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The Effect of Simulation Parameters on the Selection Approach

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Abstract

Selection approaches are used to identify the best system from a finite set of alternative systems. If it involves a small number of alternatives, then Ranking and Selection is the right procedures to be used in order to select the best system. Nevertheless, for a case of a large number of alternatives we need to change our concern from finding the best system to finding a good system with high probability by using the Ordinal Optimization procedure. Almomani and Abdul Rahman [1] has proposed a new selection approach to select a good system when the number of alternatives is very large. In this paper, we study the efficiency of Almomani and Abdul Rahman [1] selection approach base on some parameters such as the initial sample size, increment in simulation samples, total budget, and the elapsed (execution) time. In doing so, we apply their approach on the $M/M/1$ ***queuing systems, in an attempt to determine the adequate choices on these parameters in order to get the best performance for the selection approach.***

Keywords: *Simulation Optimization, Ranking and Selection, Ordinal Optimization, Optimal Computing Budget Allocation*

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1 Introduction

Selection approaches are commonly used to select the best system of a finite set of alternatives. The best system is defined as the system with a maximum or minimum mean, where the means are deduced by a statistical sampling. We consider the problem of finding the best system when the number of alternatives is finite but huge. However, with a large number of alternatives, this problem are not easy to solve, since it will need a huge computational time and a large sample. Therefore, we change our concern from estimating accurately the means for these systems to selecting a good system with high probability. This is the idea of Ordinal Optimization (OO) procedure that has been proposed by Ho et al. [2]. In order to improve the efficiency of OO procedure, Chen et al.[3] has proposed the Optimal Computing Budget Allocation ($OCBA$) procedure. This procedure reduces the computation cost by allocating the available computing budget among different systems instead of simulating equally all systems.

Ranking and Selection ($R\&S$) procedures include the Indifference-Zone (IZ) and the Subset Selection (SS), see Kim and Nelson [4], are used to select the best system when the number of alternatives is relatively small. In fact, they are used in second stage, after the OO procedure reduces the number of alternatives, so that it will appropriate for the $R\&S$ procedures. Almomani and Abdul Rahman [1] has proposed a new sequential selection approach in order to solve the selection problems for a huge number of alternatives. In this approach, the first step is to use the OO procedure to select a subset that intersects with the set of the actual best $m\%$ system. Then the $OCBA$ procedure is used to allocate the available simulation samples in a way that maximize the probability of correct selection. Finally, using the $R\&S$ procedures to select the best system from the survivors systems.

There are two measures to identify the quality of the selection approaches; the Probability of Correct Selection ($P(CS)$) and the Expected Opportunity Cost ($E(OC)$) of potentially incorrect selection, see He et al. [5]. These two measures are important, since there is a potential for incorrect selection in simulation. The $P(CS)$ is used traditionally when the goal is selecting the best system with high probability of correct selection. Meanwhile, the $E(OC)$ has become important in many applications in business and engineering that lead to a recently new selection approach in order to reduce the opportunity cost of a potentially incorrect selection. For more details, see Gupta and Miescke [6], [7], Chick and Inoue [8], [9].

In this paper, we discuss the significant differences in efficiency of the selection approach by Almomani and Abdul Rahman [1] when different simulation parameters are applied. We focus on the parameters such as; the initial sample size (t_0), increment in simulation samples (Δ), total budget (B), and the elapsed (execution) time (T), and present the numerical illustrations for each parameter. The rest of this paper is organized as follows; In Section 2, we present the problem settings. In Section 3, we review the Ranking and Selection procedures, whereas the Ordinal Optimization and Optimal Computing Budget Allocation are presented in Section 4. Next, we present the algorithm by Almomani and Abdul Rahman [1] in Section 5. The simulation issues and the efficiency of the selection approach are discussed together with a series of numerical examples in Section 6, a future research present in Section 7, and Section 8 concludes this study.

2 Problem Setting

Consider the following optimization problem

$$\min_{\theta \in \Theta} J(\theta) \tag{1}$$

where Θ is a feasible solution which is arbitrary, finite and huge. Let J be the expected performance measure of some complex stochastic system, written as $J(\theta) = E[L(\theta, \xi)]$, where θ is a vector that represents system design parameters, ξ represents all the random effect of the system and L is a deterministic function that depends on θ and ξ.

In this paper, without loss of generality, we assume that the best system is the system that has the smallest mean, which is unknown and to be inferred from simulation. Therefore, our goal is selecting the system that has the smallest sample mean. Suppose that there are n systems, and let ξ_{ij} represents an observation from the j^{th} output in system i , where $\xi_i = \{\xi_{ij}, j = 1, 2, \ldots\}$ denotes the output sequence from the system i. We assume that ξ_{ij} are independent and identically distributed ($i.i.d.$) normal with unknown means $\mu_i = E(\xi_{ij})$ and variances $\sigma_i^2 = Var(\xi_{ij})$. In addition, we assume that each $\xi_1, \xi_2, \ldots, \xi_n$ is mutually independent. Actually, the normality assumption is not a problem since simulation outputs are obtained from batch means or as an average performance. Therefore, using a Central Limit Theorem (CLT) the normality assumption holds. In practice, usually the σ_i^2 are unknown, so we estimate it using the sample variances s_i^2 for ξ_{ij}. Since we assume that the smallest mean is best, therefore if the ordered μ_i-values are denoted by $\mu_{[1]} \leq \mu_{[2]} \leq \ldots \leq \mu_{[n]}$, then the system having mean $\mu_{[1]}$ is referred to as the best system. Note that, the Correct Selection (CS) occurs when the system selected by the selection approaches is the same as the actual best system.

3 Ranking and Selection Approaches

The problem on the selection some (all) of the systems base on their ordered means, can be solved by Ranking and Selection (*R&S*) procedures. A natural strategy of the *R&S* procedures for dealing with this problem is to compare between different systems whose response is normally distributed when the number of systems in the feasible solution set is small. These procedures are usually used to select the best system or a subset that contain the best systems when the number of systems is small, with a pre-specified significance level. We consider two procedures; the indifference-zone, and the subset selection in the next two sections.

3.1 Indifference-zone (IZ)

The goal of IZ procedure is to select a system that is within δ from the best system, where δ is the Indifference Zone. In mathematical notation, we seek to achieve $P(CS) \geq 1-\alpha$ given that $|\mu_{[2]} - \mu_{[1]}| \geq \delta$. We should be careful when to specify the value of δ and $1-\alpha$, because, if δ is too small, then the number of replication samples that are required to guarantee the design is expected to be large. Therefore, δ can be thought of the smallest difference of $\mu_{[2]} - \mu_{[1]}$ and is considered worth detecting. On the other hand, if we consider a large number of $1-\alpha$ then it may require a large number of replication samples to meet the designer request. However, to use IZ procedure, the number of alternatives n should be less than or equal 20. In 1978, Rinott [10] has proposed a two stage approach when the variances are completely unknown. This procedure is called the classical *R&S* procedure and since then most of the new IZ procedures are using this procedure directly or indirectly. Also, Tamhane and Bechhofer [11] have presented a simple procedure that is valid when the variances are not be equal.

3.2 Subset selection (SS)

The main strategy of SS procedure is to screen out the search space and eliminate noncompetitive systems in order to construct a subset that contains the best system with high probability. This procedure is appropriate to select a subset that contains the actual best system when the number of alternatives is relatively large. We require that $P(\text{CS}) \geq 1-\alpha$, where CS is selecting a subset that contains the actual best system, with $1-\alpha$ as a predetermined probability.

The SS procedure dates back to Gupta [12], who presented a single stage procedure for producing a subset containing the best system with a specified probability. Extensions of this work which is relevant to the simulation setting include Sullivan and Wilson [13] who derived a two stage SS procedure that determines a subset of maximum size m that, with a specified probability will contain systems that are all within a pre-specified amount of the optimum. Another comprehensive reviews of the $R\&S$ procedures can be found in Bechhofer et al. [14], Goldsman and Nelson [15], and Kim and Nelson [4], [16], [17].

4 Ordinal Optimization and Optimal Computing Budget Allocation

In this section, we review the Ordinal Optimization (OO) procedure. Then, we argue the best way to allocate the total simulation sample in order to further enhance the efficiency of the OO procedure by using Optimal Computing Budget Allocation ($OCBA$).

4.1 Ordinal optimization (OO)

The OO procedure focuses on isolating a subset of good systems with high probability and reducing the required simulation time for discrete event simulation. The goal of OO procedure is to find a good enough system, rather than to estimate accurately the performance value of these systems. This procedure has been proposed by Ho et al. [2].

Consider the optimization problem given in equation (1). If we simulate the system to estimate the $E[L(\theta, \xi)]$, then the confidence interval of this estimator cannot be improved faster than $1/\sqrt{k}$ where k is the number of replications used in order to estimate the $J(\theta)$, see Chen et al. [3]. In fact, this is good for many problems, only when it involves with a small number of alternatives. However, this is not good enough when it involves a complex simulation problem with a large number of alternatives. Actually, each sample of $L(\theta, \xi)$ requires one simulation run, so we will need a large number of samples which is very hard and maybe impossible, when it involves with a huge number of alternative systems in the search space. However, in this situation, we could relax the objective as to get a good enough solution rather than doing extensive simulation, which is impossible in many real world applications.

Let the correct selection is to select a subset G of g systems from the search space that contains at least one of the top $m\%$ best systems. Since the search space is huge then the probability of correct selection is given by $P(CS) \approx (1 - (1 - \frac{m}{100})^g)$. Furthermore, suppose that the correct selection is to select a subset G of g systems that contains at least r of the best s systems. If we assume S to be the subset that contains the actual best s systems, then the probability of correct selection can be obtained using the hypergeometric distribution as

$$P(CS) = P\left(|G \cap S| \geq r\right) \quad = \quad \sum_{i=r}^{g} \frac{\left(\begin{array}{c} s \\ i \end{array}\right)\left(\begin{array}{c} n-s \\ g-i \end{array}\right)}{\left(\begin{array}{c} n \\ g \end{array}\right)}$$

However, since the number of alternatives is very large then the $P(CS)$ can be approximated by the binomial random variable, as

$$P(CS) \approx \sum_{i=r}^{g} \left(\begin{array}{c} g \\ i \end{array}\right) \left(\frac{m}{100}\right)^{i} \left(1 - \frac{m}{100}\right)^{g-i}$$

.

Another comprehensive review of OO procedure can be found in Deng et al. [18], Dai [19], Xiaolan [20], Deng and Ho [21], Lee et al. [22], Li et al. [23], Zhao et al. [24], and Ho et al. [25].

4.2 Optimal computing budget allocation ($OCBA$)

The $OCBA$ is used to determine the best simulation lengths for all simulation systems in order to reduce the total computation time. In fact, this procedure was proposed to improve the performance of OO procedure by determining the optimal numbers of simulation samples for each system, instead of simulating equally all the systems.

The goal of $OCBA$ is to allocate the total simulation samples from all the systems in a way that maximizes the probability of selecting the best system within a given computing budget, see Chen et al.[3], Chen [26], and Chen et al.[27]. Let B be the total sample that are available for solving the optimization problem given in (1). The target is to allocate these computed simulating samples to maximize the $P(CS)$. In mathematical notation this can be written as:

$$\max_{T_1,\ldots,T_n} \; P(CS)$$

$$s.t. \quad \sum_{i=1}^{n} T_i = B$$

$$T_i \in \mathbb{N} \qquad\qquad i = 1, 2, \ldots, n$$

where $\mathbb{N}$ is the set of non-negative integers, T_i is the number of samples allocated to system i, and $\sum_{i=1}^{n} T_i$ denotes the total computational samples. Assume that the simulation times for different systems are roughly the same. To solve this problem Chen et al. [3] proposed the following theorem.

Theorem 4.1 *Given a total number of simulated samples B to be allocated to n competing systems whose performance is depicted by random variables with means $J(\theta_1), J(\theta_2), \ldots, J(\theta_n)$, and finite variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$ respectively, as $B \longrightarrow \infty$, the approximate probability of correct selection can be asymptotically maximized when*

1. $\frac{T_i}{T_j} = \left(\frac{\sigma_i/\delta_{b,i}}{\sigma_j/\delta_{b,j}}\right)^2$; *where* $i, j \in \{1, 2, \ldots, n\}$ *and* $i \neq j \neq b$.

2. $T_b = \sigma_b \sqrt{\sum_{i=1, i\neq b}^{n} \frac{T_i^2}{\sigma_i^2}}$

where $\delta_{b,i}$ *the estimated difference between the performance of the two systems* $(\delta_{b,i} = \overline{J_b} - \overline{J_i})$, *and* $\overline{J_b} \leq \min_i \overline{J_i}$ *for all* i. *Here* $\overline{J_i} = \frac{1}{T_i} \sum_{j=1}^{T_i} L(\theta_i, \xi_{ij})$, *where* ξ_{ij} *is a sample from* ξ_i *for* $j = 1, \ldots, T_i$.

Proof: See Chen et al. [3].

More reviews on the $OCBA$ procedure can be found in Chen et al. [28], Banks [29], and Chen [30].

5 Algorithm of Almomani and Abdul Rahman

The selection approach as proposed by Almomani and Abdul Rahman [1] consists of four stages. In the first stage, using the OO procedure, a subset G is selected randomly from the search space that intersects with the set $m\%$ of actual best systems with high probability $(1 - \alpha_1)$. Then, use the $OCBA$ procedure in order to allocate the available computing budget. This is followed by the SS procedure to get a smaller subset I that contains the best system among the subset that is selected before with high probability $(1 - \alpha_2)$, where $|I| \leq 20$. Finally, using the IZ procedure to select the best system from set I with high probability $(1 - \alpha_3)$. The algorithm is described as follows:

Setup: Specify g and k where $|G| = g$, $|G^{'}| = k$, the number of initial simulation samples $t_0 \geq 2$, the indifference zone δ, and $t = t_{(1-\alpha_2/2)^{\frac{1}{g-1}}, t_0 - 1}$ from the t-distribution. Let $T_1^l = T_2^l = \ldots = T_g^l = t_0$, and determine the total computing budget B. Here, G is the selected subset from Θ, that satisfies $P(G$ contains at least one of the best $m\%$ systems$) \geq 1 - \alpha_1$, whereas $G^{'}$ is the selected subset from G, where $g \geq k$. The iteration number is represented by l.

Select a subset G of size g randomly from Θ. Take the random samples of t_0 observations y_{ij} $(j = 1, \ldots, t_0)$ for each system i in G, where $i = 1, \ldots, g$.

Initialization: Calculate the sample mean $\bar{y}_i{}^{(1)}$, and variances s_i^2, where $\bar{y}_i{}^{(1)} = \frac{\sum_{j=1}^{T_i^l} y_{ij}}{T_i^l}$ and $s_i^2 = \frac{\sum_{j=1}^{T_i^l} (y_{ij} - \bar{y}_i{}^{(1)})^2}{T_i^l - 1}$, for all $i = 1, \ldots, g$.

Arrange the systems in G in ascending order according to their sample averages; $\bar{y}_{[1]}^{(1)} \le \bar{y}_{[2]}^{(1)} \le \ldots \le \bar{y}_{[g]}^{(1)}$. Then select the best k systems from the set G, and represent this subset as $G^{'}$.

Stopping Rule: If $\sum_{i=1}^{g} T_i^l \ge B$, then stop. Otherwise, randomly select a subset $G^{''}$ of the $g-k$ alternatives from $\Theta - G^{'}$, and let $(G = G^{'} \bigcup G^{''})$.

Simulation Budget Allocation: Increase the computing budget by Δ and compute the new budget allocation, $T_1^{l+1}, T_2^{l+1}, \ldots, T_g^{l+1}$, by using Theorem 4.1.

Perform additional $\max\{0, T_i^{l+1} - T_i^l\}$ simulations for each system i, $i = 1, \ldots, g$, let $l \longleftarrow l+1$. Go to **Initialization**.

Screening: Set $I = \{i : 1 \le i \le k \text{ and } \bar{y}_i^{(1)} \ge \bar{y}_j^{(1)} - [W_{ij} - \delta]^-, \forall i \ne j\}$, where $W_{ij} = t\left(\frac{s_i^2}{T_i} + \frac{s_j^2}{T_j}\right)^{1/2}$ for all $i \ne j$, and $[x]^- = x$ if $x < 0$ and $[x]^- = 0$ otherwise.

If I contains a single index, then this system is the best system. Otherwise, for all $i \in I$, compute the second sample size $N_i = \max\{T_i, \lceil (\frac{hs_i}{\delta})^2 \rceil\}$, where $h = h(1-\alpha_3/2, t_0, |I|)$ be the Rinott [10] constant that can be obtained from tables of Wilcox [31].

Take additional $N_i - T_i$ random samples of y_{ij} for each system $i \in I$, and compute the overall sample means for $i \in I$ as $\bar{y}_i^{(2)} = \frac{\sum_{j=1}^{N_i} y_{ij}}{N_i}$

Select system $i \in I$ with the smallest $\bar{y}_i^{(2)}$ as the best.

6 Simulation Issues and the Efficiency of the Selection Approach

In this section, we argue the effects of some changes in simulation parameters such as, the initial sample size t_0, increment in simulation samples Δ, total budget B, and the elapsed (execution) time T to the selection approach as proposed by Almomani and Abdul Rahman [1]. We apply the selection approach onto the $M/M/1$ queuing systems. Furthermore, we would like to determine the appropriate values of the t_0 and Δ, since there is no clear formulation to determine these two values when the number of alternatives is very large. We also discuss the performance of Almomani and Abdul Rahman [1] selection approach when we increase the total budget B.

6.1 Different simulation parameters

6.1.1 Initial sample size (t_0)

The sample size in the first stage is called the initial sample size or t_0. It plays an important role in the performance of many selection approaches. In fact, the initial sample size cannot be too small since we might get poor estimates for the sample mean and variances. On the other hand, t_0 cannot be too large, because in the first stage there exists many non critical systems and by giving a large number of samples for these systems will result in loosing a large number of samples and also wasting a computation time. Chen et al. [3] and Chen et al. [27] suggested that as a good choice for the initial sample size, the value of t_0 should be between 10 and 20. Unfortunately, there is no clear formulation to calculate an appropriate value of the initial sample size t_0 for the selection approaches, when the number of alternatives is large.

Note that, if t_0 is too small, we might get a poor estimate of σ_i^2 (s_i^2). In certain situation, it could be that s_i^2 is much greater than σ_i^2, leading to an unnecessarily large sample size (N_i) that been used in screening step in the selection algorithm. The Rinott constant $h = h(1-\alpha_3/2, t_0, |I|)$ is determined by the desired confidence level $(1-\alpha_3/2)$, the initial sample size t_0, and the number of systems in the set I ($|I|$). From Wilcox Tables [31] we note that, constant h will increase as $|I|$ increased, and it will decrease as α_3 and t_0 decrease. Thus, the experiment design factor that under control is t_0. So, this study will look into the t_0's effect on the performance of the studied approach.

6.1.2 Increment in simulation samples (Δ)

In order to improve the performance of *OO* procedure, the *OCBA* technique is used to determine the best simulation lengths for all simulation systems and to reduce the total computation time. Here, we argue the effect of the increment in simulation samples, Δ on the performance of the selection approach. The increment in simulation samples, Δ is defined as a positive integer that represents the additional number of simulation samples in the **Simulation Budget Allocation** step in the algorithm.

In order to avoid repetition increment in the *OCBA* algorithm, the increment in simulation samples Δ cannot be too small, such that it will increase the simulation time. On the other hand, if Δ is too large will result in a waste in computation time and unnecessary higher confidence level. Chen et al. [3] and Chen et al. [27] have suggested that Δ should be between 5 and 10% of the simulated system as a good choice for the increment in simulation samples. However, so far there is no fixed values of Δ or no clear method in determine it in order to get the best performance using the *OCBA* technique.

6.1.3 Total budget (B)

In sequential stopping rule, the total budget B is used as a condition to repeat the sampling. According to Almomani and Abdul Rahman [1], they used the sequential stopping rule such that to repeat the sampling while $\sum_{i=1}^{n} T_i \leq B$, where T_i is the number of samples allocated to system i.

Clearly, the increase in the total budget B, will improve the efficiency of the selection approach by increasing the $P(CS)$ and at the same time by decreasing the $E(OC)$, where the $P(CS)$ and $E(OC)$ are measures for the selection quality. In particular, when $B \to \infty$ then $P(CS) = 1$ and $E(OC) = 0$. Whereas, by increasing the B will increase the number of simulation samples and the elapsed time. This study will look into these effects of the total budget B on the studied selection approach.

6.1.4 Elapsed time (T)

The elapsed (execution) time is related to the number of simulation samples used in selection approach to select the best system from a large number of alternatives system. In selection approach, the target is selecting the best system with a small number of simulation samples with minimum elapsed time. In our study, using different values of the initial sample size (t_0) and the increment size in simulation samples (Δ), we want to see their connections with a different values of the elapsed times.

We use "java" as the programming language to calculate the elapsed time. The elapsed time is represented by the different between the starting time and the ending time of the programme. The idea is to store the initial time of the programme in a variable called "StartTime". Once the programme ended, the time will be stored into another variable called "EndTime". Therefore, the difference between the StartTime and the EndTime will present the elapsed time of the programme.

However, the real total elapsed (execution) time should be defined as a combination of Central Processing Unit (CPU) time plus the difference between the StartTime and the EndTime. It is not representing the amount of time the user has been logged in. In our work, we take the elapsed time (denote as "Time") as a runtime for a programme which is equal to the compiler time plus the result time per millisecond, where a millisecond (from milli- and second; abbreviation: ms) is a 1/1000 of a second. For more details see Schildt [33]. We run all our programmes using a computer model of $Optiplex380$, manufactured by $Dell$ with installed memory (RAM) 2.00 GB and the Processor $Intel(R)\ Core(TM)2\ Duo\ CPU\ E7500$ @ $2.93GH_z$ $2.93GH_z$.

6.2 Numerical examples

Here, we present the numerical illustrations of the effects of changes in the simulation parameters on the Almomani and Abdul Rahman [1] selection approach as we mentioned before in section 6.1. All examples are involved with the $M/M/1$ queuing system with one server where the inter arrival times and the service times are exponentially distributed, see Ross [32].

6.2.1 Example 1

In this example, the objective is to select one of the best $m\%$ systems that has the minimum average waiting time per customer from n $M/M/1$ queuing systems. We use the Probability of Correct Selection ($P(CS)$), and the Expected Opportunity Cost ($E(OC)$) of a potentially incorrect selection as a measure of selection quality. The Opportunity Cost (OC) is defined as the difference between unknown means of the selected best system and the actual best system.

Assume that the arrival rate λ is a fixed number as $\lambda = 1$, and the service rate μ is belong to the interval $[a, b]$, where $\mu \in [7, 8]$. Suppose that we have 3000 of $M/M/1$ queuing systems, and we discretize the problem by assuming that $\mu \in \Theta = 7 + i/3000$, where $i = 0, 1, \ldots, 3000$. Therefore, the best queuing system with the minimum average waiting time, would be the 3000^{th} queuing system with $\mu_{3000} = 8.0$. For this example, we consider four different parameter settings.

In the first settings, assume that $n = 3000$, $g = 200$, $\alpha_2 = \alpha_3 = 0.005$, $\delta = 0.05$, $k = 20$ and $\Delta = 50$ (these settings are chosen arbitrarily). Suppose we want to select one of the best (1%) systems, then our target is the systems from 2971 to 3000. The correct selection here would be, selecting the system that belongs to $\{2971, 2972, \ldots, 3000\}$. Furthermore, here the analytical probability of the correct selection can be calculated as $P(CS) \geq 1 - \left(\left(1 - \frac{1}{100}\right)^{200} + 0.005 + 0.005\right) \geq 0.85$. Now, we choose t_0 as 10, 20, 30, 50, 80, 100 and 200 to see the effect of the different initial sample sizes t_0 on the selection approach. We consider the total number of simulation samples (total budget) $B = 8000$ for each value of t_0. Initially, we find that the programme cannot continue in cases of t_0 are $50, 80, 100$ and 200. With this it shows that the selection approach works just fine when $t_0 = 10, 20, 30$ whereas it fails to work when $t_0 = 50, 80, 100, 200$. We record the minimum values of B for each t_0 in Table 1, to represent the least number of B for each t_0 to able the selection approach works from first stage to the next stage.

As an extension from this we repeat the experiment for each pair of the t_0 and the minimum number of B, (as in Table 1) 10 replications and the results are presented in Table 2 to Table 8. From the tables, "Best" means the

Table 1: The minimum values of B for each value of t_0

t_0	10	20	30	50	80	100	200
$min\ B$	2000	4000	6000	10000	16000	20000	40000

index of the chosen system that being considered as the best system, $|I|$ is the number of systems from **Screening** step, $\sum_{i=1}^{g} T_i$ is the total sample size used in **Stopping Rule** step, $\sum_{i\in I} N_i$ is the total sample size used in **Screening** step and "Time" to represent the elapsed (execution) time of the programme, which is per millisecond (ms), where $ms = 1/1000$ second. The $E(OC)$ values of our approach are represented by $E(\bar{y}_b - \bar{y}_{i^*})$, whereas the analytical $E(OC)$ is defined as $E(\bar{w}_b - \bar{w}_{i^*})$. The $\bar{w}_{i^*}$ and $\bar{y}_{i^*}$ for each is the unknown average waiting time and the sample mean respectively for the actual best system i^*, where i^* =3000 and $\bar{w}_{i^*} = 0.142857143$. Whereby $\bar{w}_b$ and $\bar{y}_b$ are the unknown average waiting time and the sample means for the best system b respectively. We can calculate $\bar{w}_{i^*}$ and $\bar{w}_b$ using a formulation of $\bar{w}_i = \frac{1}{\mu_i - \lambda_i}$; where μ_i and λ_i are the service rate and the arrival rate for the system i respectively, with $i = i^*$ and b. After the simulation was performed, $\bar{y}_i$ can be calculated from the system output.

Table 2: The numerical illustration for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, t_0 = 10, B = 2000$

Run	*Best*	$\|I\|$	$\sum_{i=1}^{g} T_i$	$\sum_{i\in I} N_i$	*Time*	$E(OC)$	
						Our approach	*Analytical*
1	2973	3	14732	990	9120	-0.003508125	0.000183910
2	2971	19	23653	11763	10752	-0.002986155	0.000197552
3	2124	19	27110	14738	10023	0.000841202	0.006218588
4	2988	19	15267	3865	8930	-0.004301730	0.000081679
5	2985	14	14325	1978	9001	-0.001504849	0.000102114
6	1352	19	20619	8540	10273	-0.001535257	0.012165594
7	2979	18	19148	7075	10607	-0.004808459	0.000143000
8	2996	2	15140	1312	9109	0.000908011	0.000027216
9	2461	19	24259	13948	11072	-0.000454362	0.003763257
10	2984	11	13636	959	8626	0.000166640	0.000108927

From the result, we notice that, when the total budget B is increased the $\sum_{i=1}^{g} T_i$, $\sum_{i\in I} N_i$ and elapsed time will increase. Also, for each value of t_0 we found that the size of set I is less than 20, ($|I| \leq 20$), which follows our assumption, since we need $|I| \leq 20$ to apply the IZ procedure. There

Table 3: The numerical illustration for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, t_0 = 20, B = 4000$

Run	*Best*	$\lvert I \rvert$	$\sum_{i=1}^{g} T_i$	$\sum_{i \in I} N_i$	*Time*	$E(OC)$	
						Our approach	*Analytical*
1	2899	14	16791	1436	14599	-0.001810955	0.000690395
2	2979	3	23561	5959	16013	0.001004808	0.000143000
3	2987	12	25663	9363	16476	-0.000205038	0.000088490
4	1901	19	18298	2161	15717	-0.000520183	0.007889051
5	2980	3	19883	2450	15730	-0.000636478	0.000136184
6	2973	13	17554	2043	15745	-0.000622536	0.000183910
7	1043	19	19868	4636	15979	-0.001833651	0.014681060
8	2973	19	21245	4655	16549	-0.000514841	0.000183910
9	2974	9	17439	1028	15040	-0.000840743	0.000177090
10	1322	4	18160	867	15014	-0.002721290	0.012406287

Table 4: The numerical illustration for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, t_0 = 30, B = 6000$

Run	*Best*	$\lvert I \rvert$	$\sum_{i=1}^{g} T_i$	$\sum_{i \in I} N_i$	*Time*	$E(OC)$	
						Our approach	*Analytical*
1	1245	14	25520	4843	22105	-0.001922796	0.013027503
2	2998	9	19411	1262	20670	0.000137230	0.000013606
3	2989	9	21696	480	21216	-0.003209647	0.000074869
4	2976	2	32181	11494	23119	-0.000010734	0.000163452
5	2988	13	24416	3634	21731	0.000304445	0.000081679
6	956	17	54440	34805	28089	-0.001520444	0.015404094
7	2988	16	21506	1341	21232	-0.001677157	0.000081679
8	2204	5	28964	7414	22620	0.000365434	0.005628306
9	2974	15	44380	27554	26145	0.001073081	0.000177090
10	1234	3	20736	1173	21029	-0.003149694	0.013116654

are negative values for some values in the $E(OC)$ of our approach, whereas we know that the $E(OC)$ should be positive values. The reason for this situation is that in our approach we used OO procedure to select randomly the set G in the first stage, so in some replications, the actual best system i^* will not be selected from the first stage. It means that the actual best system i^* may not belongs to the set G. This will imply that we may not take into consideration, the addition observation in final stage, compare to the best system b. Therefore,

Table 5: The numerical illustration for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, t_0 = 50, B = 10000$

Run	*Best*	$\|I\|$	$\sum_{i=1}^{g} T_i$	$\sum_{i\in I} N_i$	*Time*	$E(OC)$	
						Our approach	*Analytical*
1	2967	19	32095	4327	34001	-0.000506349	0.000224843
2	2270	12	31496	3671	34394	-0.001562686	0.005144831
3	2990	17	30731	3226	33791	-0.001270842	0.000068059
4	2975	4	34190	6192	34429	-0.000041503	0.000170271
5	2984	2	29469	1144	33491	-0.003383476	0.000108927
6	1208	13	42173	14937	36051	0.000180282	0.013327780
7	1845	15	37330	9039	34984	-0.001122835	0.008314437
8	368	17	32984	4852	34539	-0.000924643	0.020470383
9	2989	17	32562	11691	34851	-0.002216927	0.000074869
10	2980	4	38422	8877	36320	-0.000995063	0.000136184

Table 6: The numerical illustration for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, t_0 = 80, B = 16000$

Run	*Best*	$\|I\|$	$\sum_{i=1}^{g} T_i$	$\sum_{i\in I} N_i$	*Time*	$E(OC)$	
						Our approach	*Analytical*
1	2985	15	64340	24638	56867	-0.002486243	0.000102114
2	2997	2	56076	14732	55649	-0.001599895	0.000020411
3	2977	4	59453	18303	55754	-0.000723406	0.000156634
4	2994	2	76102	37073	59295	-0.000992409	0.000040828
5	2460	15	64818	40768	57662	-0.000470404	0.003770423
6	2983	9	61452	22782	56093	-0.001146220	0.000115740
7	2995	13	44454	4946	52861	-0.002068828	0.000034021
8	317	6	42472	2862	54372	-0.000935940	0.020925136
9	1361	3	43560	5013	53677	0.000014186	0.012093531
10	2977	14	73237	36119	59906	-0.000668625	0.000156634

in certain occasions sample mean for the best system b will become less than the sample mean for the actual best system i^*, (i.e. $\bar{y}_b < \bar{y}_{i^*}$) and will result in the $E(OC)$ of our approach is equal to $E(\bar{y}_b - \bar{y}_{i^*}) < 0$. However, we can consider that the $E(OC)$ of our approach as the absolute value of the difference between the sample means of the best system b and the actual best system i^*, since our focus would be the difference means between those systems.

Table 9 summarizes the results for over 100 replications in selecting one of

Table 7: The numerical illustration for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, t_0 = 100, B = 20000$

Run	*Best*	$\|I\|$	$\sum_{i=1}^{g} T_i$	$\sum_{i \in I} N_i$	*Time*	$E(OC)$	
						Our approach	*Analytical*
1	2351	14	48406	4043	64335	0.000090873	0.004555761
2	2983	14	66142	21747	68484	0.000571347	0.00011574
3	2979	2	53990	7775	65918	0.000578487	0.000143000
4	2980	9	51088	5584	66160	-0.001119932	0.000136184
5	1522	7	60711	13179	67189	-0.000560227	0.010815637
6	2991	8	47600	1453	64678	-0.000929129	0.000061250
7	2979	5	57598	10753	66674	-0.000532607	0.00014300
8	2989	3	53696	5197	65645	-0.001005581	0.000074869
9	1781	4	75711	26651	70450	0.000203385	0.008803542
10	2012	6	79521	31931	71401	-0.000627771	0.007052911

Table 8: The numerical illustration for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, t_0 = 200, B = 40000$

Run	*Best*	$\|I\|$	$\sum_{i=1}^{g} T_i$	$\sum_{i \in I} N_i$	*Time*	$E(OC)$	
						Our approach	*Analytical*
1	2979	10	133224	46909	135335	0.000455854	0.000143000
2	2981	11	127307	40342	138964	-0.000100628	0.000129369
3	2988	13	136981	54009	141489	-0.000010765	0.000081679
4	2979	2	99268	12764	132870	-0.000663566	0.000143000
5	1249	17	93098	9196	127409	-0.001289119	0.012995109
6	1805	14	88346	11287	126256	-0.000755032	0.008619757
7	2998	2	121853	33521	133018	-0.001680106	0.000013606
8	2424	13	161544	79558	141287	0.000200142	0.004028874
9	2260	5	84533	1548	125760	-0.000655040	0.005217882
10	2987	18	158632	77372	142306	0.000372755	0.000088490

the best (1%) systems. Here we define B as the total budget and $\overline{T}$ represents the average of the elapsed (execution) time. In addition, referring to the algorithm of Almomani and Abdul Rahman [1], $\overline{\sum_{i=1}^{g} T_i}$ is the average number of the total sample size in the **Stopping Rule** step, and $\overline{\sum_{i \in I} N_i}$ is the average number of the total sample size in **Screening** step, with the $\overline{E(OC)}$ is the average number of the Expected Opportunity Cost. In this table, keep in mind that, we take the absolute values of the difference between $\bar{y}_b$ and $\bar{y}_{i^*}$ to calculate the $\overline{E(OC)}$ of our approach.

Table 9: The performance of the selection algorithm for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%$ over 100 replications for all values of t_0

t_0	10	30	50	100	200
B	2000	6000	10000	20000	40000
$\overline{T}$	9305.76	23472.83	36256.68	67532.71	138686.80
$\overline{\sum_{i=1}^{g} T_i}$	16403	28508	35203	58534	118892
$\overline{\sum_{i\in I} N_i}$	4171	8784	7872	12701	35176
Our approach $P(CS)$	75%	61%	66%	70%	68%
Analytical $P(CS)$	85%	85%	85%	85%	85%
Our approach $\overline{E(OC)}$	0.0021902	0.0014895	0.0012921	0.0009813	0.0007499
Analytical $\overline{E(OC)}$	0.0030829	0.0052927	0.0041874	0.0031094	0.0036003

From Table 9, note that the $\overline{T}$ and $\overline{\sum_{i=1}^{g} T_i}$ values are keep changing for different values of t_0 with increasing pattern as t_0. This is expected when we increase the total budget B. Furthermore, the $\overline{\sum_{i\in I} N_i}$ are approximately increasing for all cases of t_0 except when $t_0 = 50, 100$. This happened since in this approach we calculate the values of N_i after increasing the value of Δ to compute the new budget allocation for each system. However, the effects of t_0 on N_i are still not clear since the values of N_i are depending on the new budget allocation for each system that extracted by using Theorem 4.1. Besides that, Table 9 shows that the $P(CS)$ for our approach are relatively high and are quite closed to the analytical $P(CS)$. We found a high value in $P(CS)$ when $t_0 = 10$ as high as 75% compared to other values of t_0. Furthermore, the $\overline{E(OC)}$ for our approach are closed to the analytical $\overline{E(OC)}$, with the smallest difference when $t_0 = 10$.

Figure 1 shows the relation between the $E(OC)$ of our approach and the analytical values for all t_0 values, when this experiment were repeated for 100 replications. The figure considers the differences between the $\bar{y}_b$ and $\bar{y}_{i^*}$ as absolute value. Apparently the Almomani and Abdul Rahman [1] selection approach produces a very small values of the $E(OC)$ and they are mostly very closed to the analytical values in most replications. Figure 2 presents the $E(OC)$ values of our approach for each t_0. We find that the largest value in $E(OC)$ when $t_0 = 10$. Overall, the $E(OC)$ values using our approach are small.

Figure 1: Relationship between our approach $E(OC)$ and analytical $E(OC)$ for all values of t_0 when $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%$ over 100 replications

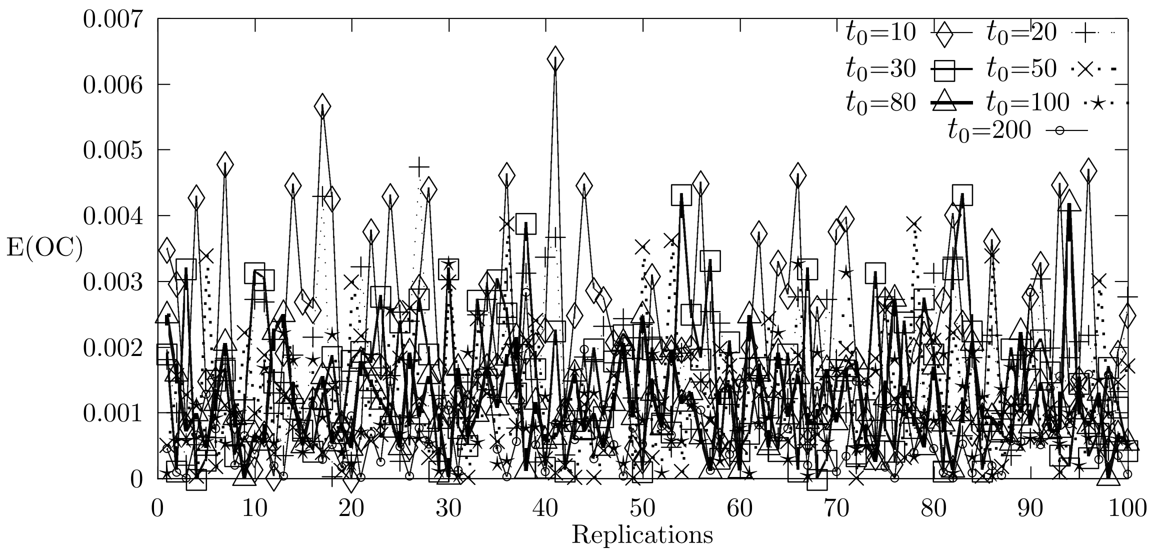

Figure 2: The $E(OC)$ values of our approach for all t_0 values when $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%$ over 100 replications

Now consider the same objective to select one of the best (1%) systems over 100 replications for the same parameter settings but with $B = 50000$ as a total budget. The results are reported in Table 10. Clearly, we can see the increasing pattern in the average of the elapsed (execution) time $\overline{T}$ when the values of t_0 are increase. Also, notice that for t_0 of $10, 20$ and 30, the values of $\overline{T}$ are very closed together. Furthermore, the values of $\overline{\sum_{i=1}^{g} T_i}$ for $t_0 = 10, 20, 30, 50, 80$ are relatively closed together since they used the same total budget. In the same time, when t_0 is 100 and 200, the values of $\overline{\sum_{i=1}^{g} T_i}$ are relatively large since these two values of t_0 are large comparing with the remainder values of t_0. The values of $\overline{\sum_{i \in I} N_i}$ are different for all values of t_0, since it depends on the new budget allocation that extracted by using Theorem 4.1. However, the $P(CS)$ for our approach are almost the same for all values of t_0 except when $t_0 = 200$. Since we used the total budget of $B = 50000$ for all t_0, then we consider that this budget is sufficient to get the best system with high $P(CS)$. Also, we find that the $P(CS)$ for our approach are approximately equal with the analytical values of $P(CS)$ for all t_0 values except when $t_0 = 200$. We assume that when $t_0 = 200$, it suggests that total budget B should be more than 50000. The $P(CS)$ for our approach in this experiment is high (better) than the $P(CS)$ in the Table 9. This is expected since we increased the total budget, B to 50000. Clearly, that the highest $P(CS)$ occurred when $t_0 = 10$ with the value as high as 84%. We also find that, the $\overline{E(OC)}$ for our approach is small for all values of t_0 where for each t_0 of 100 and 200 it becomes smaller and the values are closed to the analytical values. By comparing the values of $\overline{E(OC)}$ for our approach and the analytical between Table 9 and Table 10, we find that they are smaller in Table 10. The difference values of $\overline{E(OC)}$ between our approach and the analytical also smaller in Table 10 as compared in Table 9.

Table 10: The performance of the selection algorithm for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, B = 50000$ over 100 replications for each value of t_0

t_0	10	30	50	100	200
$\overline{T}$	31806.12	37893.73	51101.87	78518.35	140060.80
$\overline{\sum_{i=1}^{g} T_i}$	78006	70352	78752	84886	119095
$\overline{\sum_{i\in I} N_i}$	41229	32124	31523	36829	38049
Our approach $P(CS)$	84%	80%	82%	81%	77%
Analytical $P(CS)$	85%	85%	85%	85%	85%
Our approach $\overline{E(OC)}$	0.0018024	0.0016028	0.0012074	0.0009987	0.0007062
Analytical $\overline{E(OC)}$	0.0020682	0.0019704	0.0018143	0.0019885	0.0023973

Figure 3 shows the relationship between the $E(OC)$ for our approach and the analytical for a 100 replication when the difference between the $\bar{y}_b$ and $\bar{y}_{i^*}$ are considered as an absolute values. We note, the $E(OC)$ values for our approach are small and closed to the analytical values in most replications. From Figure 1, notice that most replications show high analytical values in the $E(OC)$ compared to our approach. Meanwhile, this situation is less shown in Figure 3. This is due to the usage of high total budget B in Figure 3. Figure 4 shows the $E(OC)$ values for our approach for each value of t_0. It is clear that for all values of t_0 the $E(OC)$ for our approach is very small with the largest value occurs when $t_0 = 10, 30$.

Clearly, using a total budget, $B = 50000$ we will get the desired $P(CS)$ with a minimum value of $E(OC)$. Also, we find that if we take unnecessary simulation samples, it will increase the execution time.

Figure 3: Relationship between our approach $E(OC)$ and analytical $E(OC)$ for all values of t_0 when $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, B = 50000$ over 100 replications

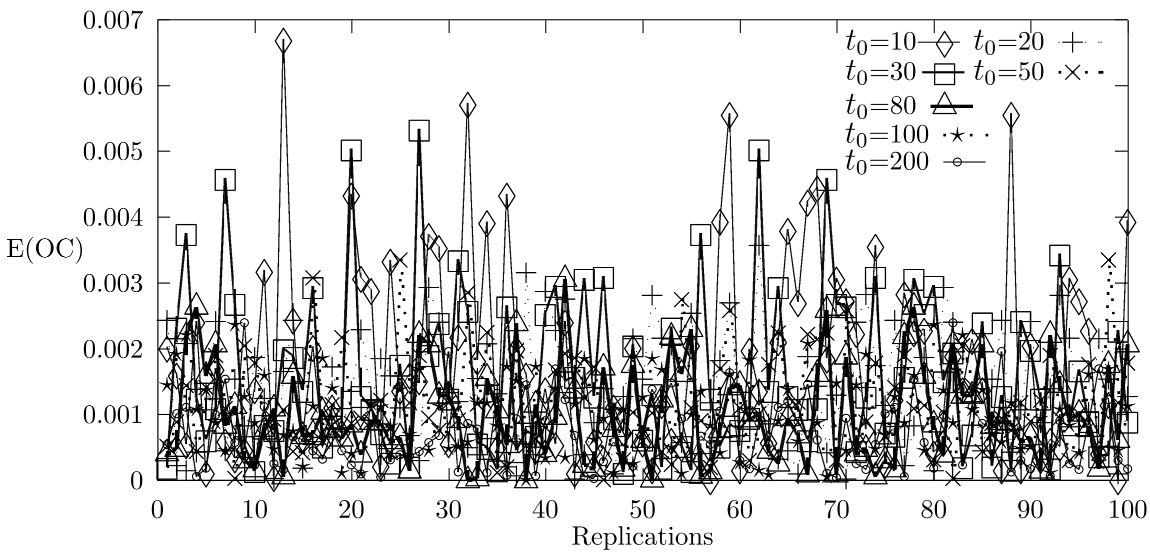

Figure 4: The $E(OC)$ values of our approach for all t_0 values when $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%, B = 50000$ over 100 replications

Now we consider the pairwise combination of $t_0 = 20, 50$ and 100, a reasonable total budget B as $10000, 15000$ and 27000 respectively and with the same parameter settings $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%$. Table 11 contains the results of this experiment, over a 100 replication for selecting one of the best (1%) systems. Compared to the results in Table 10, in this experiment we get high values of $P(CS)$ with a minimum $E(OC)$ for a case with small number of budget together with a small number of simulation samples. Thus, this last experiment was executed in short time.

Figure 5 shows the relation between the $E(OC)$ values of our approach and the analytical for 100 replications. Although in this last experiment we used relatively a small number of total budget B, we still find that the $E(OC)$ values for our approach are very small and are closed to the analytical values in most replications. However, in this experiment the number of replications that selected the best system which is faraway from the actual best system is also small. Figure 6 shows the $E(OC)$ values for our approach for each value of t_0. Clearly, we find that, for each value of t_0 the $E(OC)$ is very small with

Table 11: The performance of the selection algorithm for $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%$ over 100 replications

t_0	20	50	100
B	10000	15000	27000
$\overline{T}$	16160.54	36485.31	72012.35
$\overline{\sum_{i=1}^{g} T_i}$	20793	38981	66745
$\overline{\sum_{i\in I} N_i}$	4372	11682	21877
Our approach $P(CS)$	83%	80%	78%
Analytical $P(CS)$	85%	85%	85%
Our approach $\overline{E(OC)}$	0.0018189	0.0011006	0.0011060
Analytical $\overline{E(OC)}$	0.0018010	0.0016727	0.0022173

the largest value occurs when $t_0 = 20$.

Figure 5: Relationship the $E(OC)$ values of our approach and the analytical for each value of t_0 when $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%$ over 100 replications

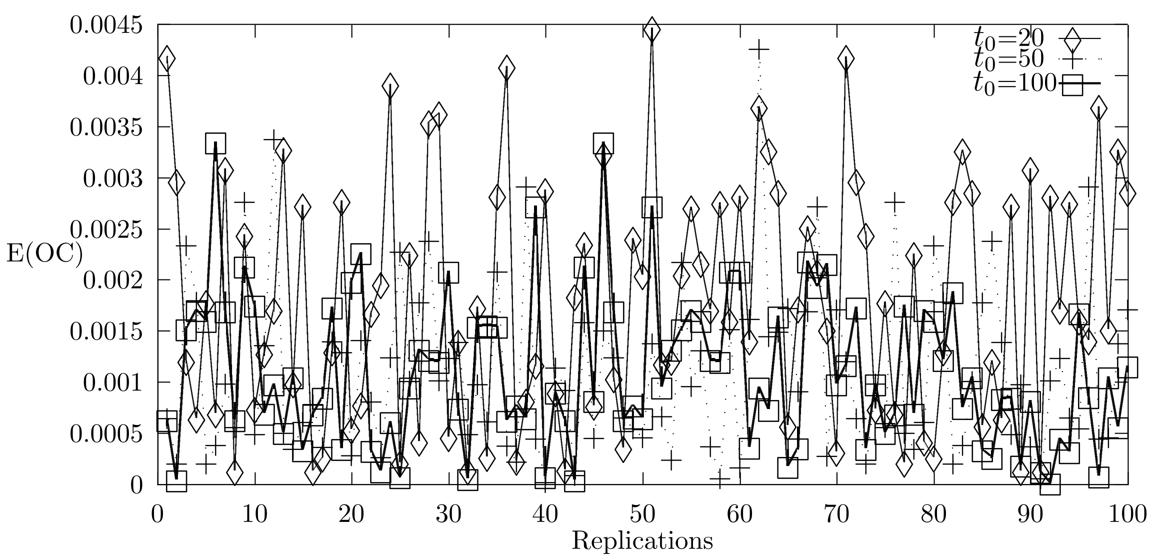

Figure 6: The $E(OC)$ values of our approach for all t_0 values when $n = 3000, g = 200, k = 20, \Delta = 50, m\% = 1\%$ over 100 replications

From all the above experiments, we can make a few conclusions on the Almomani and Abdul Rahman [1] selection approach. Initially, we find that the approach is affected by t_0 in different ways. From the numerical results, we note that the initial sample size t_0 do have affects on the B, where it needs the minimum value of B in order to make the approach works. Clearly, when we increase the budget B we will get a better performance (high $P(CS)$ and minimum $E(OC)$) for each value of t_0. In particular, when we take $B = 50000$, we get high values in $P(CS)$ and minimum values of $E(OC)$ for each value of t_0, but in the same time, the using of the unnecessary samples will cause high elapsed time. The second parameter affected by t_0 is the elapsed (execution) time, where we found that the elapsed time will increase if we increase the values of t_0. Besides that, the total sample size in **Stopping Rule** in algorithm of Almomani and Abdul Rahman [1] ($\sum_{i=1}^{g} T_i$), are also increased when t_0 are increased. We also find that for a small value of t_0 will end up with high value in $E(OC)$. On the other hand, the total sample size in **Screening** step in algorithm Almomani and Abdul Rahman [1] ($\sum_{i\in I} N_i$), is not clearly affected by t_0 since the values are calculated after we compute the new budget allocation for each system. Besides that, we also note that the $P(CS)$ values for Almomani and Abdul Rahman [1] selection approach are very closed to the analytical values for each value of t_0, and they become better (high) when we increase the total budget, B.

6.2.2 Example 2

Now we want to study the effects of the increment in simulation sample size, Δ, using the same selection approach with the same queuing systems and assumptions as in Example 1. The difference is only we would like to consider six different values of Δ, where Δ are $10, 20, 50, 80, 100$ and 200, with the same total budget $B = 10000$. Let $n = 3000$, $g = 200$, $\alpha_2 = \alpha_3 = 0.005$, $\delta = 0.05$, $k = 20$ and $t_0 = 20$ for all different values of Δ. Suppose we want to select one of the best (1%) systems, then our target is the systems from 2971 to 3000. Also the analytical probability of the correct selection can be calculated as $P(CS) \geq 1 - \left(\left(1 - \frac{1}{100}\right)^{200} + 0.005 + 0.005\right) \geq 0.85$. Table 12 contains the results of this experiment, with a 100 replication. All variables in this table are defined as the same as the previous example.

From the table, we find that the values of $\overline{T}$ and $\overline{\sum_{i=1}^{g} T_i}$ keep changing for each value of Δ with increasing pattern with exception, when $\Delta = 20$ which shows the minimum values of $\overline{T}$ and $\overline{\sum_{i=1}^{g} T_i}$. The increasing values are as expected when we increase the values of increment in simulation samples Δ, however the increasing amounts are relatively small. We also notice that, the values of $\overline{\sum_{i\in I} N_i}$ are different with each value of Δ, but we cannot see a clear relation between them, since the values of $\overline{\sum_{i\in I} N_i}$ are depending on

Table 12: The performance of the selection algorithm for $n = 3000, g = 200, k = 20, t_0 = 20, m\% = 1\%, B = 10000$ over 100 replications

Δ	10	50	80	100	200
$\overline{T}$	15358.87	16160.54	17581.20	18438.66	23558.76
$\overline{\sum_{i=1}^{g} T_i}$	16842	20793	27841	29453	51266
$\overline{\sum_{i\in I} N_i}$	7256	4372	5818	4072	7433
Our approach $P(CS)$	81%	83%	80%	79%	83%
Analytical $P(CS)$	85%	85%	85%	85%	85%
Our approach $\overline{E(OC)}$	0.0017856	0.0018189	0.0013916	0.0014443	0.0012811
Analytical $\overline{E(OC)}$	0.0015624	0.0018010	0.0013866	0.0017427	0.0012553

the new budget allocation for each system. Moreover, from Table 12 we also can see that, the values of $P(CS)$ for our selection approach are high and very closed to its analytical values for each Δ. The highest values of $P(CS)$ for our approach occurs when $\Delta = 20$ with the $P(CS) = 84\%$ compared to other values of Δ. We also notice the $\overline{E(OC)}$ for our approach are very small and are so closed with the analytical values of $\overline{E(OC)}$. However, we note here the $P(CS)$ and $\overline{E(OC)}$ are not affected by Δ.

Clearly, the appropriate increment in simulation samples, regarding to the results in Table 12, is when $\Delta = 20$. For this value of Δ, we get the minimum elapsed time where $\overline{T} = 15088.73$, and used the least simulation samples $\overline{\sum_{i=1}^{g} T_i}$ where $\overline{\sum_{i=1}^{g} T_i} = 15972$. In the same time for this $\Delta = 20$, the $P(CS)$ of our selection approach is highest as recorded as 84% which is approximately the same value for the analytical $P(CS)$. On the other hand, if we consider the $E(OC)$ as a measure of selection quality of our approach, we ended up with two best options for Δ values. The first one is $\Delta = 80$ since this value will give the smallest difference in the $\overline{E(OC)}$ between the analytical and our approach. Another option is when $\Delta = 200$ since it has the minimum $\overline{E(OC)}$. However, we cannot take these values of Δ as the best values for all exterminates in context of $E(OC)$ since the performance of Almomani and Abdul Rahman [1] selection approach is very closed together for all values of Δ.

Figure 7 shows the relation between the $E(OC)$ of our approach and the analytical, when this experiment was repeated for 100 replications, for each value of Δ. Here we take the differences between the $\bar{y}_b$ and $\bar{y}_{i^*}$ as absolute values. We can see clearly that for all values of Δ, the values of the $E(OC)$ for our approach are very small and are mostly closed to the analytical values in most replications. Moreover, the number of replications that Almomani and Abdul Rahman [1] selection approach failed to select as the best system are approximately the same for each value of Δ. Figure 8 shows the $E(OC)$ of our approach for all values of Δ. It is clear that for all values of Δ, the $E(OC)$ of our approach approximately the same.

Figure 7: Relationship between our approach $E(OC)$ and analytical $E(OC)$ for all values of Δ when $n = 3000, g = 200, k = 20, t_0 = 20, m\% = 1\%, B = 10000$ over 100 replications

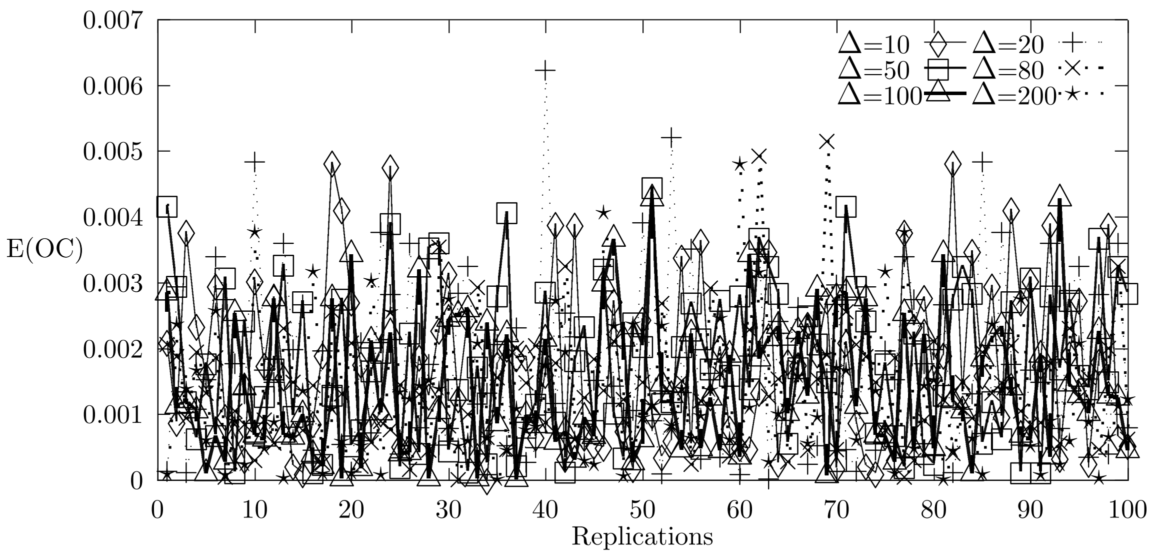

Figure 8: The $E(OC)$ values of our approach for all Δ values when $n = 3000, g = 200, k = 20, t_0 = 20, m\% = 1\%, B = 10000$ over 100 replications

From Example 2 we can conclude the following: first, when we increase the Δ, it will follow with an increase in the elapsed time values, and an increasing in the simulation sample $\overline{\sum_{i=1}^{g} T_i}$ although they are small. Secondly, there is no clear evidence to show the effects of Δ on the $\overline{\sum_{i\in I} N_i}$ and $P(CS)$. Finally, there is also no substantial effect of Δ on the expected opportunity cost $E(OC)$ of a potentially incorrect selection.

6.2.3 Example 3

Now using the same queuing systems, parameter settings and assumptions as in Example 1, we want to study the effect of the total budget B on the selection approach. We consider nine different values of total budget B, such as 10000, 15000, 25000, 40000, 50000, 80000, 100000, 150000 and 250000. Suppose we want to select one of the best (1%) systems, then our target is the systems from 2971 to 3000. Furthermore, the analytical probability of the correct selection can be calculated as $P(CS) \geq 1 - \left(\left(1 - \frac{1}{100}\right)^{200} + 0.005 + 0.005 \right) \geq 0.85$.

Figure 9 shows the relation between the total budget B and the $P(CS)$ for a 100 replication. Clearly, it shows that when we increase the total budget B the $P(CS)$ will increase, and it reaches to the maximum value of 1, when the total budget, B are 150000 and more. Figure 10 shows the relation between the total budget B and the $E(OC)$ for a 100 replication. Unfortunately, we find an odd situation here, for example when $B = 50000$ it increases the $E(OC)$, while the value should be less than the value when $B = 40000$. As a future work we will try to improve the efficiency of the selection approach in context of the expected opportunity cost $E(OC)$ of a potentially incorrect selection, in order to draw a parallel conclusion that the values of $E(OC)$ should be decreased when we increase the value of B.

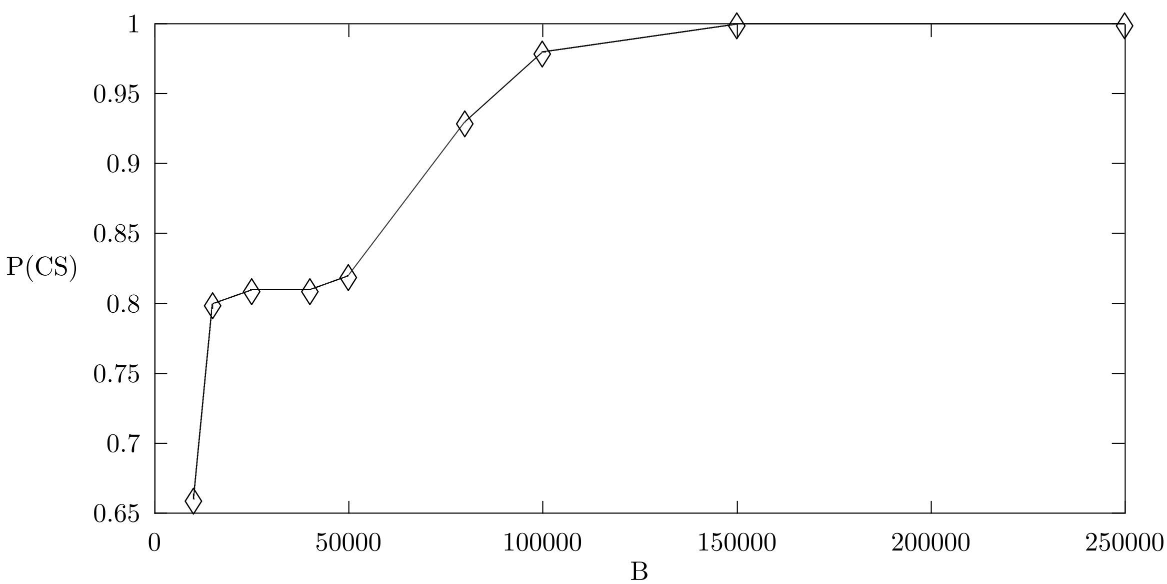

Figure 9: Relationship between the total budget B and the $P(CS)$ when $n = 3000, g = 200, k = 20, t_0 = 50, \Delta = 50, m\% = 1\%$ over 100 replications

Figure 11 shows the relation between the total budget B and the elapsed (execution) time. Obviously it shows that the increase in the total budget will increase the elapsed time. However, this will create a problem in simulation studies, because we want to achieve our goal of selecting the best system with a minimum elapsed time and at the same time a maximized the $P(CS)$.

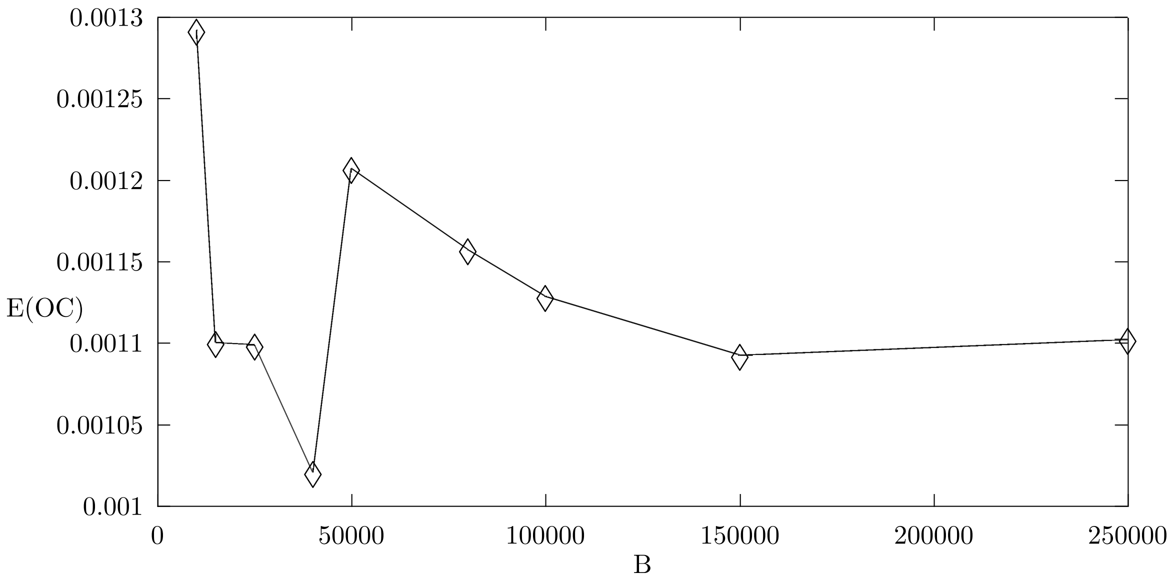

Figure 10: Relationship between the total budget B and the $E(OC)$ when $n = 3000, g = 200, k = 20, t_0 = 50, \Delta = 50, m\% = 1\%$ over 100 replications

Figure 12 and Figure 13 show the relation between total budget B with the simulation samples $\overline{\sum_{i=1}^{g} T_i}$ and $\overline{\sum_{i \in I} N_i}$ respectively, for a 100 replication. It is clear that, by increasing the total budget B, it will increase the number of simulation samples $\overline{\sum_{i=1}^{g} T_i}$ and $\overline{\sum_{i \in I} N_i}$.

This example shows the effects of the total budget B on the Almomani and Abdul Rahman [1] selection approach in context of the probability of correct selection, elapsed (execution) time and the simulation samples. Of course, when we increase the total budget B, it will increase the $P(CS)$, but at the same time, it will cause an increase in elapsed time and a number of simulation samples. However, we find an odd situation in this study regarding the effects of the total budget B on the $E(OC)$ where it should be the $E(OC) \to 0$ when $B \to \infty$ which is does not hold in this study.

7 Open Problem

This research can be used to solve real world problems such as the traffic problem, Buffer Allocation Problem, inventory system and a close networks.

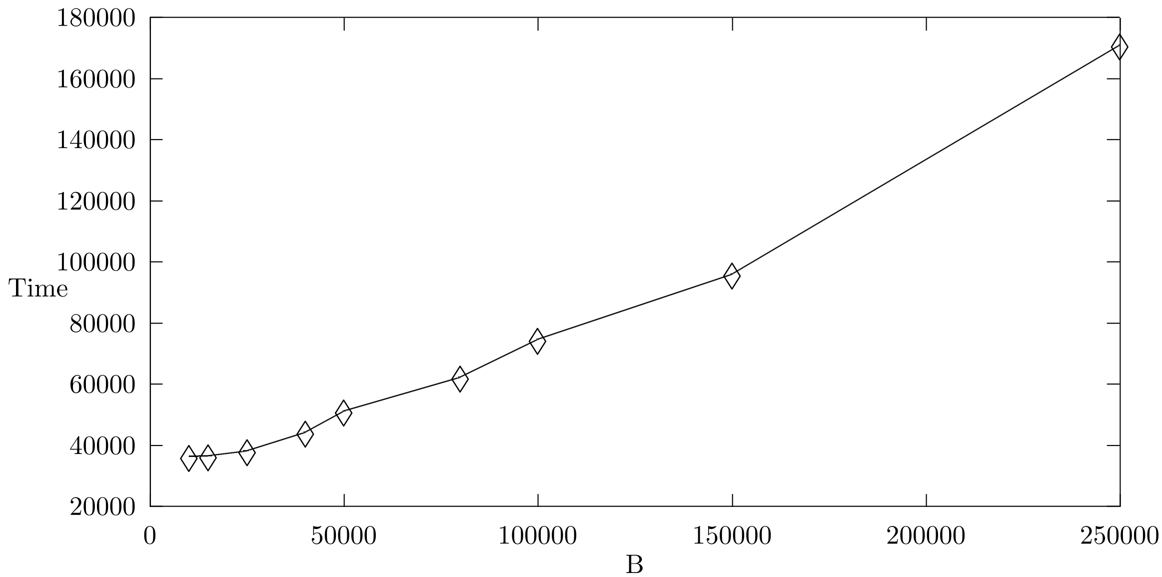

Figure 11: Relationship between the total budget B and the elapsed time (millisecond) when $n = 3000, g = 200, k = 20, t_0 = 50, \Delta = 50, m\% = 1\%$ over 100 replications

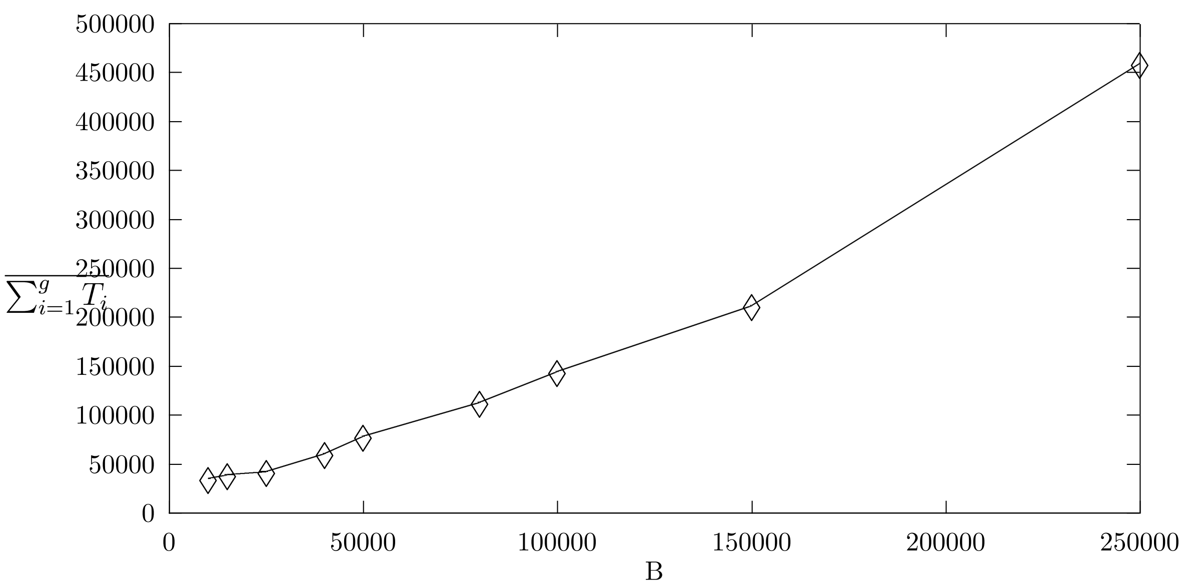

Figure 12: Relationship between the total budget B and the $\overline{\sum_{i=1}^{g} T_i}$ when $n = 3000, g = 200, k = 20, t_0 = 50, \Delta = 50, m\% = 1\%$ over 100 replications

8 Concluding Remarks

In this paper, we study the performance of the selection approach as proposed by Almomani and Abdul Rahman [1]. Our discussions are based on

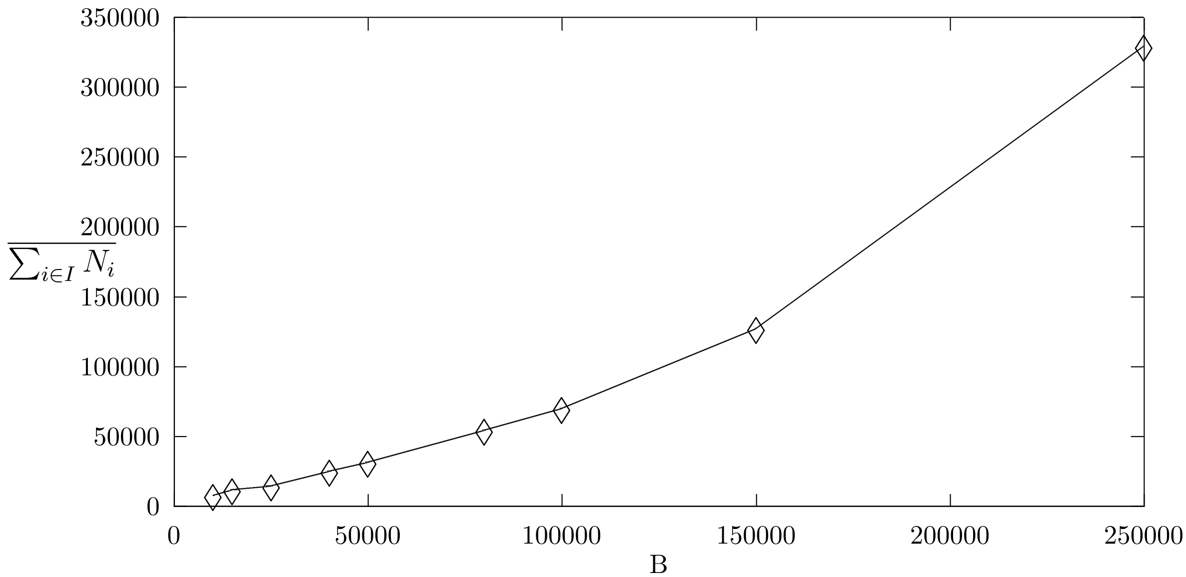

Figure 13: Relationship between the total budget B and the $\overline{\sum_{i \in I} N_i}$ when $n = 3000, g = 200, k = 20, t_0 = 50, \Delta = 50, m\% = 1\%$ over 100 replications

various simulation parameters. They are initial sample size t_0, increment in simulation samples Δ, total budget B and elapsed (execution) time T. We apply the selection approach on the $M/M/1$ queuing systems with the parameters setting as discussed previously. From the numerical results we find that, the first simulation parameter that affected the selection approach is the initial sample size t_0. When we change the initial sample size t_0 we need to use the minimum total budget B for each value of t_0. As initial sample size t_0 increase, the elapsed time T and the simulation samples T_i will increase. Meanwhile, to achieve the best performance in context of $E(OC)$ in our case study, we need a large value of t_0. However, no clear relationship between t_0 with each the simulation samples N_i, and $P(CS)$ respectively. Finally, since we find that the selection approach is sensitive to the initial sample size t_0, as a future work we will try to add a "zero-th stage" of sampling in order to determine an adequate choice of the initial sample size t_0. In our case study on the increment in simulation samples Δ, we find that it will affect the elapsed time T and simulation samples T_i. On the other hand, there are no substantial effect on the simulation samples N_i, $P(CS)$ and $E(OC)$. However, we should be careful in determining the increment values in simulation samples Δ, since if it is too small then it will require us to repeat the **Simulation Budget Allocation** step in the algorithm for many times whereas by choosing a large value of Δ will waste our computation time. Meanwhile, our study on the total budget, B, find that the simulation samples N_i, simulation samples T_i, elapsed time T

and $P(CS)$ are increased when we increase the total budget B. Note that the value of total budget B is very important in selection approach, since we want to select the best system with high $P(CS)$ and at the same time with a small number of simulation sample in short elapsed time. On the other hand, in our case study on the effect of the total budget B and the $E(OC)$, we find an odd situation. We know that when the total budget B increases the $E(OC)$ should be decreased, but our example shows the other way around. Thus in our future work we will look further into this situation. Finally, we also note that the Almomani and Abdul Rahman [1] selection approach, selects the best simulated system in a short time with a different parameter settings. Of course, selecting the best system from a huge number of alternative with a minimum elapsed time T still a challenge in the selection problems. As a future work we would like to improve the Almomani and Abdul Rahman [1] selection approach so that it will be able to select the best system with a shorter elapsed time T.

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