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# On (h-s)–Convex Functions and Hadamard-type Inequalities

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**Abstract**

*In this paper, two new classes of convex functions as a generalization of convexity which is called $(h-s)_{1,2}-$convex functions are given.*
**Keywords:** *$h-$convex, $s-$convex, Bullen's inequality.*

# 1 Introduction

The following definition is well known in the literature [8]: A function $f : I \to \mathbb{R}, \emptyset \neq I \subseteq \mathbb{R}$, is said to be convex on the interval $I$ if inequality

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) \tag{1}$$

holds for all $x, y \in I$ and $t \in [0, 1]$. $f$ is said to be concave if the inequality (1) is reversed.

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex function and $a, b \in I$ with $a < b$. The following double inequality:

$$f\left(\frac{a+b}{2}\right)\leq\frac{1}{b-a}\int_a^b f(x)\,dx\leq\frac{f(a)+f(b)}{2}\tag{2}$$

is known in the literature as Hadamard's inequality (or Hermite-Hadamard inequality) for convex functions. Keep in mind that some of the classical inequalities for means can come from (2) for convenient particular selections of the function $f$. If $f$ is concave, this double inequality hold in the inversed way.

**Remark 1.1** *[14] Note that the first inequality stronger than the second inequality in (2); i.e., the following inequality is valid for a convex function $f$ :*

$$\frac{1}{b-a}\int_a^b f(x)\,dx-f\left(\frac{a+b}{2}\right)\leq\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx.\tag{3}$$

*Indeed (3) can be written as*

$$\frac{2}{b-a}\int_a^b f(x)\,dx\leq\frac{1}{2}\left[f(a)+f(b)+2f\left(\frac{a+b}{2}\right)\right],\tag{4}$$

*which is*

$$\begin{aligned}&\frac{2}{b-a}\int_a^{\frac{a+b}{2}} f(x)\,dx+\frac{2}{b-a}\int_{\frac{a+b}{2}}^b f(x)\,dx\\ \leq\ &\frac{1}{2}\left[f(a)+f\left(\frac{a+b}{2}\right)\right]+\frac{1}{2}\left[f(b)+f\left(\frac{a+b}{2}\right)\right].\end{aligned}$$

*this immediately follows by applying the second inequality in (2) twice (on the interval $\left[a,\frac{a+b}{2}\right]$ and $\left[\frac{a+b}{2},b\right]$). By letting $a=-1$, $b=1$, we obtain the result due to Bullen (1978). Further on, we shall call (3) as Bullen's inequality.*

The inequalities (2) which have numerous uses in a variety of settings, has been came a significant groundwork in mathematical analysis and optimization. Many reports have provided new proof, extensions and considering its refinements, generalizations, numerous interpolations and applications, for example, in the theory of special means and information theory. For some results on generalizations, extensions and applications of the Hermite-Hadamard inequalities and convexity, see [1]-[15].

**Definition 1.2** *[6] We say that $f:I\to\mathbb{R}$ is Godunova-Levin function or that $f$ belongs to the class $Q(I)$ if $f$ is non-negative and for all $x,y\in I$ and $t\in(0,1)$, we have*

$$f(tx+(1-t)y)\leq\frac{f(x)}{t}+\frac{f(y)}{1-t}.\tag{5}$$

**Definition 1.3** *[1] We say that $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is a $P-$function or that $f$ belongs to the class $P(I)$ if $f$ is nonnegative and for all $x, y \in I$ and $t \in [0,1]$, we have*

$$f(tx + (1-t)y) \le f(x) + f(y). \tag{6}$$

**Definition 1.4** *[7] Let $s \in (0,1]$. A function $f : (0,\infty] \to [0,\infty]$ is said to be $s-$convex in the second sense if*

$$f(tx + (1-t)y) \le t^s f(x) + (1-t)^s f(y), \tag{7}$$

*for all $x, y \in (0,b]$ and $t \in [0,1]$. This class of $s-$convex functions is usually denoted by $K_s^2$.*

In 1978, Breckner introduced $s-$convex functions as a generalization of convex functions in [4]. Also, in that work Breckner proved the important fact that the set valued map is $s-$convex only if the associated support function is $s-$convex function in [5]. A number of properties and connections with s-convex in the first sense are discussed in paper [7]. Of course, $s-$convexity means just convexity when $s = 1$.

**Definition 1.5** *[9] Let $h : J \subseteq \mathbb{R} \to \mathbb{R}$ be a positive function, $h \not\equiv 0$. We say that $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is $h-$convex function, or that $f$ belongs to the class $SX(h, I)$, if $f$ is nonnegative and for all $x, y \in I$ and $t \in [0,1]$ we have*

$$f(tx + (1-t)y) \le h(t) f(x) + h(1-t) f(y). \tag{8}$$

If inequality (8) is reversed, then $f$ is said to be $h-$concave, i.e. $f \in SV(h, I)$. Obviously, if $h(t) = t$, then all nonnegative convex functions belong to $SX(h, I)$ and all nonnegative concave functions belong to $SV(h, I)$; if $h(t) = \frac{1}{t}$, then $SX(h, I) = Q(I)$; if $h(t) = 1$, then $SX(h, I) \supseteq P(I)$; and if $h(t) = t^s$, where $s \in (0,1)$, then $SX(h, I) \supseteq K_s^2$.

In [1], Dragomir *et al.* proved two inequalities of Hadamard-type for $P-$functions.

**Theorem 1.6** *[1] Let $f \in P(I)$, $a, b \in I$, with $a < b$ and $f \in L_1([a,b])$. Then*

$$f\left(\frac{a+b}{2}\right) \le \frac{2}{b-a}\int_a^b f(x)\,dx \le 2\left[f(a) + f(b)\right]. \tag{9}$$

In [3], Pachpatte established the new following Hadamard-type inequality for products of convex functions.

**Theorem 1.7** *[3] Let $f, g : [a, b] \to [0, \infty)$ be convex functions on $[a, b] \subset \mathbb{R}$, $a < b$. Then*

$$\frac{1}{b-a}\int_a^b f(x)\,g(x)\,dx \leq \frac{1}{3}M(a,b) + \frac{1}{6}N(a,b) \tag{10}$$

*where $M(a,b) = f(a)g(a) + f(b)g(b)$ and $N(a,b) = f(a)g(b) + f(b)g(a)$.*

In [2], Dragomir and Fitzpatrick proved a new variety of Hadamard's inequality which holds for $s-$convex functions in the second sense.

**Theorem 1.8** *[2] Suppose that $f : [0, \infty) \to [0, \infty)$ is an $s-$convex function in the second sense, where $s \in (0, 1)$, and let $a, b \in [0, \infty)$, $a < b$. If $f \in L_1([a, b])$, then the following inequalities hold:*

$$2^{s-1} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+f(b)}{s+1}. \tag{11}$$

Up until now, there are many reports on convexity and Hadamard-type inequalities. The main purpose of the present paper is to give new classes of convex functions which called $(h-s)_{1,2}-$convex functions as a generalization of ordinary convex functions and to prove new Hadamard-type inequalities for these new classes of functions. Some applications to the special meansof real numbers are given. Throughout this paper we will imply $M(a,b) = f(a)g(a) + f(b)g(b)$ and $N(a,b) = f(a)g(b) + f(b)g(a)$.

# 2 New Definitions and Results

**Definition 2.1** *Let $h : J \subset \mathbb{R} \to \mathbb{R}$ be a non-negative function, $h \not\equiv 0$. We say that $f : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}$ is an $(h-s)_1-$convex function in the first sense, or that $f$ belong to the class $SX((h-s)_1, I)$, if $f$ is non-negative and for all $x, y \in [0, \infty) = I$, $s \in (0, 1]$, $t \in [0, 1]$, we have*

$$f(tx + (1-t)y) \leq h^s(t)f(x) + (1 - h^s(t))f(y). \tag{12}$$

*If inequality (12) is reversed, then $f$ is said to be $(h-s)_1-$concav function in the first sense, i.e., $f \in SV((h-s)_1, I)$.*

**Definition 2.2** *Let $h : J \subset \mathbb{R} \to \mathbb{R}$ be a non-negative function, $h \not\equiv 0$. We say that $f : \mathbb{R}^{+} \cup \{0\} \to \mathbb{R}$ is an $(h-s)_2-$convex function in the second sense, or that $f$ belong to the class $SX((h-s)_2, I)$, if $f$ is non-negative and for all $u, v \in [0, \infty) = I$, $s \in (0,1]$, $t \in [0,1]$ we have*

$$f(tu + (1-t)v) \leq h^{s}(t)f(u) + h^{s}(1-t)f(v). \tag{13}$$

*If inequality (13) is reversed, then $f$ is said to be $(h-s)_2-$concav function in the second sense, i.e., $f \in SV((h-s)_2, I)$.*

Obviously, in (13), if $h(t) = t$, then all $s-$convex functions in the second sense belongs to $SX((h-s)_2, I)$ and all $s-$concav functions in the second sense belongs to $SV((h-s)_2, I)$, and it can be easily seen that for $h(t) = t$, $s = 1$, $(h-s)_2-$convexity reduces to ordinary convexity defined on $[0, \infty)$. Similarly, in (12), if $h(t) = t$, then all $s-$convex functions in the first sense belongs to $SX((h-s)_1, I)$ and all $s-$concav functions in the first sense belongs to $SV((h-s)_1, I)$, and it can be easily seen that for $h(t) = t$ , $s = 1$, $(h-s)_1-$convexity reduces to ordinary convexity defined on $[0, \infty)$.

The following theorem was obtained by using the $(h-s)_2$-convex function in the second sense.

**Theorem 2.3** *Let $h : J \subset \mathbb{R} \to \mathbb{R}$ be a non-negative function, $h \not\equiv 0$. $f : I = [0, \infty) \to \mathbb{R}$ is an $(h-s)_2-$convex function in the second sense, or that $f$ belong to the class $SX((h-s)_2, I)$, if $f$ is non-negative and for all $x, y \in [0, \infty) = I$, $s \in (0,1]$, $t \in [0,1]$. If $f \in L_1[a,b]$, $h \in L_1[0,1]$, we have the following inequality:*

$$\frac{1}{b-a}\int_a^b f(x)dx \leq [f(a) + f(b)] \int_0^1 h^{s}(t)\, dt. \tag{14}$$

*Proof:* By the definition of $(h-s)_2-$convex mappings in the second sense, for any $s \in (0,1]$ and $t \in [0,1]$, we obtain the following inequality for $u = a$, $y = b$

$$f(ta + (1-t)b) \leq h^{s}(t)f(a) + h^{s}(1-t)f(b). \tag{15}$$

Integrating both side of (15) with respect to $t$ on $[0,1]$, we have

$$\int_0^1 f(ta + (1-t)b)dt \leq f(a)\int_0^1 h^{s}(t)\, dt + f(b)\int_0^1 h^{s}(1-t)\, dt.$$

Use of the changing variable $ta + (1-t)b = x$, $(b-a)dt = dx$, we have

$$\frac{1}{b-a}\int_a^b f(x)dx \leq f(a)\int_0^1 h^{s}(t)\, dt + f(b)\int_0^1 h^{s}(1-t)\, dt \tag{16}$$

and, by a change of variable $u = 1-t$ in (16), which is the inequality in (14).

**Corollary 2.4** *In the inequality (14); if we choose $s = 1$, we have the inequality;*

$$\frac{1}{b-a}\int_a^b f(x)dx \leq [f(a)+f(b)]\int_0^1 h(t)\,dt$$

**Remark 2.5** *If we choose $h(t) = t$, we have the inequality;*

$$\begin{aligned}\frac{1}{b-a}\int_a^b f(x)dx &\leq f(a)\int_0^1 t^s dt + f(b)\int_0^1 (1-t)^s\,dt \\ &= \frac{f(a)+f(b)}{s+1}\end{aligned}$$

*which is the right hand side of the inequality in (11). Besides if we choose $s = 1$, we have the right hand side of the Hermite-Hadamard inequality in (2).*

**Theorem 2.6** *Let $h : J \subset \mathbb{R} \to \mathbb{R}$ be a non-negative function, $h \not\equiv 0$. $f : I = [0, \infty) \to \mathbb{R}$ is an $(h-s)_2-$convex function in the second sense, or that $f$ belong to the class $SX((h-s)_2, I)$, if $f$ is non-negative and for all $x, y \in [0, \infty) = I$, $s \in (0, 1]$, $t \in [0, 1]$. If $f \in L_1[a, b]$, $h \in L_1[0, 1]$, we have the following inequality:*

$$\frac{1}{2h^s\left(\frac{1}{2}\right)} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq [f(a)+f(b)]\int_0^1 h^s(t)\,dt. \tag{17}$$

*Proof:* By the $(h-s)_2-$convexity of $f$, we have that

$$f\left(\frac{x+y}{2}\right) \leq h^s\left(\frac{1}{2}\right) f(x) + h^s\left(\frac{1}{2}\right) f(y).$$

If we choose $x = ta + (1-t)b$, $y = tb + (1-t)a$, we get

$$f\left(\frac{a+b}{2}\right) \leq h^s\left(\frac{1}{2}\right) f(ta+(1-t)b) + h^s\left(\frac{1}{2}\right) f(tb+(1-t)a) \tag{18}$$

for all $t \in [0, 1]$. Then, integrating both side of (18) with respect to $t$ on $[0, 1]$, we have

$$f\left(\frac{a+b}{2}\right) \leq \int_0^1 \left(h^s\left(\frac{1}{2}\right) f(ta+(1-t)b) + h^s\left(\frac{1}{2}\right) f(tb+(1-t)a)\right) dt.$$

Use of the changing of variable, we have

$$\frac{1}{2h^s\left(\frac{1}{2}\right)} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx, \tag{19}$$

which is the first inequality in (17) .

To prove the second inequality in (17), we use the right side of (18) and using $(h-s)_2-$convexity of $f$, we have

$$\begin{aligned} & h^s\left(\frac{1}{2}\right)[f(ta+(1-t)b)+f(tb+(1-t)a)] \\ \leq & h^s\left(\frac{1}{2}\right)\left[h^s(t) f(a)+h^s(1-t) f(b)+h^s(t) f(b)+h^s(1-t) f(a)\right] \\ = & h^s\left(\frac{1}{2}\right)\left[h^s(t)+h^s(1-t)\right][f(a)+f(b)] \end{aligned}$$

Integrating the both side of the above inequality, we have

$$\begin{aligned} & h^s\left(\frac{1}{2}\right) \int_0^1[f(ta+(1-t)b)+f(tb+(1-t)a)]\, dt \\ = & h^s\left(\frac{1}{2}\right) \frac{2}{b-a} \int_a^b f(x)\, dx \\ \leq & h^s\left(\frac{1}{2}\right)[f(a)+f(b)] \int_0^1\left[h^s(t)+h^s(1-t)\right] dt. \end{aligned} \tag{20}$$

and, by a change of variable $u=1-t$ in (20), we obtain the inequality in (17).

**Remark 2.7** *In the inequality (17); if we choose $h(t)=t$, we have the inequality*

$$2^{s-1} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(x) dx \leq \frac{f(a)+f(b)}{s+1}$$

*which is the inequality (11).*

**Remark 2.8** *If we choose $h(t)=t$ and $s=1$, we have the inequality*

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(x) dx \leq \frac{f(a)+f(b)}{2}$$

*which is the Hermite-Hadamard inequality.*

**Theorem 2.9** *Let $h: J \subset \mathbb{R} \rightarrow \mathbb{R}$ be a non-negative function, $h \not\equiv 0$. $f, g: I=[0, \infty) \rightarrow \mathbb{R}$ are an $(h-s)_2-$convex function in the second sense, if $f, g$ are non-negative and for all $x, y \in[0, \infty)=I$, $s \in(0,1]$, $t \in[0,1]$. If $fg \in L_1[a, b]$, $h \in L_1[0,1]$, we have the following inequality;*

$$\begin{aligned} \frac{1}{b-a} \int_a^b f(x) g(x) dx \leq & f(a) g(a) \int_0^1 h^{2s}(t)\, dt+f(b) g(b) \int_0^1 h^{2s}(1-t)\, dt \\ & +[f(a) g(b)+f(b) g(a)] \int_0^1 h^s(t) h^s(1-t)\, dt . \end{aligned} \tag{21}$$

Proof: Since $f, g \in SX(h-s)_2$, we have

$$\begin{aligned} f\left(ta+(1-t)b\right) &\leq h^s(t) f(a) + h^s(1-t) f(b) \\ g\left(ta+(1-t)b\right) &\leq h^s(t) g(a) + h^s(1-t) g(b) \end{aligned}$$

for all $s \in (0,1]$, $t \in [0,1]$. Since $f$ and $g$ are non-negative,

$$\begin{aligned} & f\left(ta+(1-t)b\right) g\left(ta+(1-t)b\right) \\ \leq & \left[h^s(t) f(a) + h^s(1-t) f(b)\right]\left[h^s(t) g(a) + h^s(1-t) g(b)\right] \\ = & h^{2s}(t) f(a) g(a) + h^s(t) h^s(1-t) f(a) g(b) \\ & + h^s(t) h^s(1-t) f(b) g(a) + h^{2s}(1-t) f(b) g(b). \end{aligned}$$

Then if we integrate the both side of the above inequality with respect to $t$ on $[0,1]$, we have the inequality in (21).

In the next corollary we will also make use of the Beta function of Euler type, which is for $x, y > 0$ defined as

$$\beta(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$

**Corollary 2.10** *In the inequality (21), if we choose $h(t) = t$ and $s \in (0,1)$, we have*

$$\begin{aligned} \frac{1}{b-a}\int_a^b (fg)(x)dx &\leq f(a) g(a) \int_0^1 t^{2s} dt + f(b) g(b) \int_0^1 (1-t)^{2s} dt \\ &\quad + \left[f(a) g(b) + f(b) g(a)\right] \int_0^1 t^s (1-t)^s dt \\ &= \frac{M(a,b)}{2s+1} + N(a,b)\,\beta(s+1, s+1) \\ &= \frac{M(a,b)}{2s+1} + N(a,b)\frac{\Gamma^2(s+1)}{\Gamma(2s+2)}. \end{aligned}$$

**Remark 2.11** *In the inequality (21), if we choose $h(t) = t$ and $s = 1$, we have*

$$\begin{aligned} \frac{1}{b-a}\int_a^b (fg)(x)dx &\leq f(a) g(a) \int_0^1 t^2 dt + f(b) g(b) \int_0^1 (1-t)^2 dt \\ &\quad + \left[f(a) g(b) + f(b) g(a)\right] \int_0^1 t(1-t)\,dt \\ &= \frac{M(a,b)}{3} + \frac{N(a,b)}{6} \end{aligned}$$

*which is the inequality in (10).*

**Theorem 2.12** *Let $h : J \subset \mathbb{R} \to \mathbb{R}$ be a non-negative function, $h \not\equiv 0$. $f : I = [0, \infty) \to \mathbb{R}$ is an $(h-s)_2-$convex function in the second sense, if $f$ is non-negative and for all $x, y \in [0, \infty) = I$, $s \in (0,1]$, $t \in [0,1]$. If $f \in L_1[a,b]$, $h \in L_1[0,1]$, we have*

$$\frac{1}{b-a}\int_a^b f(x)dx \leq \left[\frac{f(a)+f(b)}{2} + f\left(\frac{a+b}{2}\right)\right]\int_0^1 h^s(t)\,dt. \tag{22}$$

*Proof:* By the $(h-s)_2-$convexity of $f$, we have

$$\begin{aligned} f\left(ta+(1-t)\frac{a+b}{2}\right) &\leq h^s(t)\,f(a) + h^s(1-t)\,f\left(\frac{a+b}{2}\right) \\ f\left(t\frac{a+b}{2}+(1-t)\,b\right) &\leq h^s(t)\,f\left(\frac{a+b}{2}\right) + h^s(1-t)\,f(b) \end{aligned}$$

If we integrate the both side of the above inequalities with respect to $t$ on $[0,1]$, and use of the changing of variable, we get

$$\frac{2}{b-a}\int_a^{\frac{a+b}{2}} f(x)\,dx \leq f(a)\int_0^1 h^s(t)\,dt + f\left(\frac{a+b}{2}\right)\int_0^1 h^s(1-t)\,dt$$

and

$$\frac{2}{b-a}\int_{\frac{a+b}{2}}^b f(x)\,dx \leq f\left(\frac{a+b}{2}\right)\int_0^1 h^s(t)\,dt + f(b)\int_0^1 h^s(1-t)\,dt.$$

By adding the above inequalities and taking into account the $\int_0^1 h^s(t)\,dt = \int_0^1 h^s(1-t)\,dt$ for $s \in (0,1]$, we get

$$\frac{2}{b-a}\int_a^b f(x)dx \leq \left[f(a)+f(b)+2f\left(\frac{a+b}{2}\right)\right]\int_0^1 h^s(t)\,dt$$

which completes the proof.

**Corollary 2.13** *If in (22), we choose $h(t) = t$, we have*

$$\frac{1}{b-a}\int_a^b f(x)dx \leq \frac{1}{s+1}\left[\frac{f(a)+f(b)}{2} + f\left(\frac{a+b}{2}\right)\right].$$

*Besides if we set $s = 1$, we have the inequality (4).*

# 3 Open Problem

It is well known that if $f$ is a convex function on the interval $I \subset \mathbb{R}$, then the Hadamard's inequality holds for the convex functions. It has already been proved a lot of this type inequalities for different kinds of convex functions. So, there is one question as follows:

Under what conditions, the composition $f \circ g$ is $(h-s)_{1,2}-$convex function on $I$? Can we prove Hadamard type inequalities for composition $f \circ g$.

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