1-ax^{a}t\right)^{-n-\left(\frac{\sigma+s}{a}\right)}}{E^{\gamma,\delta,q}_{\alpha,\beta,p}\left[P_k\left\{x\left(1-ax^{a}t\right)^{\frac{-1}{a}}\right\}\right]}\,.\,A^{(\alpha,\beta,\gamma,\delta,\sigma)}_{qn,pn}\left[x\left(1-ax^{a}t\right)^{\frac{-1}{a}};a,k,s\right]. \tag{3.7}$$

But by replacing n by m + n in (1.8) , we have

$$\theta^{m+n}\left[x^{\sigma} E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{-P_k\,(x)\right\}\right] = \frac{(n+m)!\,x^{\sigma+a(n+m)}}{E^{\gamma,\delta,q}_{\alpha,\beta,p}\left\{P_k\,(x)\right\}}\; A^{(\alpha,\beta,\gamma,\delta,\sigma)}_{q(m+n),p(m+n)}(x;a,k,s)\,,$$

substituting the last equation in (3.7), we obtain

$$\sum_{m=0}^{\infty} \frac{t^m (n+m)! \,.\, x^{\sigma+a(n+m)}}{m!\,n! \quad E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}} A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)$$

$$= \frac{x^{\sigma+an}\left(1-ax^a t\right)^{-n-\left(\frac{\sigma+s}{a}\right)}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\left(1-ax^a t\right)^{\frac{-1}{a}}\right)\right\}} \cdot A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}\left[x\left(1-ax^a t\right)^{\frac{-1}{a}};a,k,s\right]$$

Now replacing t by tx^{-a}, then we have

$$\sum_{m=0}^{\infty} \binom{n+m}{m} A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)t^m$$

$$= \frac{(1-at)^{-n-\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x(1-at)^{\frac{-1}{a}}\right)\right\}} \cdot A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}\left[x(1-at)^{\frac{-1}{a}};a,k,s\right]$$

which ends the proof.

(V)

$$\sum_{n=0}^{\infty} \binom{n+m}{m} A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)t^n$$

$$= \frac{(1+at)^{-1+\left(\frac{s+\sigma}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x(1+at)^{\frac{1}{a}}\right)\right\}} A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma)}\left[x(1+at)^{\frac{1}{a}};a,k,s\right] \tag{3.8}$$

Proof:

Multiplying (3.5) by $x^{\sigma}/E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}$, then operating by θ^m for both sides yields

$$\sum_{n=0}^{\infty} \frac{x^{\sigma}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}} \,.\, A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)t^n = (1+at)^{-1+\left(\frac{s+\sigma}{a}\right)} x^{\sigma} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x(1+at)^{\frac{1}{a}}\right)\right\},$$

thus

$$\sum_{n=0}^{\infty} \theta^m \left\{\frac{x^{\sigma}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}} \,.\, A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)t^n\right\}$$

$$= (1+at)^{-1+\left(\frac{\sigma+s}{a}\right)} \theta^m \left[x^{\sigma} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x(1+at)^{\frac{1}{a}}\right)\right\}\right] \tag{3.9}$$

Replacing n by m in equation (1.8), we get

$$\frac{m!\ \ x^{\sigma+am}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)=\theta^m\left[x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right], \tag{3.10}$$

and replacing m by $m+n$ in (3.10) implies

$$\frac{(m+n)!\ x^{\sigma+am+an}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)$$

$$=\theta^{m+n}\left[x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right]=\theta^m\left[\theta^n\left(x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right)\right]$$

$$=n!\,\theta^m\left[\frac{x^{\sigma+an}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}.A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)\right], \tag{3.11}$$

and replacing σ by $\sigma-an$ in (3.11), yields

$$\frac{(m+n)!\qquad x^{\sigma+am}}{n!\ \ .\ \ E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}\cdot A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)$$

$$=\theta^m\left[\frac{x^{\sigma-an+an}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}.A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)\right]. \tag{3.12}$$

Substituting the result in (3.9), then we have

$$\sum_{n=}^{\infty}\frac{(m+n)!\qquad x^{\sigma+am}}{n!\ \ E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}\ .\ A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)t^n$$

$$=(1+at)^{-1+\left(\frac{\sigma+s}{a}\right)}\theta^m\left[x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\,x\,(1+at)^{\frac{1}{a}}\right\}\right].$$

Now making use of (3.10) implies

$$\sum_{n=0}^{\infty}\binom{n+m}{n}A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma-an)}(x;a,k,s)t^n$$

$$=\frac{(1+at)^{-1+\left(\frac{\sigma+s}{a}\right)}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\ x\,(1+at)^{\frac{1}{a}}\right\}}\ A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma)}\left[x\,(1+at)^{\frac{1}{a}};a,k,s\right]$$

which proves the relation.

Now using (1.15) – (1.19) we can also obtain the following generating relations

(VI)

$$\sum_{n=0}^{\infty} m^n \, A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)\, z^m = \frac{(1-az)^{-\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k \; x(1-az)^{\frac{-1}{a}}\right\}}$$

$$\times \sum_{m=0}^{\infty} m!\, S(n,m)\, A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma)}\left[x(1-az)^{\frac{-1}{a}};a,k,s\right]\left(\frac{z}{1-az}\right)^m \tag{3.13}$$

Proof:

Comparing (3.6) and (1.18), we get

$$f(x,t) = (1-at)^{-\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\} / E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k \; x(1-at)^{\frac{-1}{a}}\right\},$$

$$g(x,t) = (1-at)\,, \quad h(x,t) = x(1-at)^{\frac{-1}{a}} \quad \text{and} \quad \varepsilon_n(x) \to A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s),$$

hence equation (1.19) yields

$$\sum_{m=0}^{\infty} m^n \, A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma)}\left[x(1+az)^{\frac{-1}{a}};a,k,s\right]\left(\frac{z}{1+az}\right)^m$$

$$= (1+az)^{\left(\frac{\sigma+s}{a}\right)} \frac{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k \; x(1+az)^{\frac{-1}{a}}\right\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}} \times \sum_{m=0}^{\infty} m!\, S(n,m)\, A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)\, z^m$$

Replacing $z \mapsto \frac{z}{1-az}$, $x \mapsto \frac{x}{(1-az)^{\frac{1}{a}}}$, then the last equation implies

$$\sum_{m=0}^{\infty} m^n \, A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)\, z^m = (1-az)^{-\left(\frac{\sigma+s}{a}\right)} \frac{E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k \; x(1-az)^{\frac{-1}{a}}\right\}}$$

$$\times \sum_{m=0}^{\infty} m!\; S(n,m)\, A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma)}\left[x(1-az)^{\frac{-1}{a}};a,k,s\right]\left(\frac{z}{1-az}\right)^m .$$

(VII)

$$\sum_{m=0}^{\infty} m^n \, A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma-am)}(x;a,k,s)\, z^m = \frac{(1+az)^{-1+\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k \; x(1+az)^{\frac{1}{a}}\right\}}$$

$$\times\sum_{m=0}^{\infty} m!\ S(n,m)\, A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma-am)}\left[x(1+az)^{\frac{1}{a}};a,k,s\right]\left(\frac{z}{1+az}\right)^{m} \tag{3.14}$$

Proof:

Comparing (3.8) and (1.18), we get

$$f(x,t)=(1+at)^{-1+m+\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\} \,/\, E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\ x(1+at)^{\frac{1}{a}}\right\}$$

$$g(x,t)=(1+at) \quad , \quad h(x,t)=x(1+at)^{\frac{1}{a}} \text{ and } \varepsilon_n(x)\mapsto A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s).$$

Now equation (1.19) yields

$$\sum_{m=0}^{\infty} m^{n} A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma)}\left[x(1-az)^{\frac{1}{a}};a,k,s\right]\left(\frac{z}{1-az}\right)^{m}$$

$$=(1-az)^{+1-m-\left(\frac{\sigma+s}{a}\right)}\frac{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\ x(1-az)^{\frac{1}{a}}\right\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}}\cdot\sum_{m=0}^{\infty} m!\,S(n,m)\,A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)z^{m}$$

Replacing σ *by* $\sigma-am$ *and for* $z\mapsto\frac{z}{1+az}$, $x\mapsto x(1+az)^{\frac{1}{a}}$, the result becomes

$$\sum_{m=0}^{\infty} m^{n} A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma-am)}(x;a,k,s)z^{m}=\frac{(1+az)^{-1+m+\left(\frac{\sigma-am+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\ x(1+az)^{\frac{1}{a}}\right\}}$$

$$\times\sum_{m=0}^{n} m!\ S(n,m)\,A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma-am)}\left[x(1+az)^{\frac{1}{a}};a,k,s\right]\left(\frac{z}{1+az}\right)^{m}.$$

Hence,

$$\sum_{m=0}^{\infty} m^{n} A_{q(m+n),p(m+n)}^{(\alpha,\beta,\gamma,\delta,\sigma-am)}(x;a,k,s)z^{m}=\frac{(1+az)^{-1+\left(\frac{\sigma+s}{a}\right)} E_{\alpha,\beta,p}^{\gamma,\delta,q}\{P_k(x)\}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\ x(1+az)^{\frac{1}{a}}\right\}}$$

$$\times\sum_{m=0}^{n} m!\ S(n,m)\,A_{qm,pm}^{(\alpha,\beta,\gamma,\delta,\sigma-am)}\left[x(1+az)^{\frac{1}{a}};a,k,s\right]\left(\frac{z}{1+az}\right)^{m}$$

4. Finite Summation Formulae

Now we obtained two finite summation formulae for equation (1.8) as follows

(I)

$$A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)=\sum_{m=0}^{n}\frac{1}{m!}a^m\left(\frac{\sigma}{a}\right)_m A_{q(n-m),p(n-m)}^{(\alpha,\beta,\gamma,\delta,0)}(x;a,k,s) \tag{4.1}$$

Proof:

$$A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)=\frac{1}{n!}x^{-\sigma-an}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}\theta^n\left[x\,.\,x^{\sigma-1}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}\right]$$

$$=\frac{1}{n!}x^{-\sigma-an}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}.x\sum_{m=0}^{n}\binom{n}{m}\theta^{n-m}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}\theta_1^m\left(x^{\sigma-1}\right)$$

$$=\frac{1}{n!}x^{-\sigma-an+1}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}\sum_{m=0}^{n}\frac{n!\,x^{a(n-m)}}{m!(n-m)!}\prod_{i=o}^{n-m-1}\left(S+xD+ia\right)$$

$$\times E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}.x^{am}\left[(1+xD)(1+a+xD).........(1+(m-1)a+xD)\right]x^{\sigma-1}$$

$$=\frac{1}{n!}x^{-\sigma+1}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}\sum_{m=0}^{n}\frac{n!}{m!(n-m)!}$$
$$\times\prod_{i=o}^{n-m-1}\left(S+xD+ia\right)\times E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}.a^m\left(\frac{\sigma}{a}\right)_m x^{\sigma-1}. \tag{4.2}$$

Now putting $\sigma=0$ and replacing $n\ by\ (n-m)$ in equation (1.8), we get

$$A_{q(n-m),p(n-m)}^{(\alpha,\beta,\gamma,\delta,0)}(x;a,k,s)=\frac{1}{(n-m)!}x^{-a(n-m)}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}\theta^{n-m}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}$$

thus, we have

$$\frac{1}{(n-m)!}\theta^{n-m}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}=\frac{x^{a(n-m)}}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}}A_{q(n-m),p(n-m)}^{(\alpha,\beta,\gamma,\delta,0)}(x;a,k,s),$$

the above equation can be written as

$$\frac{1}{(n-m)!}\prod_{i=0}^{n-m-1}\left(S+ia+xD\right)\left[E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right] = \frac{1}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}\,A_{q(n-m),p(n-m)}^{(\alpha,\beta,\gamma,\delta,0)}(x;a,k,s). \tag{4.3}$$

Using equations (4.2) and (4.3), we get the result

$$A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)$$

$$=\frac{1}{n!}x^{-\sigma+1}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_x\left(x\right)\right\}\sum_{m=0}^{n}\frac{n!}{m!}\times\frac{1}{E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}}\times A_{q(n-m),p(n-m)}^{(\alpha,\beta,\gamma,\delta,0)}(x;a,k,s).a^m\left(\frac{\sigma}{a}\right)_m x^{\sigma-1}$$

Hence

$$A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)=\sum_{m=0}^{n}\frac{a^m}{m!}\left(\frac{\sigma}{a}\right)_m A_{q(n-m),p(n-m)}^{(\alpha,\beta,\gamma,\delta,0)}(x;a,k,s)$$

which completes the proof.

(II)

$$A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)=\sum_{n=0}^{\infty}\frac{a^m}{m!}\left(\frac{\sigma-\xi}{a}\right)_m A_{q(n-m),p(n-m)}^{(\alpha,\beta,\gamma,\delta,\xi)}(x;a,k,s) \tag{4.4}$$

Proof:

$$\sum_{n=0}^{\infty}x^{an}A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)\,t^n = x^{-\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}e^{t\theta}\left[x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right]$$

$$=\left(1-ax^a t\right)^{-\left(\frac{\sigma+s}{a}\right)}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}\times E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\;x\left(1-ax^a t\right)^{\frac{-1}{a}}\right\}$$

$$=\left(1-ax^a t\right)^{-\left(\frac{\xi+s}{a}\right)}\sum_{m=0}^{\infty}\left(\frac{\sigma-\xi}{a}\right)_m\frac{\left(ax^a t\right)^m}{m!}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\;x\left(1-ax^a t\right)^{\frac{-1}{a}}\right\}$$

$$=\sum_{m=0}^{\infty}\left(\frac{\sigma-\xi}{a}\right)_m\frac{\left(ax^a t\right)^m}{m!}x^{-\xi}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}e^{t\theta}\left[x^{\xi}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right]$$

Also from the proof of equation (3.1), we have

$$\sum_{m=0}^{\infty}x^{an}\frac{x^{-\sigma-an}}{n!}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}\theta^n\left[x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right]t^n$$
$$=x^{-\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}e^{t\theta}\left[x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right]$$

so that

$$\sum_{n=0}^{\infty} x^{an} A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)t^n$$

$$=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{\sigma-\xi}{a}\right)_m \frac{\left(ax^a t\right)^m}{m!}\frac{x^{-\xi}}{(n-m)!} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}\theta^{n-m}\left[x^{\xi} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right]t^{n-m}$$

$$=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{\sigma-\xi}{a}\right)_m \frac{\left(ax^a\right)^m}{m!}\frac{x^{-\xi}}{(n-m)!}\cdot t^n E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}\theta^{n-m}\left[x^{\xi} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right].$$

Now equating coefficient of t^n implies

$$x^{an} A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)$$
$$=\sum_{m=0}^{\infty}\left(\frac{\sigma-\xi}{a}\right)_m \frac{a^m}{m!}\frac{x^{am}}{(n-m)!}x^{-\xi} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}\theta^{n-m}\left[x^{\xi} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right],$$

hence

$$A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)$$
$$=\sum_{m=0}^{\infty}\left(\frac{\sigma-\xi}{a}\right)_m \frac{a^m\quad x^{-\xi-a(n-m)}}{m!(n-m)!} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k\left(x\right)\right\}\times\theta^{n-m}\left[x^{\xi} E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k\left(x\right)\right\}\right].$$

Using (1.8) , we get

$$A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)=\sum_{m=0}^{\infty}\left(\frac{\sigma-\xi}{a}\right)_m \frac{a^m}{m!} A_{q(n-m),p(n-m)}^{(\alpha,\beta,\gamma,\delta,\xi)}(x;a,k,s)$$

which completes the proof of (4.4).

Open Problem : One can try to obtain results of bilateral generating function relations and other related results by using the general class of polynomials $A_{qn,pn}^{(\alpha,\beta,\gamma,\delta,\sigma)}(x;a,k,s)=\frac{x^{-\sigma-an}}{n!}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{P_k(x)\right\}\theta^n\left[x^{\sigma}E_{\alpha,\beta,p}^{\gamma,\delta,q}\left\{-P_k(x)\right\}\right]$.

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