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An approach to multicriteria optimization based on aggregation: application to manufacturing systems

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Abstract

The field of multicriteria optimization knows important development which has led of many methods. These methods are different according to the criteria aggregation step, based on specific parameters. In this paper, we present a multi criteria optimization method based on aggregation which allows ranking alternatives with no specifications on the criteria. The proposed approach is applied to an example of manufacturing systems. The result is an optimal ranking of manufacturing orders to improve productivity.

Keywords: *Optimization, multicriteria, aggregation, manufacturing orders, productivity.*

1 Introduction

The decision support is a science devoted to clarifying the understanding of a decision problem and its resolution, it becomes multicriteria when the problem involves multiple objectives, often contradictory [1].

Before the emergence of multicriteria analysis, decision problems were treated like the optimization of an objective function. This approach had the merit to lead to well-posed mathematical problems but that did not always reflect reality because [2]:

- Comparison of several possible alternatives is rarely following a single criterion;
- In many cases, preferences on a test are difficult to model by a function;
- When there are multiple objectives, it is impossible to reach them all at once.

In this paper, we present a multi criteria optimization method based on aggregation which allows ranking alternatives. In the second section, we present the issues related to multicriteria optimization and classification of various existing approaches in this field. In the third section, we present our approach for multicriteria optimization based on aggregation of alternatives. The validation of our method was obtained by its application in an industrial company and in the fourth section we discuss the results of our experiment. Section 5 and 6 summarize conclusions and open problems of the paper.

2 Multicriteria optimization

The multicriteria optimization [3] is to choose best among a given set of possible alternatives, in the presence of multiple criteria that measure the quality of alternatives.

2.1 Problematic

According to [2], [4] and [5], the real problems can be formulated using methods of multicriteria analysis, according to three basic formulations: problems of choice, denoted P_α, the problem of sorting or assignment noted P_β and the ranking problem denoted P_γ.

2.2 Classification of multicriteria optimization methods

According to [6], we distinguish two different procedures giving place to two distinct approaches to multicriteria aggregation: approach AC and CA.

- Approach AC "*Aggregate and Compare*»: It is to summarize the value of any alternative by an overall mark *v(a)* calculated from its vector performance. This note is meant to summarize the overall value of the alternative and serves as a basis for comparison of multi alternatives.
- Approach CA "*Compare and Aggregate*»: It is necessary to compare first, criterion by criterion, the performance of alternatives. Thus, for each pair of alternatives *(a,b)* and each criterion *j*, we can define a binary index $\phi_j(a_j,b_j)$ where partial preference ϕ_j is an increasing function for the first argument, decreasing for the second argument. Then, the preference *P(a,b)* is defined by aggregation of partial preference indices.

3 Proposed optimization method

In this section, we present our optimization method based on the aggregation of potential alternatives. The method takes place in the framework of the optimization context in holonic manufacturing systems [7]. The approach is based on the CA approach "Compare and aggregate" defined in the previous section and in this context we deal with the problematic P_γ , giving a ranking between all alternatives.

3.1 Proposed approach

Our proposed approach includes the three sequential steps [1] of any multicriteria method. We specify the fourth step and we add two steps to complete the process.

So, the six steps are:

- Step 1: List the possible potential alternatives;
- Step 2: List the criteria to be considered;
- Step 3: Quantification and classification of criteria from the strongest to weakest, to have an initial table of performance;
- Step 4: Application of the CA approach: for each criterion above, we proceed to a classification evaluation e_{ij}. As a result, we get groups of alternatives ranked according to the considered criteria;
- Step 5: We repeat step 4, for all criteria;
- Step 6: Aggregating alternatives which present indifference into one alternative.

3.2 Description of the proposed method

The proposed method allows us to move from a multi-criteria analysis to a mono criterion analysis.

3.2.1 Input elements

- All potential alternatives A = $\{a_1, a_2, ..., a_n\}$ $i = 1,2, ..., n$. Alternatives are the solutions in the context of the production optimization;
- Different criteria, C_j where $j = 1,2, ..., m$. The criteria are chosen from the performance measures in manufacturing (unit cost, productivity and quality).
- Assessments or judgments, e_{ij} where $i = 1,2, ..., n$; $j = 1,2, ..., m$.

3.2.2 Application of the method

As already explained in the previous paragraph, we compare our alternatives following each criterion greater weight to smaller (Fig.1), through successive iterations to obtain a classification of alternatives.

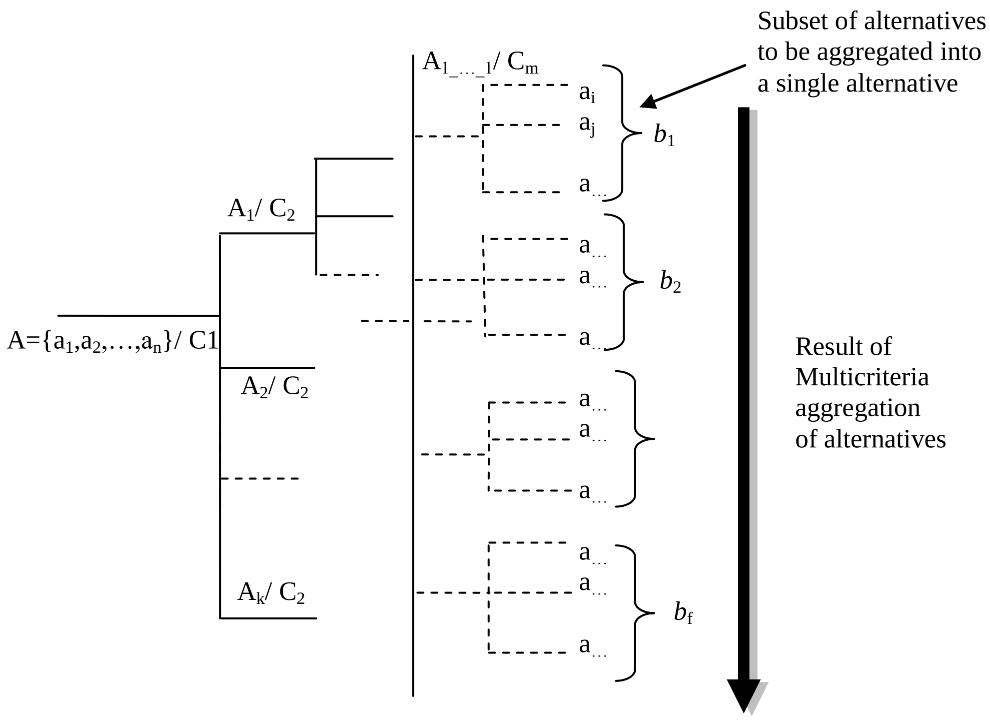

Fig. 1 Proposed method of multicriteria aggregation

The first iteration is to classify the alternatives according to the first criterion. The classification process is reapplied to each subset to the following criteria.

Finally, we get alternatives classes, which will be aggregated.

3.2.3 Output elements

After application of the method, we have the following results:

- The set B = $\{b_1, b_2, ..., b_f\}$, new set of aggregated alternatives.
- Evaluation / judgment e'_{ij} is defined by a function f such that $e'_{ij} = f(b_i)$
- The new performance table will be of the form:

Alternatives	Criteria					
	c_1	c_2	c_3	…	…	c_m
b_1	e'_{11}	e'_{12}	e'_{13}	…	…	e'_{1m}
b_2	e'_{21}	…	…	…	…	…
b_3	…	…	…	…	…	…
…	…	…	…	…	…	…
b_f	e'_{f1}	…	…	…	…	e'_{fm}

where the set $\{b_1, b_2, b_3, \ldots, b_f\}$ represents the best classification alternatives.

4 Application to manufacturing system

We applied our approach to an industrial enterprise specialized in welded mesh. The company produces many products such as welded mesh, metallic beams and reinforcing mesh. In this paper, we consider the welded mesh product, specially its intermediate product, the semi finished product ‘drawn wire’.

The company adopts the mixed flow for its production (pull and push): it builds and stores its standard product (welded mesh 4.5) continuously, and manufactures other products when an external command comes from a client.

Our goal of multicriteria optimization is to improve the productivity and in this section we will discuss the results obtained through the application of our method of aggregation of multicriteria optimization.

4.1 Application of the optimization approach

4.1.1 Definition of potential alternatives

During a given period, the company received a set of commands. After the programming phase, we identified thirteen (13) potential alternatives representing 13 manufacturing orders noted mo_i. The set of possible alternatives is:

A={mo_1, mo_2, mo_3, mo_4, mo_5, mo_6, mo_7, mo_8, mo_9, mo_10, mo_11, mo_12, mo_13}.

4.1.2 Identification of criteria

After site survey and in the context of optimization and evaluation of performances, we identified the following criteria:

• Product type, noted type. The intermediate product is noted ‘FT’.

• The diameter, noted diam, which takes the value from 4.5 to 10 mm;

• The quantity, noted qte, which represents the quantity ordered by the customer and measured in tons;

• The raw material, noted mp;

• The production time, noted t_prod, which represents the machine time necessary to manufacture the ordered quantity of the product;

• Resource preparation time, noted t_res, represents the time necessary to prepare and adjust resources to manufacture a specific product;

• The type of order, noted t_cmd. According to the production flow, we adopt the type '1' for internal command and '2' for external command. The priority is gives to the external command;

• The delivery time, noted t_deliv, represents the time specified by the customer when ordering;

• The cycle time, noted t_cycle, represents the time required to manufacture a product, it is equal to the sum of production time and resources preparation time (t_prod+t_res).

4.1.3 Evaluation criteria

The evaluation criteria on potential alternatives are given in the performance table (Table 1).

Table 1: Performance matrix.

ident	type	diam	qte	mp	t_prod	t_res	t_cmd	t_deliv	t_cycle
mo_1	FT	4,5	50	5,5	40	5	2	10	45
mo_2	FT	6	50	8	20	5	2	5	25
mo_3	FT	4,5	100	5,5	80	5	1	15	85
mo_4	FT	8	40	6	8	5	2	3	13
mo_5	FT	4,5	100	5,5	80	5	2	5	85
mo_6	FT	10	50	12	4	5	2	2	9
mo_7	FT	4,5	50	5,5	40	5	1	15	45
mo_8	FT	8	200	6	40	5	2	4	45
mo_9	FT	10	150	8	15	5	2	3	20
mo_10	FT	6	100	8	40	5	2	6	45
mo_11	FT	8	200	10	40	5	2	5	45
mo_12	FT	10	40	12	3	5	2	2	8
mo_13	FT	4,5	50	5,5	40	5	2	10	45

The next step in the process is to aggregate the alternatives by applying the approach CA (Compare and Aggregate).

4.2 Application of the aggregation method

4.2.1 Definition of criteria aggregation

In the context of the production optimization and among the measures of performance, we choose the following criteria ranked from strongest to weakest:

- Type of command (C1)
- Delivery time (C2);
- Diameter (C3).

The set of ordered criteria C is C={C1,C2,C3}.

4.2.2 Classification of alternatives

First iteration: We apply the criterion C1 and we obtain the following table:

ident	type	diam	qte	mp	t_prod	t_res	t_cmd	t_deliv	t_cycle
mo_1	FT	4,5	50	5,5	40	5	2	10	45
mo_2	FT	6	50	8	20	5	2	5	25
mo_4	FT	8	40	6	8	5	2	3	13
mo_5	FT	4,5	100	5,5	80	5	2	5	85
mo_6	FT	10	50	12	4	5	2	2	9
mo_8	FT	8	200	6	40	5	2	4	45
mo_9	FT	10	150	8	15	5	2	3	20
mo_10	FT	6	100	8	40	5	2	6	45
mo_11	FT	8	200	10	40	5	2	5	45
mo_12	FT	10	40	12	3	5	2	2	8
mo_13	FT	4,5	50	5,5	40	5	2	10	45
mo_3	FT	4,5	100	5,5	80	5	1	15	85
mo_7	FT	4,5	50	5,5	40	5	1	15	45

After applying the criterion C1, we got 2 subsets A1 and A2:

- A1={mo_1,mo_2,mo_4,mo_5,mo_6,mo_8,mo_9,mo_10,mo_11,mo_12, mo_13};
- A2={mo_3,mo_7}

2nd iteration: We apply the criterion C2: Delivery time.

ident	type	diam	qte	mp	t_prod	t_res	t_cmd	t_deliv	t_cycle
mo_6	FT	10	50	12	4	5	2	2	9
mo_12	FT	10	40	12	3	5	2	2	8
mo_4	FT	8	40	6	8	5	2	3	13
mo_9	FT	10	150	8	15	5	2	3	20
mo_8	FT	8	200	6	40	5	2	4	45
mo_2	FT	6	50	8	20	5	2	5	25
mo_5	FT	4,5	100	5,5	80	5	2	5	85
mo_11	FT	8	200	10	40	5	2	5	45
mo_10	FT	6	100	8	40	5	2	6	45
mo_1	FT	4,5	50	5,5	40	5	2	10	45
mo_13	FT	4,5	50	5,5	40	5	2	10	45
mo_3	FT	4,5	100	5,5	80	5	1	15	85
mo_7	FT	4,5	50	5,5	40	5	1	15	45

After applying the criterion C2, we obtain the following subsets:

- A1_1={mo_6,mo_12};
- A1_2={mo_4,mo_9};
- A1_3={mo_8};
- A1_4={mo_2,mo_5,mo_11};
- A1_5={mo_10};
- A1_6={mo_1,mo_13};
- A2_1={mo_3,mo_7};

3rd iteration: We apply the criterion C3: Diameter

ident	type	diam	qte	mp	t_prod	t_res	t_cmd	t_deliv	t_cycle
mo_6	FT	10	50	12	4	5	2	2	9
mo_12	FT	10	40	12	3	5	2	2	3
mo_9	FT	10	150	12	15	5	2	3	15
mo_4	FT	8	40	10	8	5	2	3	13
mo_8	FT	8	200	10	40	5	2	4	40
mo_11	FT	8	200	10	40	5	2	5	40
mo_2	FT	6	50	8	20	5	2	5	25
mo_5	FT	4,5	100	5,5	80	5	2	5	85
mo_10	FT	6	100	8	40	5	2	6	45
mo_1	FT	4,5	50	5,5	40	5	2	10	40
mo_13	FT	4,5	50	5,5	40	5	2	10	40
mo_3	FT	4,5	100	5,5	80	5	1	15	80
mo_7	FT	4,5	50	5,5	40	5	1	15	40

After applying the criterion C3, we obtain the following subsets:

- A1_1_1={mo_6,mo_12};
- A1_2_1={mo_9}; A1_2_2={mo_4}
- A1_3_1={mo_8};
- A1_4_1={mo_11}; A1_4_2={mo_2}; A1_4_3={mo_5};
- A1_5_1={mo_10};
- A1_6_1={mo_1,mo_13};
- A2_1_1={mo_3,mo_7};

4.2.3 Aggregation of alternatives

Subsets obtained by applying the criteria contain alternatives with indifference (noted I) on all criteria. These alternatives are to be aggregated. These subsets are:

- A1_1_1={mo_6,mo_12};
- A1_6_1={mo_1,mo_13}
- A2_1_1={mo_3,mo_7};

4.2.4 Evaluation of performance

The performance e'_{ij} is defined by a function f such that $e'_{ij} = f(b_i)$

1- For attributes type, diam, mp, t_cmd and t_deliv, the new performance is calculated using the formula 1:

• a I b then f(a_b) = f(a) = f(b) (1)

Example:

ident	*type*	*diam*	*mp*	*t_cmd*	*t_deliv*
mo_6	FT	10	12	2	2
mo_12	FT	10	12	2	2
mo_6_12	FT	10	12	2	2

2- For attributes qte and t_prod, the new performance is the sum of the aggregate performance measures.

Example:

Ident	*qte*	*t_prod*
mo_6	50	4
mo_12	40	3
mo_6_12	90	7

3- For the attribute t_res, the new performance is calculated following two cases, using the formula 2 (i, ii):

- a I b then f(a_b) = f(a) = f(b) (2.i)
- a Ig b then f(b)=0, where g is the criterion diameter (2.ii)

Examples:

Alternatives with indifference on all criteria

ident	*t_res*
mo_6	5
mo_12	5
mo_6_12	5

Alternatives with indifference on the criterion diameter

ident	*t_res*
mo_6	5
mo_12	0
mo_9	0

After calculating the new performance of the criterion -time of preparation of resources (t_res), we obtain the intermediate performance table (Table 2) before aggregation.

Table 2: Intermediate performance table after optimization

ident	type	diam	qte	mp	t_prod	t_res	t_cmd	t_deliv
mo_6	FT	10	50	12	4	5	2	2
mo_12	FT	10	40	12	3	0	2	2
mo_9	FT	10	150	12	15	0	2	3
mo_4	FT	8	40	10	8	5	2	3
mo_8	FT	8	200	10	40	0	2	4
mo_11	FT	8	200	10	40	0	2	5
mo_2	FT	6	50	8	20	5	2	5
mo_5	FT	4,5	100	5,5	80	5	2	5
mo_10	FT	6	100	8	40	5	2	6
mo_1	FT	4,5	50	5,5	40	5	2	10
mo_13	FT	4,5	50	5,5	40	0	2	10
mo_3	FT	4,5	100	5,5	80	0	1	15
mo_7	FT	4,5	50	5,5	40	0	1	15

We obtain the optimal ranking: mo_6, mo_12, mo_9, mo_4, mo_8, mo_11, mo_2, mo_5, mo_10, mo_1, mo_13, mo_3, mo_7.

After calculating all the performance measures and aggregation, we obtain the final performance table (Table 3) as follows:

Table 3: Final performance table after aggregation

ident	type	diam	qte	mp	t_prod	t_res	t_cmd	t_deliv	t_cycle
mo_6_12	FT	10	90	12	7	5	2	2	12
mo_9	FT	10	150	12	15	0	2	3	15
mo_4	FT	8	40	10	8	5	2	3	13
mo_8	FT	8	200	10	40	0	2	4	40
mo_11	FT	8	200	10	40	0	2	5	40
mo_2	FT	6	50	8	20	5	2	5	25
mo_5	FT	4,5	100	5,5	80	5	2	5	85
mo_10	FT	6	100	8	40	5	2	6	45
mo_1_13	FT	4,5	100	5,5	80	5	2	10	80
mo_3_7	FT	4,5	150	5,5	120	0	1	15	120

4.3 Analysis of results

To evaluate the contribution of our multicriteria optimization method, we calculated the time of execution of orders following two ways:

1. Passage of manufacturing orders (mo) in FIFO (First In First Out).
2. Passage of manufacturing orders (mo) after optimization.

The completion time, denoted dr_i, measured in days, is calculated using the following formula 3:

$$dr_i = (t\_prod + t\_res)/24 + dr_i\text{-}1 \quad \text{with } dr_0 = 0 \qquad (3)$$

4.3.1 Passage of manufacturing orders in FIFO

The completion times are calculated from the initial performance table (Table 1) following the formula (3):

ident	Completion time
mo_1	1,88
mo_2	2,92
mo_3	6,46
mo_4	7,01
mo_5	10,55
mo_6	10,92
mo_7	12,80
mo_8	14,67
mo_9	15,51
mo_10	17,38
mo_11	19,26
mo_12	19,59
mo_13	21,46

The thirteen manufacturing orders from the mo_1 to mo_13, using the FIFO strategy, will require a time equal to 21.46 days (Fig. 2).

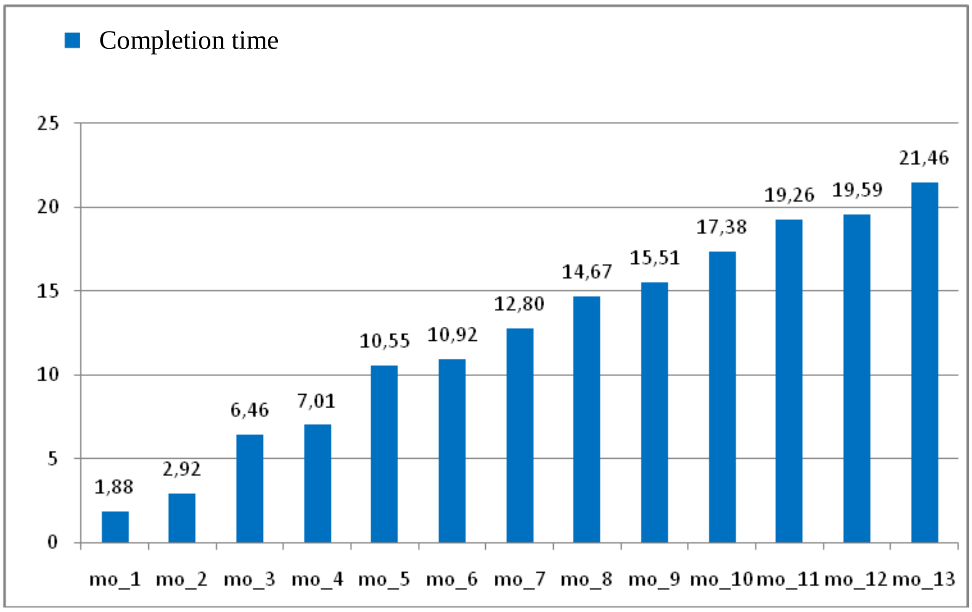

Fig.2 Passage of manufacturing orders in FIFO.

4.3.2 Passage of manufacturing orders after optimization

The completion time are calculated from the intermediate performance table before aggregation (Table 2), following the formula (3) :

ident	Completion time
mo_6	0,38
mo_12	0,51
mo_9	1,13
mo_4	1,67
mo_8	3,34
mo_11	5,01
mo_2	6,05
mo_5	9,59
mo_10	11,46
mo_1	13,34
mo_13	15,01
mo_3	18,34
mo_7	20,01

The thirteen manufacturing orders from mo_1 to mo_13, after applying the optimization method will require a time equal to 20.01 days (Fig.3).

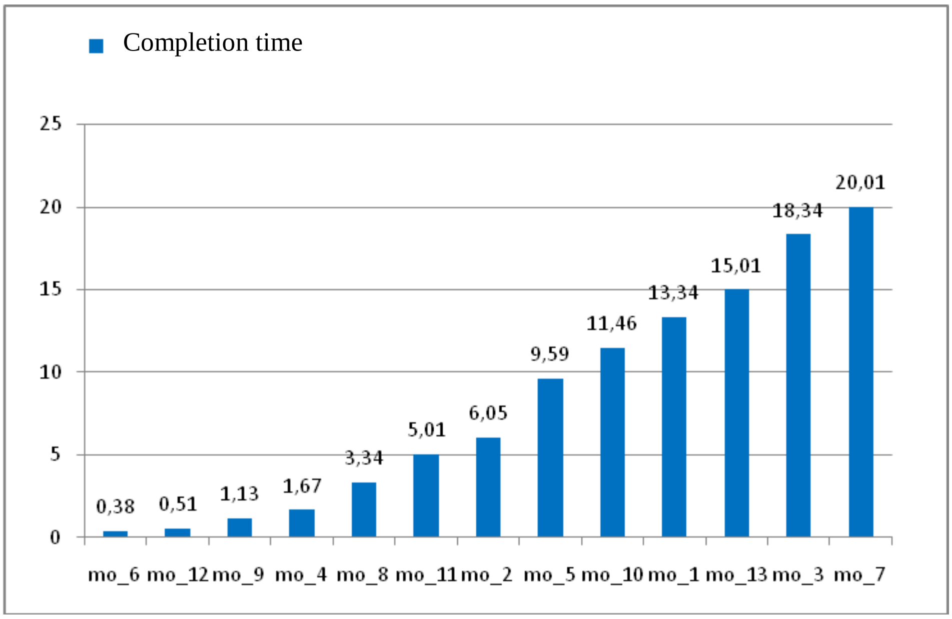

Fig.3 Passage of manufacturing orders after optimization.

4.3.3 Comparison of results

We can conclude with the following table:

Table 4 : Comparison of the two results

	machine time (hour)	resource preparation time (hour)	Manufacturing time (day)
FIFO	450	65	21,46
Optimization	450	30	20,01

By applying our multicriteria optimization method based on the aggregation, we reduced the cycle time approximately of 35 hours. This reduces time is important in the functioning of the industrial enterprise. So, the result allowed enhancing the productivity.

The obtained ranking mo_6, mo_12, mo_9, mo_4, mo_8, mo_11, mo_2, mo_5, mo_10, mo_1, mo_13, mo_3, mo_7 represents the optimal solution.

5 Conclusion

In this paper, we presented a multicriteria optimization method based on the aggregation principle.

The adopted multicriteria approach includes the first steps of any multicriteria method and add steps to complete the process. The aggregation principle is based on the CA approach, without assigning weights to criteria and without assigning a specific form of criteria. This strategy allows avoiding the subjective choices for this assignment.

The validation of our method was obtained by its application in an industrial company specializing in the manufacture of welded mesh. In the context of the production optimization and among the measures of performance, we choose three criteria and we described the method in detail. Finally, to evaluate the contribution of our multicriteria optimization method, we calculated the execution time of manufacturing orders following the classical strategy FIFO and after optimization in our approach. The comparison between the obtained results showed a reduction of the completion time.

The proposed method gives good results in manufacturing systems, provides an optimal ranking of manufacturing orders, enhancing the productivity.

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6 Open Problem

In this article, we presented a multicriteria optimization approach based on aggregation and applied to a manufacturing system.

For a wide application of our optimization method, we have two issues. The first one concerns the domain of multicriteria analysis. We adopt an approach without assigning a specific form of criteria and without assigning any weight to criteria. So, we can:

- Introduce specific forms of criteria, such as quasi and pseudo criterion [2] with their respective thresholds (indifference and preference).
- Consider others multicriteria methods in the same problematic P_γ, for example variants of Electre or PROMETHEE ([5] and [8]), giving an outranking graph of alternatives. In this case, specific forms and weight could be assigning for criteria. It will be interesting to compare the results and to choose the most appropriate method for enhancing the productivity.

The second issue of the proposed approach deals with the application domain, which is the system manufacturing. In this context, we can:

- Consider the parallel machining lines, which exist in real industrial enterprises.

- Apply the approach to the real-time control in order to take account the system interruptions (such as machine breakdowns, deadlocks, out of stock). The expected result is an optimal ranking of manufacturing orders, to compare with initial process without interruptions. It will be important to manage this aspect, for increasing productivity.

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