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Random coincidence and fixed point theorem for hybrid maps in separable metric spaces

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Abstract

In this paper, we establish a random coincidence and random fixed point theorem for hybrid contractions consisting of two single-valued and two multivalued mappings in separable metric spaces. Some special results are stated. The results improve and extend some known results.

Keywords: *Common random fixed points, (IT) commuting mappings, Orbitally complete, Random coincidence, Separable metric space.*

1 Introduction and preliminaries

A random version of a fixed point theorem for multivalued contraction mapping of Nadler (1969) [21] was given by Itoh (1977) [14]. Random fixed point theorems for contraction mappings on separable complete metric spaces have been proved by several authors (see e.g. [1]-[13], [19]- [22], [25]), and many authors.

Throughout this paper, let (X, d) be a separable metric space and (Ω, Σ) is a measurable space. Let 2^X be a family of all subsets of X, $CB(X)$ denote the family of all non-empty bounded closed subsets of X and H denotes the Housdorff metric on $CB(X)$ induced by the metric d on X, that is for $A, B \in CB(X)$,

$$H(A, B) = \max\{\sup_{a\in A} d(a, B), \sup_{b\in B} d(A, b)\},$$

where $d(x, E)$ is the distance from a point $x \in X$ to a subset $E \subset X$, that is $d(x, E) = \inf\{d(x, y) : y \in E\}$. A mapping $T : \Omega \to 2^X$ is called measurable if $T^{-1}(C) = \{w \in \Omega : T(w) \cap C \neq \phi\} \in \Sigma$ for all open subsets C of X.
A mapping $\xi : \Omega \to X$ is called a measurable selector of a measurable mapping $T : \Omega \to 2^X$ if ξ is measurable and $\xi(w) \in T(w)$ for each $w \in \Omega$. A mapping $f : \Omega \times X \to X$ is said to be random mapping if for each $x \in X$, the mapping $f(., x) : \Omega \to X$ is measurable. A mapping $T : \Omega \times X \to CB(X)$ is said to be random multivalued mapping if for each $x \in X$, $T(., x)$ is measurable. A measurable mapping $\xi : \Omega \to X$ is called a random fixed point of the random multivalued mapping $T : \Omega \times X \to CB(X)$, $(f : \Omega \times X \to X)$ if $\xi(w) \in T(w, \xi(w))$, $(\xi(w) = f(w, \xi(w)))$ for each $w \in \Omega$. A measurable mapping $\xi : \Omega \to X$ is called a random coincidence point of $T : \Omega \times X \to CB(X)$ and $f : \Omega \times X \to X$ if $f(w, \xi(w)) \in T(w, \xi(w))$ for each $w \in \Omega$. We denote the set of all coincidence points of the pair (f, T) by $C(f, T)$.

Definition 1.1 *[16],[6] The random mappings $f : \Omega \times X \to X$ and $T : \Omega \times X \to CB(X)$ are compatible if and only if $f(w, T(w, \xi(w))) \in CB(X)$ for each $\xi(w) \in X$, $w \in \Omega$ and $\lim_{n\to\infty} H(f(w, T(w, \xi_n)), T(w, f(w, \xi_n))) = 0$, whenever ξ_n is a sequence in X such that $\lim_{n\to\infty} T(w, \xi_n) = M \in CB(X)$, $\lim_{n\to\infty} f(w, \xi_n) = t \in M$.*

Definition 1.2 *[17] Random operators $f : \Omega \times X \to X$ and $T : \Omega \times X \to CB(X)$ are weakly compatible if $f(w, \xi(w)) \in T(w, \xi(w))$, for some measurable mappings ξ, then $T(w, f(w, \xi(w))) = f(w, T(w, \xi(w)))$ for every $w \in \Omega$.*

Motivated by definition of (IT)-commuting on metric spaces given in [14] and [24], we define the (IT)-commuting mappings in random spaces.

Definition 1.3 *[14] Random mappings $f : \Omega \times X \to X$ and $T : \Omega \times X \to CB(X)$ are (IT)-commuting at $\xi(w) \in X$ if $f(w, T(w, \xi(w))) \subseteq T(w, f(w, \xi(w)))$, $w \in \Omega$,where $\xi : \Omega \to X$ is a measurable mapping.*

Remark 1.4 *It is well known that the compatible maps T, f are weakly compatible (see [17]). In addition, (IT)-commutativity of a a hybrid pair T and f at a coincidence point $x \in X$ is more general than its compatibility and weak compatibility at the same point. The following example shows that and (see also, Example 1 in [23]).*

Example 1.5 *Let $X = [0, \infty), \Omega = [0, 1]$ and Σ be the sigma algebra of Lebesgue's measurable subset of $[0, 1]$ with usual metric and suppose: $f(w, \xi(w)) = 2\xi(w)$, $T(w, \xi(w)) = [\xi(w) + 2(1 - w^2), \infty)$, then*

$$\begin{aligned} f(w, 2(1-w^2)) &= 4(1-w^2), \\ T(w, 2(1-w^2)) &= [2(1-w^2) + 2(1-w^2), \infty) = [4(1-w^2), \infty), w \in \Omega, \end{aligned}$$

then $f(w, 2(1-w^2)) \in T(w, 2(1-w^2))$ *and*

$$f(w, T(w, 2(1-w^2))) = [8(1-w^2), \infty) \subset [6(1-w^2), \infty) = T(w, f(w, 2(1-w^2))),$$

hence f *and* T *are (IT)-commuting at the coincidence point* $\xi(w) = 2(1-w^2)$ *also the pair* (f, T) *is not weakly compatible since* $f(w, T(w, 2(1-w^2))) \neq T(w, f(w, 2(1-w^2)))$.

Jhade et al. [15] studied the coincidence and fixed point theorems for a multivalued mapping T and self mapping f under the following nonexpansive type condition:

$$\begin{aligned} H(Tx, Ty) &\leq a(x,y)d(fx, fy) + b(x,y)\max\{d(fx, Tx), d(fy, Ty)\} \\ &+ c(x,y)\max\{d(fx, fy), d(fx, Tx), d(fy, Ty)\} \\ &+ e(x,y)\max\{d(fx, fy), d(fx, Tx), d(fy, Ty), d(fx, Ty)\}, \quad (1) \end{aligned}$$

where a, b, c, e are nonnegative functions from $X \times X \to [0, 1)$, such that $\beta = \inf_{x,y\in X} e(x,y) > 0$ and$\gamma = \inf_{x,y\in X}(1 + b(x,y) + e(x,y)) > 0$ with $\sup_{x,y\in X}(a + b + c + 2e)(x,y) = 1$.

Motivated by the work of [15], we present the stochastic version of coincidence and common fixed point theorems by extending the contractive condition (1) to a pair of multivalued random mappings T, S and a pair of self random mappings f, g. The contractive condition is defined as follows:

Condition A: Let $T, S : \Omega \times X \to CB(X)$ be two random multivalued mappings and $f, g : \Omega \times X \to X$ are self random mappings satisfying the following condition:

$$\begin{aligned} H(T(w,x), S(w,y)) &\leq a(w)d(f(w,x), g(w,y)) + b(w)\max\{d(f(w,x), T(w,x)), d(g(w,y), S(w,y))\} \\ &+ c(w)\max\{d(f(w,x), g(w,y)), d(f(w,x), T(w,x)), d(g(w,y), S(w,y))\} \\ &+ e(w)\max\{d(f(w,x), g(w,y)), d(f(w,x), T(w,x)), d(g(w,y), S(w,y)) \\ &, d(f(w,x), S(w,y))\}, \end{aligned}$$

for all $x, y \in X$ and for all $w \in \Omega$, where $a(w), b(w), c(w), e(w) : \Omega \to (0, 1)$ are measurable mappings such that, $a(w) + b(w) + c(w) + 2e(w) = 1$.

We also give the following definition:

Definition 1.6 *Let* (X, d) *be a separable metric space, Let* $T, S : \Omega \times X \to CB(X)$ *be two random multivalued mappings and* $f, g : \Omega \times X \to X$ *are self random mappings. Suppose that* $\xi_0(w) \in X$. *Then the set*

$$O(f, g, T, S; \xi_0) = \begin{cases} y_i : y_i = f(w, \xi_i(w)) \in T(w, \xi_{i-1}(w)), & \text{for i=2n+1;} \\ y_i = g(w, \xi_i(w)) \in S(w, \xi_{i-1}(w)), & \text{for i=2n+2.} \end{cases} \quad (2)$$

for $n \geq 0$ *and for all* $w \in \Omega$ *is called an orbit of* (T, S, f, g) *at* ξ_0. *A metric space* X *is called* (T, S, f, g)*- orbitally complete if and only if every Cauchy sequence in the orbit of* (T, S, f, g) *at* ξ_0 *is convergent in* X.

2 Main Results

In this section, we will obtain a random coincidence and a common random fixed point of (IT)-commuting random mappings under a condition A.

Theorem 2.1 *Let X be a separable metric space. Let $T, S : \Omega \times X \to CB(X)$ be two continuous random multivalued mappings and $f, g : \Omega \times X \to X$ are self random mappings satisfying Condition A such that for $w \in \Omega$, $T(w, X) \subseteq f(w, X)$ and $S(w, X) \subseteq g(w, X)$. If there exists $\xi_0(w) \in X$ such that $f(w, X) \cap g(w, X)$ is (T, S, f, g)-orbitally complete at ξ_0 for each $w \in \Omega$, then*
(I) The pair (f, T) has a random coincidence point, i.e. there is $\eta(w)$ such that $f(w, \eta(w)) \in T(w, \eta(w)), w \in \Omega$.
(II) The pair (g, S) has a random coincidence point, i.e. there is $\zeta(w)$ such that $g(w, \zeta(w)) \in S(w, \zeta(w)), w \in \Omega$.
Further:
(III) T and f have a common random fixed point $f(w, \eta(w))$ provided that $f(w, \eta(w))$ is a random fixed point of f for some $\eta(w) \in C(f, T)$ and T and f are (IT)-commuting at $\eta(w)$.
(IV) S and g have a common random fixed point $g(w, \zeta(w))$ provided $g(w, \zeta(w))$ is a random fixed point of g for some $\zeta(w) \in C(g, S)$ and S and g are (IT)-commuting at $\zeta(w)$.
(V) T, S, f and g have a common random fixed point provided (I) and (II) both are true.

proof. Let $\xi_0 : \Omega \to X$ be arbitrary measurable function on Ω. Since $T(w, X) \subseteq f(w, X)$ and T is continuous, then for every $u \in X$, the map $(w, x) \to d(u, T(w, x))$ is a caratheodory function (that is measurable in $w \in \Omega$, continues in $x \in X$). Thus it is jointly measurable. Hence, since $\xi_0 : \Omega \to X$ is measurable, $w \to d(u, T(w, \xi_0(w)))$ is measurable too, therefore $w \to T(w, \xi_0(w))$ is weakly measurable by Wagner ([26], p. 868). By Kuratowski ([18], Theorem 8), there exists a measurable map $\xi_1 : \Omega \to X$ such that $y_1 = f(w, \xi_1(w)) \in T(w, \xi_0(w))$ for $w \in \Omega$ and similarly since S is continuous and $S(w, X) \subseteq g(w, X)$, for $\xi_1 : \Omega \to X$ we can choose another function $\xi_2 : \Omega \to X$ such that for $w \in \Omega$, $g(w, \xi_2(w)) \in S(w, \xi_1(w))$ such that $y_1 \neq y_2$ and $d(y_1, y_2) \leq H(T(w, \xi_0(w)), S(w, \xi_1(w)))$.
In general, we can define a sequence of functions $\{y_n(w)\}$, $w \in \Omega$ as follows:

$$\begin{aligned} y_{2n+1}(w) &= f(w, \xi_{2n+1}(w)) \in T(w, \xi_{2n}(w)), \\ y_{2n+2}(w) &= g(w, \xi_{2n+2}(w)) \in S(w, \xi_{2n+1}(w)), \end{aligned} \tag{3}$$

From Condition A, we have

$$\begin{aligned}
d(y_{2n+1}(w),y_{2n+2}(w)) &\leq H(T(w,\xi_{2n}(w)),S(w,\xi_{2n+1}(w)))\\
&\leq a(w)d(f(w,\xi_{2n}(w)),g(w,\xi_{2n+1}(w)))\\
&+ b(w)\max\{d(f(w,\xi_{2n}(w)),T(w,\xi_{2n}(w))),d(g(w,\xi_{2n+1}(w)),S(w,\xi_{2n+1}(w)))\}\\
&+ c(w)\max\{d(f(w,\xi_{2n}(w)),g(w,\xi_{2n+1}(w))),d(f(w,\xi_{2n}(w)),T(w,\xi_{2n}(w)))\\
&, d(g(w,\xi_{2n+1}(w)),S(w,\xi_{2n+1}(w)))\}\\
&+ e(w)\max\{d(f(w,\xi_{2n}(w)),g(w,\xi_{2n+1}(w))),d(f(w,\xi_{2n}(w)),T(w,\xi_{2n}(w)))\\
&, d(g(w,\xi_{2n+1}(w)),S(w,\xi_{2n+1}(w))),d(f(w,\xi_{2n}(w)),S(w,\xi_{2n+1}(w)))\},
\end{aligned}$$

It follows by (3) that

$$\begin{aligned}
d(y_{2n+1}(w),y_{2n+2}(w)) &\leq a(w)d(y_{2n}(w),y_{2n+1}(w))\\
&+ b(w)\max\{d(y_{2n}(w),y_{2n+1}(w)),d(y_{2n+1}(w),y_{2n+2}(w))\}\\
&+ c(w)\max\{d(y_{2n}(w),y_{2n+1}(w)),d(y_{2n}(w),y_{2n+1}(w))\\
&, d(y_{2n+1}(w),y_{2n+2}(w))\}\\
&+ e(w)\max\{d(y_{2n}(w),y_{2n+1}(w)),d(y_{2n}(w),y_{2n+1}(w))\\
&, d(y_{2n+1}(w),y_{2n+2}(w)),d(y_{2n}(w),y_{2n+2}(w))\},
\end{aligned}$$

using triangle inequality, we get

$$\begin{aligned}
d(y_{2n+1}(w),y_{2n+2}(w)) &\leq a(w)d(y_{2n}(w),y_{2n+1}(w))\\
&+ b(w)\max\{d(y_{2n}(w),y_{2n+1}(w)),d(y_{2n+1}(w),y_{2n+2}(w))\}\\
&+ c(w)\max\{d(y_{2n}(w),y_{2n+1}(w)),d(y_{2n}(w),y_{2n+1}(w))\\
&, d(y_{2n+1}(w),y_{2n+2}(w))\}\\
&+ e(w)\max\{d(y_{2n}(w),y_{2n+1}(w)),d(y_{2n}(w),y_{2n+1}(w))\\
&, d(y_{2n+1}(w),y_{2n+2}(w))\\
&, d(y_{2n}(w),y_{2n+1}(w))+d(y_{2n+1}(w),y_{2n+2}(w))\}, \qquad (4)
\end{aligned}$$

If for some n, $d(y_{2n+1}(w),y_{2n+2}(w)) > d(y_{2n}(w),y_{2n+1}(w))$, (4) becomes:

$d(y_{2n+1}(w),y_{2n+2}(w)) < (a(w)+b(w)+c(w)+2e(w))d(y_{2n+1}(w),y_{2n+2}(w)).$

A contradiction. Thus

$$d(y_{2n+1}(w),y_{2n+2}(w)) \leq d(y_{2n}(w),y_{2n+1}(w)). \qquad (5)$$

Again from Condition A, we have

$$\begin{aligned}
d(f(w,\xi_{2n-1}),S(w,\xi_{2n})) &\leq H(T(w,\xi_{2n-2}(w)),S(w,\xi_{2n}(w)))\\
&\leq a(w)d(f(w,\xi_{2n-2}(w)),g(w,\xi_{2n}(w)))\\
&+ b(w)\max\{d(f(w,\xi_{2n-2}(w)),T(w,\xi_{2n-2}(w))),d(g(w,\xi_{2n}(w)),S(w,\xi_{2n}(w)))\}\\
&+ c(w)\max\{d(f(w,\xi_{2n-2}(w)),g(w,\xi_{2n}(w))),d(f(w,\xi_{2n-2}(w)),T(w,\xi_{2n-2}(w)))\\
&, d(g(w,\xi_{2n}(w)),S(w,\xi_{2n}(w)))\}\\
&+ e(w)\max\{d(f(w,\xi_{2n-2}(w)),g(w,\xi_{2n}(w))),d(f(w,\xi_{2n-2}(w)),T(w,\xi_{2n-2}(w)))\\
&, d(g(w,\xi_{2n}(w)),S(w,\xi_{2n}(w))),d(f(w,\xi_{2n-2}(w)),S(w,\xi_{2n}(w)))\},
\end{aligned}$$

using (3), we obtain

$$\begin{aligned}
d(f(w,\xi_{2n-1}),S(w,\xi_{2n})) &\leq a(w)d(y_{2n-2}(w),y_{2n}(w))\\
&+ b(w)\max\{d(y_{2n-2}(w),y_{2n-1}(w)),d(y_{2n}(w),y_{2n+1}(w))\}\\
&+ c(w)\max\{d(y_{2n-2}(w),y_{2n}(w)),d(y_{2n-2}(w),y_{2n-1}(w))\\
&, d(y_{2n}(w),y_{2n+1}(w))\}\\
&+ e(w)\max\{d(y_{2n-2}(w),y_{2n}(w)),d(y_{2n-2}(w),y_{2n-1}(w))\\
&, d(y_{2n}(w),y_{2n+1}(w)),d(y_{2n-2}(w),y_{2n+1}(w))\},
\end{aligned}$$

by triangle inequality and (5), we obtain

$$\begin{aligned}
d(f(w,\xi_{2n-1}),S(w,\xi_{2n})) &\leq 2a(w)d(y_{2n-2}(w),y_{2n-1}(w))+b(w)d(y_{2n-2}(w),y_{2n-1}(w))\\
&+ 2c(w)d(y_{2n-2}(w),y_{2n-1}(w))\\
&+ e(w)\max\{2d(y_{2n-2}(w),y_{2n-1}(w)),3d(y_{2n-2}(w),y_{2n-1}(w))\}\\
&= (2a(w)+b(w)+2c(w)+3e(w))d(y_{2n-2}(w),y_{2n-1}(w)), \qquad (6)
\end{aligned}$$

since $a(w) + b(w) + c(w) + 2e(w) = 1$, then

$$d(f(w,\xi_{2n-1}),S(w,\xi_{2n})) \leq (2-b(w)-e(w))d(y_{2n-2}(w),y_{2n-1}(w)). \qquad (7)$$

Again, using Condition A,

$$\begin{aligned}
d(y_{2n}(w),y_{2n+1}(w)) &\leq H(T(w,\xi_{2n-1}(w)),S(w,\xi_{2n}(w)))\\
&\leq a(w)d(f(w,\xi_{2n-1}(w)),g(w,\xi_{2n}(w)))\\
&+ b(w)\max\{d(f(w,\xi_{2n-1}(w)),T(w,\xi_{2n-1}(w))),d(g(w,\xi_{2n}(w)),S(w,\xi_{2n}(w)))\}\\
&+ c(w)\max\{d(f(w,\xi_{2n-1}(w)),g(w,\xi_{2n}(w))),d(f(w,\xi_{2n-1}(w)),T(w,\xi_{2n-1}(w)))\\
&, d(g(w,\xi_{2n}(w)),S(w,\xi_{2n}(w)))\}\\
&+ e(w)\max\{d(f(w,\xi_{2n-1}(w)),g(w,\xi_{2n}(w))),d(f(w,\xi_{2n-1}(w)),T(w,\xi_{2n-1}(w)))\\
&, d(g(w,\xi_{2n}(w)),S(w,\xi_{2n}(w))),d(f(w,\xi_{2n-1}(w)),S(w,\xi_{2n}(w)))\}. \qquad (8)
\end{aligned}$$

Substituting (7) into the last term of (8) and using (5), we get

$$\begin{aligned} d(y_{2n}(w),y_{2n+1}(w)) &\leq [a(w)+b(w)+c(w)+e(w)(2-b(w)-e(w))]d(y_{2n-2}(w),y_{2n-1}(w)) \\ &= [1-e(w)b(w)-e^2(w)]d(y_{2n-2}(w),y_{2n-1}(w)) \\ &= kd(y_{2n-2}(w),y_{2n-1}(w)) \\ &\leq k^n d(y_0(w),y_1(w)) \end{aligned}$$

where $k = 1-(e(w)b(w)+e^2(w)) < 1$. In general,

$$d(y_n(w),y_{n+1}(w)) \leq k^{\frac{n}{2}} d(y_0(w),y_1(w)), w \in \Omega.$$

Now, for positive integer $m > n \geq 1$ we have

$$\begin{aligned} d(y_n(w),y_m(w)) &\leq d(y_n(w),y_{n+1}(w))+d(y_{n+1}(w),y_{n+2}(w))+...+d(y_{m-1}(w),y_m(w)) \\ &\leq [(\sqrt{k})^n+(\sqrt{k})^{n+1}+...+(\sqrt{k})^{m-1}]d(y_0(w),y_1(w)) \end{aligned}$$

It follows that $\{y_n(w)\}$ is a Cauchy sequence such that $\{y_n(w)\} \subset O(f,g,T,S;\xi_0)$ $\cap f(w,X) \cap g(w,X)$. Since $f(w,X) \cap g(w,X)$ is (T,S,f,g)-orbitally complete at ξ_0, so that, there exists $\xi(w) \in f(w,X) \cap g(w,X)$ such that $\{y_n(w)\} \to \{\xi(w)\}$ as $n \to \infty$ for $w \in \Omega$ and consequently the subsequences $\{y_{2n+1}(w)\}$ and $\{y_{2n+2}(w)\}$ converge to $\{\xi(w)\}$ for $w \in \Omega$. Let $\eta(w) \in f^{-1}(w,\xi(w))$ and $\zeta(w) \in g^{-1}(w,\xi(w))$, then $\eta(w),\zeta(w) \in X$ and

$$\xi(w) = f(w,\eta(w)) = g(w,\zeta(w)).$$

Now, we will show that $f(w,\eta(w)) \in T(w,\eta(w)), w \in \Omega$,

$$\begin{aligned} d(f(w,\eta(w)),T(w,\eta(w))) &\leq d(f(w,\eta(w)),g(w,\xi_{2n+2}(w)))+d(g(w,\xi_{2n+2}(w)),T(w,\eta(w))) \\ &\leq d(f(w,\eta(w)),g(w,\xi_{2n+2}(w)))+H(T(w,\eta(w)),S(w,\xi_{2n+1}(w))) \\ &\leq d(f(w,\eta(w)),g(w,\xi_{2n+2}(w)))+a(w)d(f(w,\eta(w)),g(w,\xi_{2n+1}(w))) \\ &+ b(w)\max\{d(f(w,\eta(w)),T(w,\eta(w))),d(g(w,\xi_{2n+1}(w)),S(w,\xi_{2n+1}(w)))\} \\ &+ c(w)\max\{d(f(w,\eta(w)),g(w,\xi_{2n+1}(w))),d(f(w,\eta(w)),T(w,\eta(w))) \\ &, d(g(w,\xi_{2n+1}(w)),S(w,\xi_{2n+1}(w)))\} \\ &+ e(w)\max\{d(f(w,\eta(w)),g(w,\xi_{2n+1}(w))),d(f(w,\eta(w)),T(w,\eta(w))) \\ &, d(g(w,\xi_{2n+1}(w)),S(w,\xi_{2n+1}(w))),d(f(w,\eta(w)),S(w,\xi_{2n+1}(w)))\}, \end{aligned}$$

by (3), we obtain

$$\begin{aligned} d(f(w,\eta(w)),T(w,\eta(w))) &\leq d(f(w,\eta(w)),y_{2n+2}(w))+a(w)d(f(w,\eta(w)),y_{2n+1}(w)) \\ &+ b(w)\max\{d(f(w,\eta(w)),T(w,\eta(w))),d(y_{2n+1}(w),y_{2n+2}(w))\} \\ &+ c(w)\max\{d(f(w,\eta(w)),y_{2n+1}(w)),d(f(w,\eta(w)),T(w,\eta(w))) \\ &, d(y_{2n+1}(w),y_{2n+2}(w))\} \\ &+ e(w)\max\{d(f(w,\eta(w)),y_{2n+1}(w)),d(f(w,\eta(w)),T(w,\eta(w))) \\ &, d(y_{2n+1}(w),y_{2n+2}(w)),d(f(w,\eta(w)),y_{2n+2}(w))\}. \end{aligned}$$

Taking the limit as $n \to \infty$, and using $\xi(w) = f(w, \eta(w))$ we obtain

$$\begin{aligned}
d(f(w,\eta(w)),T(w,\eta(w))) &\leq d(f(w,\eta(w)),f(w,\eta(w)))+a(w)d(f(w,\eta(w)),f(w,\eta(w))) \\
&+ b(w)\max\{d(f(w,\eta(w)),T(w,\eta(w))),d(f(w,\eta(w)),f(w,\eta(w)))\} \\
&+ c(w)\max\{d(f(w,\eta(w)),f(w,\eta(w))),d(f(w,\eta(w)),T(w,\eta(w))) \\
&, d(f(w,\eta(w)),f(w,\eta(w)))\} \\
&+ e(w)\max\{d(f(w,\eta(w)),f(w,\eta(w))),d(f(w,\eta(w)),T(w,\eta(w))) \\
&, d(f(w,\eta(w)),f(w,\eta(w))),d(f(w,\eta(w)),f(w,\eta(w)))\}.
\end{aligned}$$

It follows that

$$d(f(w,\eta(w)),T(w,\eta(w))) \leq (b(w)+c(w)+e(w))d(f(w,\eta(w)),T(w,\eta(w))),$$

which leads to,

$$f(w,\eta(w)) \in T(w,\eta(w)), w \in \Omega. \tag{9}$$

Similarly, we can show that $g(w,\zeta(w)) \in S(w,\zeta(w)), w \in \Omega$,

$$\begin{aligned}
d(g(w,\zeta(w)),S(w,\zeta(w)) &\leq d(g(w,\zeta(w)),f(w,\xi_{2n+1}(w)))+d(f(w,\xi_{2n+1}(w)),S(w,\zeta(w))) \\
&\leq d(g(w,\zeta(w)),f(w,\xi_{2n+1}(w)))+H(T(w,\xi_{2n}(w)),S(w,\zeta(w))) \\
&\leq d(g(w,\zeta(w)),f(w,\xi_{2n+1}(w)))+a(w)d(f(w,\xi_{2n}(w)),g(w,\zeta(w))) \\
&+ b(w)\max\{d(f(w,\xi_{2n}(w)),T(w,\xi_{2n}(w))),d(g(w,\zeta(w)),S(w,\zeta(w)))\} \\
&+ c(w)\max\{d(f(w,\xi_{2n}(w)),g(w,\zeta(w))),d(f(w,\xi_{2n}(w)),T(w,\xi_{2n}(w))) \\
&, d(g(w,\zeta(w)),S(w,\zeta(w)))\} \\
&+ e(w)\max\{d(f(w,\xi_{2n}(w)),g(w,\zeta(w))),d(f(w,\xi_{2n}(w)),T(w,\xi_{2n}(w))) \\
&, d(g(w,\zeta(w)),S(w,\zeta(w))),d(f(w,\xi_{2n}(w)),S(w,\zeta(w)))\},
\end{aligned}$$

using (3), we obtain

$$\begin{aligned}
d(g(w,\zeta(w)),S(w,\zeta(w)) &\leq d(g(w,\zeta(w)),y_{2n+1}(w))+a(w)d(y_{2n}(w),g(w,\zeta(w))) \\
&+ b(w)\max\{d(y_{2n}(w),y_{2n+1}(w)),d(g(w,\zeta(w)),S(w,\zeta(w)))\} \\
&+ c(w)\max\{d(y_{2n}(w),g(w,\zeta(w))),d(y_{2n}(w),y_{2n+1}(w)) \\
&, d(g(w,\zeta(w)),S(w,\zeta(w)))\} \\
&+ e(w)\max\{d(y_{2n}(w),g(w,\zeta(w))),d(y_{2n}(w),y_{2n+1}(w)) \\
&, d(g(w,\zeta(w)),S(w,\zeta(w))),d(y_{2n}(w),S(w,\zeta(w)))\},
\end{aligned}$$

Again taking the limit as $n \to \infty$, and using $\xi(w) = g(w,\zeta(w))$

$$\begin{aligned}
d(g(w,\zeta(w)),S(w,\zeta(w)) &\leq d(g(w,\zeta(w),g(w,\zeta(w)))+a(w)d(g(w,\zeta(w),g(w,\zeta(w))) \\
&+ b(w)\max\{d(g(w,\zeta(w),g(w,\zeta(w))),d(g(w,\zeta(w)),S(w,\zeta(w)))\} \\
&+ c(w)\max\{d(g(w,\zeta(w),g(w,\zeta(w)))),d(g(w,\zeta(w),g(w,\zeta(w))) \\
&, d(g(w,\zeta(w)),S(w,\zeta(w)))\} \\
&+ e(w)\max\{d(g(w,\zeta(w),g(w,\zeta(w)))),d(g(w,\zeta(w),g(w,\zeta(w)))) \\
&, d(g(w,\zeta(w)),S(w,\zeta(w))),d(g(w,\zeta(w),S(w,\zeta(w))))\}.
\end{aligned}$$

It follows that

$$d(g(w,\zeta(w)), S(w,\zeta(w)) \leq (b(w)+c(w)+e(w))d(g(w,\zeta(w)), S(w,\zeta(w))$$

Hence,

$$g(w,\zeta(w)) \in S(w,\zeta(w)), w \in \Omega. \tag{10}$$

So, we prove that the pairs (f,T) and (g,S) have a random coincidence point. Now, from the hypotheses (III) of the theorem, we have $f(w,\eta(w))$ is a random fixed point of f for some $\eta(w) \in C(f,T)$, i.e. $f(w,f(w,\eta(w))) = f(w,\eta(w))$. Since T and f are (IT)-commuting at $\eta(w)$, then

$$\begin{aligned} f(w,\eta(w)) \in\ & T(w,\eta(w)) \\ & \Rightarrow f(w,f(w,\eta(w))) \in f(w,T(w,\eta(w))) \subseteq T(w,f(w,\eta(w))) \\ & \Rightarrow f(w,\eta(w)) = f(w,f(w,\eta(w))) \in T(w,f(w,\eta(w))), w \in \Omega. \end{aligned}$$

Hence, T and f have a common random fixed point $f(w,\eta(w))$.
Similarly, from (IV), we have $g(w,\zeta(w))$ is a fixed point of g for some $\zeta(w) \in C(g,S)$, i.e. $g(w,g(w,\zeta(w))) = g(w,\zeta(w))$. Since S and g are (IT)-commuting at $\zeta(w)$, we have

$$\begin{aligned} g(w,\zeta(w)) \in\ & S(w,\zeta(w)) \\ & \Rightarrow g(w,g(w,\zeta(w))) \in g(w,S(w,\zeta(w))) \subseteq S(w,g(w,\zeta(w))) \\ & \Rightarrow g(w,\zeta(w)) = g(w,g(w,\zeta(w))) \in S(w,g(w,\zeta(w))), w \in \Omega. \end{aligned}$$

It follows that S and g have a common random fixed point $g(w,\zeta(w))$.
Finally, (V) is obtained immediately from (III) and (IV).

Corollary 2.2 *Let X be a separable metric space. Let $T : \Omega \times X \to CB(X)$ be continuous random multivalued mapping and $f : \Omega \times X \to X$ is self random mapping such that $T(w,X) \subseteq f(w,X)$ and satisfying the following condition:*

$$\begin{aligned} H(T(w,x),T(w,y)) \leq\ & a(w)d(f(w,x),f(w,y))+b(w)\max\{d(f(w,x),T(w,x)),d(f(w,y),T(w,y))\} \\ +\ & c(w)\max\{d(f(w,x),f(w,y)),d(f(w,x),T(w,x)),d(f(w,y),T(w,y))\} \\ +\ & e(w)\max\{d(f(w,x),f(w,y)),d(f(w,x),T(w,x)),d(f(w,y),T(w,y)) \\ ,\ & d(f(w,x),T(w,y))\}, w \in \Omega, \end{aligned}$$

for all $x,y \in X$ and for all $w \in \Omega$, where $a(w),b(w),c(w),e(w) : \Omega \to (0,1)$ are measurable mappings such that, $a(w)+b(w)+c(w)+2e(w) = 1$. If there exists $\xi_0 \in X$ such that $f(w,X)$ is (T,f)-orbitally complete at ξ_0, then the pairs (f,T) has a random coincidence point. Further T and f have a common fixed point $f(w,\eta(w))$ provided that $f(w,\eta(w))$ is a fixed point of f for some $\eta(w) \in C(f,T)$ and T and f are (IT)-commuting at $\eta(w)$.

proof. Put $T = S$ and $f = g$ in Theorem 2.1, then the Corollary 2.2 follows from Theorem 2.1.

Remark 2.3 *Corollary 2.2 is a stochastic version and extension of Theorem 2 in [15].*

If we put $f = g = I(w, x) = x$ (the identity random mapping on X) in Theorem 2.1, we obtain the following corollary:

Corollary 2.4 *Let X be a separable metric space. Let $T, S : \Omega \times X \to CB(X)$ be two random multivalued mappings satisfying the following condition*

$$\begin{aligned} H(T(w,x), S(w,y)) &\leq a(w)d(x,y) + b(w)\max\{d(x,T(w,x)), d(y,S(w,y))\} \\ &+ c(w)\max\{d(x,y), d(x,T(w,x)), d(y,S(w,y))\} \\ &+ e(w)\max\{d(x,y), d(x,T(w,x)), d(y,S(w,y)) \\ &, d(x,S(w,y))\}, w \in \Omega, \end{aligned}$$

for all $x, y \in X$ and for all $w \in \Omega$, where $a(w), b(w), c(w), e(w) : \Omega \to (0,1)$ are measurable mappings such that, $a(w) + b(w) + c(w) + 2e(w) = 1$. If there exists $\xi_0 \in X$ such that is $T(w,X) \cap S(w,X)$-orbitally complete at ξ_0, then the pair T, S have a common random fixed point.

Also, if we put $T = S$ in Corollary 2.4 we also obtain a random version of Corollary 2 in [15].

Corollary 2.5 *Let X be a separable metric space. Let $T : \Omega \times X \to CB(X)$ be random multivalued mapping satisfying the following condition*

$$\begin{aligned} H(T(w,x), T(w,y)) &\leq a(w)d(x,y) + b(w)\max\{d(x,T(w,x)), d(y,T(w,y))\} \\ &+ c(w)\max\{d(x,y), d(x,T(w,x)), d(y,T(w,y))\} \\ &+ e(w)\max\{d(x,y), d(x,T(w,x)), d(y,T(w,y)) \\ &, d(x,T(w,y))\}, w \in \Omega, \end{aligned}$$

for all $x, y \in X$ and for all $w \in \Omega$, where $a(w), b(w), c(w), e(w) : \Omega \to (0,1)$ are measurable mappings such that, $a(w)+b(w)+c(w)+2e(w) = 1$. If there exists $\xi_0 \in X$ such that is $T(w,X)$-orbitally complete at ξ_0, then T has a random fixed point.

3 Open Problem

In this section, we should present an open problem:
Let (X, d) be a separable complete metric space and (Ω, Σ) is a measurable space, $CB(X)$ denotes the family of all nonempty closed bounded subsets of

X and $K(X)$ denotes the family of all nonempty compact subsets of X. Can be the Theorem (2.1) extended to four multivalued mappings $S, T : \Omega \times X \to CB(X)$ and $F, G : \Omega \times X \to K(X)$ under the conditions given in the Theorem. Moreover, are the results in this paper hold if we replace a separable metric space X by a cone random metric space?

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