Cartesian product and homomorphism of interval-valued fuzzy linear space

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Abstract

The aim of this paper is to introduce the notion of cartesian product and homomorphism of interval-valued fuzzy linear space and to provide some results on it.

Keywords: fuzzy field, fuzzy linear space, interval-valued fuzzy field, interval-valued fuzzy linear space.

1 Introduction

S.Vijayabalaji and S.Sivaramakrishnan[18] introduced the notion of interval-valued fuzzy field and interval-valued fuzzy linear space.

In this paper we introduce the notion of cartesian product and homomorphism of interval-valued fuzzy linear space and provide results on it.

2 Preliminaries

In the following we provide the essential definitions and results necessary for the development of our theory.

Definition 2.1[20]. An interval number on $[0, 1]$, say $\bar{a}$, is a closed sub interval of $[0, 1]$, that is $\bar{a} = [a^-, a^+]$, where $0 \leq a^- \leq a^+ \leq 1$. Let $D[0, 1]$ denote the family of all closed sub-intervals of $[0, 1]$, that is, $D[0, 1] = \{\bar{a} = [a^-, a^+] : a^- \leq a^+ \text{ and } a^-, a^+ \in [0, 1]\}$.

Definition 2.2[20]. Let $a_i = [a^-_i, a^+_i] \in D[0, 1]$ for all $i \in \Omega$, $\Omega$ be an index set. Define

\[
\begin{align*}
(a) \inf_i \{a_i : i \in \Omega\} &= \inf_{i \in \Omega} a^-_i, \\
(b) \sup_i \{a_i : i \in \Omega\} &= \sup_{i \in \Omega} a^+_i.
\end{align*}
\]

In particular, whenever $\bar{a} = [a^-, a^+]$, $\bar{b} = [b^-, b^+]$ in $D[0, 1]$, we define

\[
\begin{align*}
(i) \bar{a} \leq \bar{b} & \text{ if and only if } a^- \leq b^- \text{ and } a^+ \leq b^+ \\
(ii) \bar{a} = \bar{b} & \text{ if and only if } a^- = b^- \text{ and } a^+ = b^+ \\
(iii) \bar{a} < \bar{b} & \text{ if and only if } a^- < b^- \text{ and } a^+ < b^+. \\
(iv) \min_i \{\bar{a}, \bar{b}\} &= \min\{a^-, b^-, a^+, b^+\} \\
(v) \max_i \{\bar{a}, \bar{b}\} &= \max\{a^-, b^-, a^+, b^+\}.
\end{align*}
\]

Definition 2.3[20]. Let $X$ be a set. A mapping $\overline{A} : X \to D[0, 1]$ is called an interval-valued fuzzy subset (briefly, an i-v fuzzy subset) of $X$, where $\overline{A}(x) = [A^-(x), A^+(x)]$, for all $x \in X$, $A^-$ and $A^+$ are fuzzy subsets in $X$ such that $A^-(x) \leq A^+(x)$ for all $x \in X$.

Definition 2.4[6]. Let $V$ denote a vector space of dimension $n$ over a field $F$. A fuzzy subspace is a fuzzy subset $\mu$ of $V$ such that $\mu(\alpha x + \beta y) \geq \min\{\mu(x), \mu(y)\}, x, y \in V, \alpha, \beta \in F(\text{Field})$.

Definition 2.5[17]. Let $V$ denote a vector space over a field $F$. Let $\overline{A} : X \to D[0, 1]$ be an interval-valued fuzzy subset of $V$. Then $\overline{A}$ is said to be an interval-valued fuzzy subspace (or shortly i-v fuzzy subspace) if, $\overline{A}(\alpha x + \beta y) \geq \min_i \{\overline{A}(x), \overline{A}(y)\}, x, y \in V$ and $\alpha, \beta \in F(\text{field})$. 
Definition 2.6[4]. Let f be a mapping from a set X into a set Y. Let B be an interval-valued fuzzy set in Y. Then the inverse image of B, i.e., \( f^{-1}[B] \) is the interval-valued fuzzy set in X with the membership function given by \( \overline{\mu}_{f^{-1}[B]}(x) = \overline{\mu}_B(f(x)), \forall x \in X \).

Definition 2.7[4]. Let f be a mapping from a set X into a set Y. Let A be an interval-valued fuzzy set in X. Then the image of A, i.e., \( f[A] \) is the interval-valued fuzzy set in Y with the membership function defined by

\[
\overline{\mu}_{f[A]}(y) = \begin{cases} 
  \sup_{z \in f^{-1}(y)} \overline{\mu}_A(z), & \text{if } f^{-1}(y) \neq \phi, \forall y \in Y, \\
  [0,0], & \text{otherwise,}
\end{cases}
\]

where \( f^{-1}(y) = \{ x : f(x) = y \} \) and [0,0] denotes the interval-valued fuzzy empty set.

Definition 2.8[11]. Let \( A=(\tilde{\mu}_A, \tilde{\lambda}_A) \) and \( B=(\tilde{\mu}_B, \tilde{\lambda}_B) \) be interval-valued intuitionistic fuzzy sets on \( L \). Then generalized cartesian product \( A \times B \) is defined as follow: \( A\times B=(\tilde{\mu}_A \times \tilde{\mu}_B, \tilde{\lambda}_A \times \tilde{\lambda}_B) \), where \( (\tilde{\mu}_A \times \tilde{\mu}_B)(x,y)=\min\{\tilde{\mu}_A(x), \tilde{\mu}_B(y)\} \) and \( (\tilde{\lambda}_A \times \tilde{\lambda}_B)(x,y)=\max\{\tilde{\lambda}_A(x), \tilde{\lambda}_B(y)\} \).

Definition 2.9[18]. Let X be a field and \( \overline{A} \) be an interval-valued fuzzy set on X. If the following conditions hold:

(i) \( \overline{A}(x+y) \geq \min\{\overline{A}(x), \overline{A}(y)\}, x, y \in X; \)
(ii) \( \overline{A}(-x) = \overline{A}(x), x \in X; \)
(iii) \( \overline{A}(xy) \geq \min\{\overline{A}(x), \overline{A}(y)\}, x, y \in X; \)
(iv) \( \overline{A}(x^{-1}) = \overline{A}(x), (x \neq 0) \in X. \)

Then \( \overline{A} \) is said to be an interval-valued fuzzy field on X or briefly i-v fuzzy field on X, denoted by \( (\overline{A}, X) \).

Example 2.10[18]. Consider a field \( Z_5 = \{0, 1, 2, 3, 4\} \) with following Cayley tables:

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Let $A: Z_5 \to D[0,1]$ be an interval-valued fuzzy set defined by

$$
A(x) = \begin{cases} 
[0.8, 0.9], & \text{if } x = 0 \\
[0.6, 0.7], & \text{otherwise}
\end{cases}
$$

Clearly $A$ is an interval-valued fuzzy field on $Z_5$.

**Definition 2.11**[18]. Let $X$ be a field and $(\overline{A}, X)$ be an interval-valued fuzzy field of $X$. Let $Y$ be an linear space over $X$ and $\overline{V}$ be an interval-valued fuzzy set of $Y$. Suppose the following conditions hold:

1. $\overline{V}(x + y) \geq \min\{\overline{V}(x), \overline{V}(y)\}, x, y \in Y$;
2. $\overline{V}(-x) = -\overline{V}(x), x \in Y$;
3. $\overline{V}(\lambda x) \geq \min\{\overline{A}(\lambda), \overline{V}(x)\}, \lambda \in X, x \in Y$;
4. $\overline{A}(1) \geq \overline{V}(0)$.

Then $(\overline{V}, Y)$ is called an interval-valued fuzzy linear space or briefly i-v fuzzy linear space over $(\overline{A}, X)$.

**Example 2.12**[18]. Consider a linear space $Z_3 = \{0, 1, 2\}$ with the following Cayley tables:

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Here $Z_3$ is a subset of $Z_5$ and

Let $\overline{A}: Z_5 \to D[0,1]$ and
Let $\overline{V} : Z_3 \rightarrow D[0, 1]$ be a fuzzy subsets defined by

$$\overline{A}(x) = \begin{cases} [0.8, 0.9], & \text{if } x = 0 \\ [0.6, 0.7], & \text{otherwise} \end{cases}$$

and

$$\overline{V}(x) = \begin{cases} [0.5, 0.6], & \text{if } x = 0 \\ [0.3, 0.4], & \text{otherwise} \end{cases}$$

Clearly $\overline{V}$ is an interval-valued fuzzy linear space of $Z_3$ over $Z_5$.

**Definition 2.13**[18]. Let $(\overline{V}, Y)$ and $(\overline{W}, Y)$ be two interval-valued fuzzy linear space over an interval-valued fuzzy field $(\overline{A}, X)$. If $\overline{W} \subset \overline{V}$, then $(\overline{W}, Y)$ is said to be an interval-valued fuzzy linear subspace of $(\overline{V}, Y)$.

**Theorem 2.14**[18]. If $(\overline{A}, X)$ is an interval-valued fuzzy field of $X$, then

(i) $\overline{A}(0) \geq \overline{A}(x)$, $x \in X$;

(ii) $\overline{A}(1) \geq \overline{A}(x)$, $x(\neq 0) \in X$.

**Remark 2.15**[18]. If $(\overline{A}, X)$ is an interval-valued fuzzy field of $X$, then $\overline{A}(0) \geq \overline{A}(1)$.

**Remark 2.16**[18]. If $(\overline{V}, Y)$ is an interval-valued fuzzy linear space over $(\overline{A}, X)$, then

(i) $\overline{A}(0) \geq \overline{V}(0)$;

(ii) $\overline{V}(0) \geq \overline{V}(x), x \in Y$;

(iii) $\overline{A}(1) \geq \overline{V}(x), x \in Y$.

**Theorem 2.17**[18]. Let $(\overline{A}, X)$ be an interval-valued fuzzy field of $X$, and $Y$ a linear space over $X$. Assume $\overline{V}$ is an interval-valued fuzzy set of $Y$. Then $(\overline{V}, Y)$ is an interval-valued fuzzy linear space over $(\overline{A}, X)$ iff

(i) $\overline{V}(\lambda x + \mu y) \geq \min \{ \min \overline{A}(\lambda), \overline{V}(x) \}, \min \overline{A}(\mu), \overline{V}(y) \}$, $\lambda, \mu \in X$ and $x, y \in Y$.

(ii) $\overline{A}(1) \geq \overline{V}(x), x \in Y$.

**Theorem 2.18**[18]. The intersection of a family of interval-valued fuzzy linear spaces is an interval-valued fuzzy linear space.
3 Main results

We now introduce the notion of cartesian product of two interval-valued fuzzy linear spaces in the following theorem.

Theorem 3.1. Let $(\overline{A},X)$ be an interval-valued fuzzy field of $X$. Let $(\overline{V}_1,Y_1), (\overline{V}_2,Y_2)$ be interval-valued fuzzy linear spaces over $(\overline{A},X)$. Then $(\overline{V}_1 \times \overline{V}_2, Y_1 \times Y_2)$ is an interval-valued fuzzy linear space over $(\overline{A},X)$.

Proof. Let $\overline{V} = \overline{V}_1 \times \overline{V}_2$.
Let $u = (u_1,u_2), v = (v_1,v_2) \in Y_1 \times Y_2$, and $\lambda, \mu \in X$
(i) $\overline{V}(\lambda u + \mu v)$
\[= (\overline{V}_1 \times \overline{V}_2)(\lambda u_1 + \mu v_1, \lambda u_2 + \mu v_2)\]
\[= \min_{j=1,2} \overline{V}_j(\lambda u_j + \mu v_j)\]
\[\geq \min_{j=1,2} \{ \min[\overline{V}_j(\lambda u_j), \overline{V}_j(\mu v_j)] \}\]
\[\geq \min_{j=1,2} \{ \min[\overline{A}(\lambda), \overline{V}_j(u_j)], \min[\overline{A}(\mu), \overline{V}_j(v_j)] \}\]
\[= \min \{ \min[\overline{A}(\lambda), \min_{j=1,2} \overline{V}_j(u_j)], \min[\overline{A}(\mu), \min_{j=1,2} \overline{V}_j(v_j)] \}\]
\[= \min \{ \min[\overline{A}(\lambda), \overline{V}(u)], \min[\overline{A}(\mu), \overline{V}(v)] \}\]
(ii) $\overline{A}(1) \geq \overline{V}_j(u_j)$ for all $j=1,2$.
So,
$\overline{A}(1) \geq \min_{j=1,2} \overline{V}_j(u_j) = \overline{V}(u)$ for all $u \in Y_1 \times Y_2$.

Hence $(\overline{V}_1 \times \overline{V}_2, Y_1 \times Y_2)$ is an interval-valued fuzzy linear space over $(\overline{A},X)$. $\square$

Theorem 3.2. Let $X_1$ and $X_2$ be fields and $f : X_1 \rightarrow X_2$ be a homomorphism. Suppose that $(\overline{A}_1,X_1)$ is an interval-valued fuzzy field of $X_1$ and $(\overline{A}_2,X_2)$ is an interval-valued fuzzy field of $X_2$. Then
(i) $(f(\overline{A}_1), X_2)$ is an interval-valued fuzzy field of $X_2$.
(ii) $(f^{-1}(\overline{A}_2), X_1)$ is an interval-valued fuzzy field of $X_1$.

Proof. (i) Let $u, v \in X_2$.
(a) If either $f^{-1}(u) = \phi$ or $f^{-1}(v) = \phi$, then their $f(\overline{A}_1)(u) = 0$ or $f(\overline{A}_1)(v) = 0$
So, $f(\overline{A}_1)(u + v) \geq 0 = \min \{ f(\overline{A}_1)(u), f(\overline{A}_1)(v) \}$
Suppose that neither $f^{-1}(u) = \phi$ nor $f^{-1}(v) = \phi$.
Then $f^{-1}(u + v) \neq \phi$.
Let $r \in f^{-1}(u)$ and $s \in f^{-1}(v)$.
Then $r + s \in f^{-1}(u + v)$, so $r + s \in \{ w : w \in f^{-1}(u + v) \}$.
Therefore $\{ r + s : r \in f^{-1}(u), s \in f^{-1}(v) \} \subseteq \{ w : w \in f^{-1}(u + v) \}$ (3.1)
Now, $f(\overline{A}_1)(u + v)$
\[ f(\overline{A}_1)(u) = \sup_{w \in f^{-1}(u+v)} \overline{A}_1(w). \]
\[ \geq \sup_{r \in f^{-1}(u), s \in f^{-1}(v)} \overline{A}_1(r + s), \text{ by the expression (3.1)} \]
\[ \geq \sup_{r \in f^{-1}(u), s \in f^{-1}(v)} \min \{ \overline{A}_1(r), \overline{A}_1(s) \} \]
\[ = \min \{ \sup_{r \in f^{-1}(u)} \overline{A}_1(r), \sup_{s \in f^{-1}(v)} \overline{A}_1(s) \} \]
\[ = \min \{ f(\overline{A}_1)(u), f(\overline{A}_1)(v) \}. \]
\[ \Rightarrow f(\overline{A}_1)(u + v) \geq \min \{ f(\overline{A}_1)(u), f(\overline{A}_1)(v) \}. \]

(b) \[ f(\overline{A}_1)(-u) = \sup \{ \overline{A}(r) : f(r) = -u \} = \sup \{ \overline{A}(r) : f(r) = u \} \]
\[ = \sup \{ \overline{A}(-r) : f(-r) = u \} = \sup \{ \overline{A}(s) : f(s) = u \} \]
\[ = f(\overline{A}_1)(u). \]

(c) As in (a), \[ f(\overline{A}_1)(uv) \geq \min \{ f(\overline{A}_1)(u), f(\overline{A}_1)(v) \}. \]
(d) As in (b), if \( u \neq 0 \), then \( f(\overline{A}_1)(u^{-1}) = f(\overline{A}_1)(u) \)

Hence \( (f(\overline{A}_1), X_2) \) is an interval-valued fuzzy field of \( X_2 \).

(ii) Let \( r, s \in X_1 \).
\[ f^{-1}(\overline{A}_2)(r + s) = \overline{A}_2(f(r + s)) \]
\[ = \overline{A}_2(f(r) + f(s)) \text{ (since } f \text{ is homomorphism)} \]
\[ \geq \min \{ \overline{A}_2(f(r)), \overline{A}_2(f(s)) \} \]
\[ = \min \{ f^{-1}(\overline{A}_2)(r), f^{-1}(\overline{A}_2)(s) \} \]

and
\[ f^{-1}(\overline{A}_2)(-r) = \overline{A}_2(f(-r)) = \overline{A}_2(-f(r)) \]
\[ = \overline{A}_2(f(r)) \]
\[ = f^{-1}(\overline{A}_2)(r). \]

Similarly, \( f^{-1}(\overline{A}_2)(rs) \geq \min \{ f^{-1}(\overline{A}_2)(r), f^{-1}(\overline{A}_2)(s) \} \) and
\( f^{-1}(\overline{A}_2)(r^{-1}) = f^{-1}(\overline{A}_2)(r) \) if \( r \neq 0 \).

Hence \( (f^{-1}(\overline{A}_2), X_1) \) is an interval-valued fuzzy field of \( X_1 \). \[ \square \]

**Theorem 3.3.** Let \( Y \) and \( Z \) be linear spaces over the field \( X \), and \( f \) a linear transformation of \( Y \) into \( Z \). Let \( (\overline{A}, X) \) be an interval-valued fuzzy field of \( X \), and \( (\overline{W}, Z) \) be an interval-valued fuzzy linear space over \( (\overline{A}, X) \). Then \( (f^{-1}(\overline{W}), Y) \) is an interval-valued fuzzy linear space over \( (\overline{A}, X) \).

**Proof.** (i) For all \( \lambda, \mu \in X \) and \( u, v \in Y \),
\[ f^{-1}(\overline{W})(\lambda u + \mu v) = \overline{W}(f(\lambda u + \mu v)) \]
\[ \geq \min\{\min[\overline{A}(\lambda), \overline{W}(f(u))], \min[\overline{A}(\mu), \overline{W}(f(v))]\} \]
\[ = \min\{\min[\overline{A}(\lambda), f^{-1}(\overline{W})(u)], \min[\overline{A}(\mu), f^{-1}(\overline{W})(v)]\} \]

(ii) Since \((\overline{W}, Z)\) is an interval-valued fuzzy linear space over \((\overline{A}, X)\), for all \(u \in Y\), \(\overline{A}(1) \geq \overline{W}(f(u)) = f^{-1}(\overline{W})(u)\)
Therefore, \((f^{-1}(\overline{W}), Y)\) is an interval-valued fuzzy linear space over \((\overline{A}, X)\).

**Theorem 3.4.** Let \(Y\) and \(Z\) be linear spaces over the field \(X\), and \(f\) a linear transformation of \(Y\) into \(Z\). Let \((\overline{A}, X)\) be an interval-valued fuzzy field of \(X\) and \((\overline{V}, Y)\) be an interval-valued fuzzy linear space over \((\overline{A}, X)\). Then \((f(\overline{V}), Z)\) is an interval-valued fuzzy linear space over \((\overline{A}, X)\).

**Proof.** For all \(\lambda, \mu \in X\) and \(u, v \in Z\), if either \(f^{-1}(u)\) or \(f^{-1}(v)\) is empty,
then either \(f(\overline{V})(u) = 0\) or \(f(\overline{V})(v) = 0\),
so \(f(\overline{V})(\lambda u + \mu v) \geq 0 = \min\{\min[\overline{A}(\lambda), f(\overline{V})(u)], \min[\overline{A}(\mu), f(\overline{V})(v)]\}\),
\(\lambda, \mu \in X\) and \(x, y \in Y\), the inequality \((i)\) of Theorem 2.17 is satisfied.

Suppose neither \(f^{-1}(u)\) nor \(f^{-1}(v)\) is empty, then \(f^{-1}(\lambda u + \mu v) \neq \phi\).
Let \(r \in f^{-1}(u), s \in f^{-1}(v)\).
Then
\[ f(\lambda r + \mu s) = f(r) + \mu f(s) = \lambda u + \mu v. \]

So,
\[ f(\overline{V})(\lambda u + \mu v) \]
= \[ \sup_{w \in f^{-1}(\lambda u + \mu v)} \overline{V}(w) \]
\[ \geq \sup_{r \in f^{-1}(u), s \in f^{-1}(v)} \overline{V}(\lambda r + \mu s) \]
\[ \geq \sup_{r \in f^{-1}(u), s \in f^{-1}(v)} \min\{\min[\overline{A}(\lambda), \overline{V}(r)], \min[\overline{A}(\mu), \overline{V}(s)]\} \]
\[ = \min\{\min[\overline{A}(\lambda), \sup_{r \in f^{-1}(u)} \overline{V}(r)], \min[\overline{A}(\lambda), \sup_{s \in f^{-1}(v)} \overline{V}(s)]\} \]
\[ = \min\{\min[\overline{A}(\lambda), f(\overline{V})(u)], \min[\overline{A}(\mu), f(\overline{V})(v)]\} \]

Obviously, for any \(u \in Z\), \(\overline{A}(1) \geq f(\overline{V})(u)\).
Thus \((f(\overline{V}), Z)\) is an interval-valued fuzzy linear space over \((\overline{A}, X)\), which ends
the proof. \(\square\)

**Theorem 3.5.** If \((\overline{V}, Y)\) is an interval-valued fuzzy linear space over \((\overline{A}, X)\),
then the nonempty set \(X_V = \min\{a \in X: \overline{A}(a) \geq \overline{V}(u), \forall u \in Y\}\) is a subfield
of \(X\). Also, \((\overline{A}, X_V)\) is an interval-valued fuzzy field of \(X_V\) and \((\overline{V}, Y)\) is an
interval-valued fuzzy linear space over \((\overline{A}, X_V)\).
Proof. Let \(a, b \in X_V\).
Then for all \(u \in Y\),
\[\overline{A}(a - b) \geq \min\{\overline{A}(a), \overline{A}(b)\} \geq \overline{V}(u)\]
and for \(a, b \neq 0\),
\[\overline{A}(ab^{-1}) \geq \min\{\overline{A}(a), \overline{A}(b)\} \geq \overline{V}(u)\]
Therefore \(a - b \in X_V\) and \(ab^{-1} \in X_V\) if \(a, b \neq 0\).
Therefore \(X_V\) is a subfield of \(X\).

The second part is trivial.

\[\square\]

4 Open Problem

We suggest some Open Problems as follows:

(i) Construction of interval-valued fuzzy norm on interval-valued fuzzy linear space.

(ii) Construction of interval-valued fuzzy inner product on interval-valued fuzzy linear space.

(iii) Interrelating interval-valued fuzzy norm and interval-valued fuzzy inner product space.

References


