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Measurable soft sets

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Abstract

The soft set is a mapping from a parameter to the crisp subset of universe. Molodtsov introduced the concept of soft sets as a generalized tool for modeling complex systems involving uncertain or not clearly defined objects. In this paper the concept of measurable soft sets are introduced and their properties are discussed. The open problem of this paper is to develop measurable functions and lebesgue integral in soft set theory context.

Keywords: *soft set, outer measure of a soft set, measurable soft set.*

1 Introduction

Many disciplines, including engineering, economics, medical science and social science are highly dependent on the task of modeling and computing uncertain data. When the uncertainty is highly complicated and difficult to characterize, classical mathematical approaches are often insufficient to derive effective or useful models. Testifying to the importance of uncertainties that cannot be defined by classical mathematics, researchers are introducing alternative theories every day. In addition to classical probability theory, some of the most important results on this topic are fuzzy sets [21], intuitionistic fuzzy sets [3], vague sets [9], interval-valued fuzzy sets [4, 11] and rough sets [17]. But each of these theories has its inherent limitations as pointed out by Molodtsov[16]. Molodtsov[16] introduced soft set theory as a completely new approach for modeling vagueness and uncertainty. This so-called soft set theory is free from the above mentioned difficulties as it has enough parameters. In soft set theory, the problem of setting membership function simply doesn't arise. This makes the theory convenient and easy to apply in practice. Soft set theory has potential applications in various fields including smoothness of functions, game theory, operations research, Riemann integration, probability theory and measurement theory. Most of these applications have already been demonstrated by Molodtsov[16].

In recent years, soft set theory have been developed rapidly and focused by many researchers in theory and practice. Maji et al. [15] defined several operations on soft sets and made a theoretical study on the theory of soft sets. Aktas and Cagman[1]compared soft sets to the related concepts of fuzzy sets and rough sets. They also defined the notion of soft groups. Jun [12] applied soft set to the theory of BCK/BCI algebra and introduced the concept of soft BCK/BCI algebra. Jun and park [13] discussed the applications of soft sets in ideal theory of BCK/BCI algebra. Feng et al. [7] defined soft semi rings and several related notions in order to establish a connection between soft sets and semi rings. Furthermore based on [15], Ali et al.[2]introduced some new operations on soft sets and by improving the notion of complement of soft set, proved that certain De Morgan's laws hold in soft set theory. Qin and Hong [18] introduced the notion of soft equality and established lattice structures and soft quotient algebras of soft sets. Chen et al. [6] presented a new definition of soft set parametrizationreduction and compared this definition to the related concept of attribute reduction in rough set theory. Kong et al. [14] introduced the notion of normal parameter reduction of soft sets and constructed a reduction algorithm based on the importance degree of parameters. Gong et al. [10] proposed the notion of bijective soft sets and implement some operations of classical rough sets. Xiao et al. [20] initiated the concept of exclusive disjunctive soft sets, which is an extended concept of bijective soft set. Babhita and Sunil [5] introduced the concept of soft set relations as a soft subset of the Cartesian product of soft sets.

In this paper a new approach called measurable soft sets is presented. Mathematically, this so called notion of measurable soft sets may seem different from the classical measure theory but the underlying concepts are very similar. This new type of soft sets satisfies all the basic properties of measurable sets. The organization of the paper is as follows: In section 2, basic notions of soft sets are given. Section 3 focuses on the study of outer measure of soft sets. In section 4, we present the concept of measurable soft sets. The last section we summarize all the contributions and give our future plan.

2 Preliminaries

We recall some definitions and notions related to soft set theory.

Let U be an universe set and E be a set of parameters with respect to U. Usually parameters are attributes and properties of the objects in U. Let $\hat{P}(U)$ denotes the power set of U

Definition 2.1 ([16]) A pair $(\xi, \tilde{A})$ is called a *soft set* over U, where $\tilde{A} \subseteq E$ and $\xi : \tilde{A} \rightarrow \hat{P}(U)$ is a mapping.

In other words, a soft set over U can be regarded as a parameterized family of subsets of U, which gives an approximation(soft) description of the objects in U. For $\hat{e} \in \tilde{A}$, $\xi(\hat{e})$ may be considered as the set of $\hat{e}$ -approximate elements of the soft set $(\xi, \tilde{A})$.

Definition 2.2 ([15]) For two soft sets $(\xi, \tilde{A})$ and $(\zeta, \tilde{B})$ over a common universe *U*, we say that $(\xi, \tilde{A})$ is a soft subset of $(\zeta, \tilde{B})$ if

(i) $\tilde{A} \subseteq \tilde{B}$

(ii) $\xi(\hat{e}) \subseteq \zeta(\hat{e})$ *for* $\hat{e} \in \tilde{A}$.

We write $(\xi, \tilde{A}) \tilde{\subseteq} (\zeta, \tilde{B})$.

Definition 2.3 ([15]) The *union* of two soft sets $(\xi, \tilde{A})$ and $(\zeta, \tilde{B})$ over a common universe *U* is the soft set $(\vartheta, \tilde{C})$ where

$\tilde{C} = \tilde{A} \cup \tilde{B}$ and $\forall \hat{e} \in \tilde{C}$

$$\vartheta(\hat{e}) = \begin{cases} \xi(\hat{e}) & if\ \hat{e} \in \tilde{A} - \tilde{B} \\ \zeta(\hat{e}) & if\ \hat{e} \in \tilde{B} - \tilde{A} \\ \xi(\hat{e}) \cup \zeta(\hat{e}) & if\ \hat{e} \in \tilde{A} \cap \tilde{B} \end{cases}$$

We write $(\xi, \tilde{A}) \tilde{\cup} (\zeta, \tilde{B}) = (\vartheta, \tilde{C})$.

Definition 2.4 ([2]) The *intersection* of two soft sets $\left(\xi,\tilde{A}\right)$ and $\left(\zeta,\tilde{B}\right)$ over a common universe U is the soft set $\left(\varsigma,\tilde{C}\right)$ where
$\tilde{C}=\tilde{A}\cup\tilde{B}$ and $\forall\hat{e}\in\tilde{C}$

$$\varsigma(\hat{e})=\begin{cases}\xi(\hat{e}) & if\ \hat{e}\in\tilde{A}-\tilde{B}\\ \zeta(\hat{e}) & if\ \hat{e}\in\tilde{B}-\tilde{A}\\ \xi(\hat{e})\cap\zeta(\hat{e}) & if\ \hat{e}\in\tilde{A}\cap\tilde{B}\end{cases}$$

We write $\left(\xi,\tilde{A}\right)\tilde{\cap}\left(\zeta,\tilde{B}\right)=\left(\varsigma,\tilde{C}\right)$.

Definition 2.5 ([19]) The *complement* of a soft set $\left(\xi,\tilde{A}\right)$ is denoted by $\left(\xi,\tilde{A}\right)^{\tilde{c}}$ and is defined by $\left(\xi,\tilde{A}\right)^{\tilde{c}}=\left(\xi^{c},\tilde{A}\right)$, *where* $\xi^{c}:\tilde{A}\rightarrow\hat{P}(U)$ is a mapping defined by $\xi^{c}\left(\hat{e}\right)=U-\xi\left(\hat{e}\right)$ *for* $\hat{e}\in\tilde{A}$.

Definition 2.6 ([19]) The *difference* between two soft sets $\left(\xi,\tilde{A}\right)$ *and* $\left(\zeta,\tilde{A}\right)$ is denoted by $\left(\xi,\tilde{A}\right)\Theta\left(\zeta,\tilde{A}\right)$ and is defined by the soft set $\left(\psi,\tilde{A}\right)$, where $\psi\left(\hat{a}\right)=\xi\left(\hat{a}\right)-\zeta\left(\hat{a}\right)$ *for* $\hat{a}\in\tilde{A}$.

Definition 2.7 ([15]) A soft set $\left(\xi,\tilde{A}\right)$ is called a *null* soft set denoted by ϕ_{soft} if for all $\hat{e}\in\tilde{A}$, $\xi\left(\hat{e}\right)=\phi(null\ set)$.

Definition 2.8 ([15]) A soft set $\left(\xi,\tilde{A}\right)$ is called an *absolute* soft set denoted by U_{soft} if for all $\hat{e}\in\tilde{A}$, $\xi\left(\hat{e}\right)=U$.

For the remaining parts of the paper, we consider $\Re$ (the set of all real numbers) as an universe set and E as an finite set of parameters. We use the symbol $\hat{P}\left(\Re;E\right)$ to denote the collection of all soft sets over $\Re$ via $\tilde{A}\subseteq E$.

3 Outer measure of a soft set

Definition3.1: The outer measure of a soft set $\left(\xi,\tilde{A}\right)$ is denoted by $M^*\left(\xi,\tilde{A}\right)$ and is defined by $M^*\left(\xi,\tilde{A}\right)=\sum_{\hat{a}\in\tilde{A}} m^*\left(\xi\left(\hat{a}\right)\right), where\ m^*\left(\xi\left(\hat{a}\right)\right)$ denotes the lebesgue outer measure of $\xi\left(\hat{a}\right)\subseteq\Re$ for each $\hat{a}\in\tilde{A}$.

In other words, $M^*\left(\xi,\tilde{A}\right)=\sum_{\hat{a}\in\tilde{A}} m^*\left(\xi\left(\hat{a}\right)\right)=\sum_{\hat{a}\in\tilde{A}}\inf\sum_i l\left(I_i^{\hat{a}}\right)$,where the infimum is taken over all countable collections $\left\{I_i^{\hat{a}}\right\}$ of open intervals in $\Re$ such that $\xi(\hat{a})\subseteq\bigcup_i I_i^{\hat{a}}$ for each $\hat{a}\in\tilde{A}$.

Example 3.2:Let

$A=\left\{\hat{e}_1(which\,denotes\,an\,open\,set\,in\,\Re),\hat{e}_2(which\,denotes\,a\,closed\,set\,in\,\Re)\right\}$.

Let $\left(\xi,\tilde{A}\right)$ be a soft set, where $\xi:\tilde{A}\to\hat{P}\left(\Re\right)$ be defined by

$\xi\left(\hat{e}_1\right)=(-2,2),\ \xi\left(\hat{e}_2\right)=\left[2,5\right]$.

$$Then\ M^*\left(\xi,\tilde{A}\right)=\sum_{\hat{a}\in\tilde{A}} m^*\left(\xi\left(\hat{a}\right)\right)=m^*\left(\xi\left(\hat{e}_1\right)\right)+m^*\left(\xi\left(\hat{e}_2\right)\right)$$

$$=\left(2-(-2)\right)+\left(5-2\right)$$

$$=7$$

Theorem 3.3:Let $\left(\xi,\tilde{A}\right),\left(\zeta,\tilde{A}\right)\in\hat{P}\left(\Re;\tilde{E}\right)$.Then

(i) $M^*\left(\xi,\tilde{A}\right)\geq 0$

(ii) $M^*\left(\phi_{soft}\right)=0$

(iii) if $\left(\xi,\tilde{A}\right)\tilde{\subseteq}\left(\zeta,\tilde{A}\right)$, then $M^*\left(\xi,\tilde{A}\right)\leq M^*\left(\zeta,\tilde{A}\right)$

(iv) if for each $\hat{a}\in\tilde{A}$, $\xi(\hat{a})$ is a singleton set then $M^*\left(\xi,\tilde{A}\right)=0$.

(v) $M^*\left(\left(\xi,\tilde{A}\right)\oplus x\right)=M^*\left(\varsigma,\tilde{A}\right)$, where $\left(\varsigma,\tilde{A}\right)$ is a soft set defined by $\varsigma(\hat{a})=\left\{y+x:y\in\xi(\hat{a})\right\}$ for $\hat{a}\in\tilde{A}$

Proof:-(i)-(iii) are straight forward.

(iv) Let $\xi(\hat{a})=\left\{x^a\right\}$, where $x^a\in\Re$ and $I_n^{\hat{a}}=\left(x^{\hat{a}}-\frac{1}{n},x^{\hat{a}}+\frac{1}{n}\right)$ for each $\hat{a}\in\tilde{A}$

Then clearly $\xi(\hat{a})\subseteq\bigcup_n I_n^{\hat{a}}$ .Since $l\left(I_n^{\hat{a}}\right)=\frac{2}{n}$, we have

$$M^*\left(\xi,\tilde{A}\right)=\sum_{\hat{a}\in\tilde{A}}\inf\sum_n l(I_n^{\hat{a}})$$
$$=\sum_{\hat{a}\in\tilde{A}}\lim_{n\to\infty}\sum_n\frac{2}{n}$$
$$=0\qquad ,$$

(v) Let $\varepsilon>0$ be given. Then foreach $\hat{a}\in\tilde{A},\ \exists$ a countable collection $\left\{I_n^{\hat{a}}\right\}$ of open intervals such that

$\xi(\hat{a})\subseteq\bigcup_n I_n^{\hat{a}}$ and $\sum_n l(I_n^{\hat{a}})\le m^*\left(\xi(\hat{a})\right)+\varepsilon$. As $\varsigma(\hat{a})=\xi(\hat{a})+x\subseteq\bigcup_n\left(I_n^{\hat{a}}+x\right)$, we have

$$m^*\left(\varsigma(\hat{a})\right)\le\sum_n l\left(I_n^{\hat{a}}+x\right)$$
$$=\sum_n l\left(I_n^{\hat{a}}\right)\le m^*\left(\xi(\hat{a})\right)+\varepsilon$$

Since $\varepsilon>0$ is arbitrary, we have $m^*\left(\varsigma(\hat{a})\right)\le m^*\left(\xi(\hat{a})\right)$.

The reverse inequality follows by considering $\xi(\hat{a})=\left[\xi(\hat{a})+x\right]-x$ and using the above.

Thus we get $m^*\left(\varsigma(\hat{a})\right)=m^*\left(\xi(\hat{a})\right)$ *and so* $M^*\left(\left(\xi,\tilde{A}\right)\oplus x\right)=M^*\left(\xi,\tilde{A}\right)$

Theorem 3.4 Let $\left\{\left(\xi_n,\tilde{A}\right)\right\}$ be a countable collection of soft sets. Then

$$M^*\left(\bigcup_n\left(\xi_n,\tilde{A}\right)\right)\le\sum_n M^*\left(\xi_n,\tilde{A}\right).$$

Proof:-If $M^*\left(\xi_n,\tilde{A}\right)=\infty$ for some $n\in N$, then the inequality holds trivially.

Let us assume that $M^*\left(\xi_n,\tilde{A}\right)<\infty$ *for each* $n\in N$.

We have $M^*\left(\xi_n,\tilde{A}\right)=\sum_{\hat{a}\in\tilde{A}}m^*\left(\xi_n\left(\hat{a}\right)\right)$. For each fixed n and for a given $\varepsilon>0$, there exists a countable collection $\left\{I_{n,i}^{\hat{a}}\right\}_i$ of open intervals for each $\hat{a}\in\tilde{A}$ such that $\xi_n(\hat{a})\subseteq\bigcup_i I_{n,i}^{\hat{a}}$ .satisfying $\sum_i l(I_{n,i}^{\hat{a}})<m^*\left(\xi_n(\hat{a})\right)+2^{-n}\varepsilon$.

Now $\bigcup_n\xi_n(\hat{a})\subseteq\bigcup_n\bigcup_i I_{n,i}^{\hat{a}}$. Moreover the collection $\left\{I_{n,i}^{\hat{a}}\right\}_{n,i}$ forms a countable collection of open intervals and covers $\bigcup_n\xi_n(\hat{a})$.

Then $m^*\left(\bigcup_n\xi_n(\hat{a})\right)\le\sum_n\sum_i l\left(I_{n,i}^{\hat{a}}\right)<\sum_n\left(m^*\left(\xi_n(\hat{a})\right)+2^{-n}\varepsilon\right)=m^*\left(\xi_n(\hat{a})\right)+\varepsilon$

As $\varepsilon > 0$ is arbitrary, so $m^*\left(\bigcup_n \xi_n(\hat{a})\right) \le m^*\left(\xi_n(\hat{a})\right)$. consequently,

$$M^*\left(\bigcup_n \left(\xi_n, \tilde{A}\right)\right) = \sum_{\hat{a}\in\tilde{A}} m^*\left(\bigcup_n \left(\xi_n(\hat{a})\right)\right) \le \sum_{\hat{a}\in\tilde{A}} m^*\left(\xi_n(\hat{a})\right) = \sum_n M^*\left(\xi_n, \tilde{A}\right).$$

Theorem 3.5 $M^*\left(\xi, \tilde{A}\right) = 0$ if $\xi(\hat{a})$ is countable for each $\hat{a} \in \tilde{A}$.

Proof: It follows from the fact that any countable subset of $\Re$ has outer measure zero.

Definition 3.6 A soft set $\left(\xi, \tilde{A}\right)$ is called a soft G_δ-set if each of $\xi(\hat{a})$ is a G_δ-set for each $\hat{a} \in \tilde{A}$. In other words a soft set $\left(\xi, \tilde{A}\right)$ is called a soft G_δ-set if $\xi(\hat{a})$ can be expressed as a countable intersection of open sets in $\Re$ for each $\hat{a} \in \tilde{A}$.

Example 3.7 The soft set $\left(\xi, \tilde{A}\right)$ defined by

$\xi(\hat{a}) = [c_{\hat{a}}, d_{\hat{a}}] = \bigcap_{n=1}^{\infty}\left(c_{\hat{a}} - \frac{1}{n}, d_{\hat{a}} + \frac{1}{n}\right)$ *for* $\hat{a} \in \tilde{A}$ (*where* $c_{\hat{a}}, d_{\hat{a}} \in \Re$) is a soft G_δ-set.

Theorem 3.8 For a given soft set $\left(\xi, \tilde{A}\right)$, there exist a soft G_δ-set $\left(\zeta, \tilde{A}\right) \supset \left(\xi, \tilde{A}\right)$ such that $M^*\left(\xi, \tilde{A}\right) = M^*\left(\zeta, \tilde{A}\right)$.

Proof: Since for each $\hat{a} \in \tilde{A}$, $\zeta(\hat{a})$ is a G_δ-set, there exists a countable collection of open sets O_n $(n \in N)$ *in* $\Re$ such that $\zeta(\hat{a}) = \bigcap_{n=1}^{\infty} O_n$. Now as $\left(\zeta, \tilde{A}\right) \supset \left(\xi, \tilde{A}\right)$, we have $\xi(\hat{a}) \subseteq \zeta(\hat{a}) = \bigcap_{n=1}^{\infty} O_n$.

Then for each $n \in N$, there exist an open set O_n satisfying $m^*(O_n) < m^*(\xi(\hat{a})) + \frac{1}{n}$.

Now *Now* $\zeta(\hat{a}) = \bigcap_{n=1}^{\infty} O_n \subseteq O_n \Rightarrow m^*\left(\zeta(\hat{a})\right) \le m^*(O_n) < m^*(\xi(\hat{a})) + \frac{1}{n}$

Letting $n \to \infty$, we get $m^*\left(\zeta(\hat{a})\right) = m^*\left(\xi(\hat{a})\right)$. Consequently $M^*\left(\xi, \tilde{A}\right) = M^*\left(\zeta, \tilde{A}\right)$.

4 Measureable soft sets

Definition 4.1 A soft set $(\xi,\tilde{A})$ is said to be soft measurable if for each soft set $(\zeta,\tilde{A})$, we have $M^*(\zeta,\tilde{A})=M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A}))+M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A})^{\tilde{c}})$.

Here $(\zeta,\tilde{A})$ is called test soft set.

Theorem 4.2 If $(\xi,\tilde{A})$ is soft measurable, then so $(\xi,\tilde{A})^{\tilde{c}}$.

Proof:-Follows directly from definition.

Theorem 4.3 If the soft set $(\xi,\tilde{A})$ has outer measure zero, then it is soft measureable.

Proof:-Let us choose $(\zeta,\tilde{A})$ as a test soft set. Then

$$(\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A})\subseteq(\xi,\tilde{A})\Rightarrow M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A}))\le M^*(\xi,\tilde{A})=0,$$

$$(\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A})^{\tilde{c}}\subseteq(\zeta,\tilde{A})\Rightarrow M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A})^{\tilde{c}})\le M^*(\zeta,\tilde{A}).$$

Thus

$$M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A}))+M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A})^{\tilde{c}})\le M^*(\zeta,\tilde{A})\text{......................}(4.3.1)$$

$$Again\ M^*(\zeta,\tilde{A})=\sum_{\hat{a}\in\tilde{A}}m^*(\zeta(\hat{a}))=\sum_{\hat{a}\in\tilde{A}}m^*((\zeta(\hat{a})\cap\xi(\hat{a}))\cup(\zeta(\hat{a})\cap\xi(\hat{a})^c))$$

$$\le\sum_{\hat{a}\in\tilde{A}}(m^*(\zeta(\hat{a})\cap\xi(\hat{a}))+m^*(\zeta(\hat{a})\cap\xi(\hat{a})^c))$$

$$=M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A}))+M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A})^{\tilde{c}})$$

$$So\ M^*(\zeta,\tilde{A})\le M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A}))+M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A})^{\tilde{c}})\text{..................}(4.3.2)$$

From (4.3.1) and (4.3.2) we get $M^*(\zeta,\tilde{A})=M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A}))+M^*((\zeta,\tilde{A})\tilde{\cap}(\xi,\tilde{A})^{\tilde{c}})$. Hence $(\xi,\tilde{A})$ is soft measurable.

Theorem 4.4 If $(\xi_1,\tilde{A})\ and\ (\xi_2,\tilde{A})$ are soft measurable, then so is $(\xi_1,\tilde{A})\tilde{\cup}(\xi_2,\tilde{A})$.

Proof:-Let us choose $(\zeta,\tilde{A})$ as a test soft set. Then since $(\xi_1,\tilde{A})$ is soft measurable, we have

$$M^*(\zeta,\tilde{A})$$

$$=M^*\left((\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})\right)+M^*\left((\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})^{\tilde{c}}\right)$$

$$=M^*\left((\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})\right)+M^*\left((\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})^{\tilde{c}}\tilde{\cap}(\xi_2,\tilde{A})\right)+M^*\left((\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})^{\tilde{c}}\tilde{\cap}(\xi_2,\tilde{A})^{\tilde{c}}\right)$$

$$=M^*\left((\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})\right)+M^*\left((\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})^{\tilde{c}}\tilde{\cap}(\xi_2,\tilde{A})\right)+M^*\left((\zeta,\tilde{A})\tilde{\cap}\left((\xi_1,\tilde{A})\tilde{\cup}(\xi_2,\tilde{A})\right)^{\tilde{c}}\right)$$

$$\geq M^*\left((\zeta,\tilde{A})\tilde{\cap}\left((\xi_1,\tilde{A})\tilde{\cup}(\xi_2,\tilde{A})\right)\right)+M^*\left((\zeta,\tilde{A})\tilde{\cap}\left((\xi_1,\tilde{A})\tilde{\cup}(\xi_2,\tilde{A})\right)^{\tilde{c}}\right)$$

$$\left(\text{since } (\zeta,\tilde{A})\tilde{\cap}\left((\xi_1,\tilde{A})\tilde{\cup}(\xi_2,\tilde{A})\right)=\left[(\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})\right]\tilde{\cup}\left[(\zeta,\tilde{A})\tilde{\cap}(\xi_1,\tilde{A})^{\tilde{c}}\tilde{\cap}(\xi_2,\tilde{A})\right]\right)$$

The reverse inequality easily follows. Hence the theorem.

Theorem 4.5 If $(\xi_1,\tilde{A})$ *and* $(\xi_2,\tilde{A})$ are soft measurable, then so is $(\xi_1,\tilde{A})\,\tilde{\cap}(\xi_2,\tilde{A})$.

Proof:-

$$(\xi_1,\tilde{A})\ \textit{and}\ (\xi_2,\tilde{A})\ \textit{are soft measureable} \Rightarrow (\xi_1,\tilde{A})^{\tilde{c}}\ \textit{and}\ (\xi_2,\tilde{A})^{\tilde{c}}\ \textit{are soft measureable}$$

$$\Rightarrow (\xi_1,\tilde{A})^{\tilde{c}}\tilde{\cup}(\xi_2,\tilde{A})^{\tilde{c}}\ \textit{is soft measureable}$$

$$\Rightarrow \left((\xi_1,\tilde{A})\,\tilde{\cap}(\xi_2,\tilde{A})\right)^{\tilde{c}}\ \textit{is soft measureable}$$

$$\Rightarrow \left(\left((\xi_1,\tilde{A})\,\tilde{\cap}(\xi_2,\tilde{A})\right)^{\tilde{c}}\right)^{\tilde{c}}\ \textit{is soft measureable}$$

$$\Rightarrow (\xi_1,\tilde{A})\,\tilde{\cap}(\xi_2,\tilde{A})\ \textit{is soft measureable}$$

Definition 4.6 Two soft sets $(\xi_1,\tilde{A})$ *and* $(\xi_2,\tilde{A})$ are said to be disjoint if $\xi_1(\hat{a})\,\tilde{\cap}\,\xi_2(\hat{a})=\phi$ for each $\hat{a}\in\tilde{A}$.

Thus we can say that two soft sets $(\xi_1,\tilde{A})$ *and* $(\xi_2,\tilde{A})$ are said to be disjoint if $(\xi_1,\tilde{A})\,\tilde{\cap}(\xi_2,\tilde{A})=\phi_{soft}$.

Theorem 4.7 Let $\left\{\left(\xi_n,\tilde{A}\right)\right\}_n$ be a sequence of disjoint measurable soft sets. Then for any soft set $\left(\zeta,\tilde{A}\right)$, we have $M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\bigcup_{i=1}^{n}\left(\xi_i,\tilde{A}\right)\right)\right)=\sum_{i=1}^{n}M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_i,\tilde{A}\right)\right)$

Proof:-We will prove theorem by using the method of induction.
For n=1, the proof is obvious.
Let us assume that the given statement is true for n=k-1(>=2).
Then

$$M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\bigcup_{i=1}^{k-1}\left(\xi_i,\tilde{A}\right)\right)\right)=\sum_{i=1}^{k-1}M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_i,\tilde{A}\right)\right)$$

$$\Rightarrow M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_k,\tilde{A}\right)\right)+M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\bigcup_{i=1}^{k-1}\left(\xi_i,\tilde{A}\right)\right)\right)=\sum_{i=1}^{k-1}M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_i,\tilde{A}\right)\right)+M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_k,\tilde{A}\right)\right)$$

$$\Rightarrow M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\left(\bigcup_{i=1}^{k}\left(\xi_i,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_k,\tilde{A}\right)^{\tilde{c}}\right)\right)+M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\left(\bigcup_{i=1}^{k}\left(\xi_i,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_k,\tilde{A}\right)\right)\right)=\sum_{i=1}^{k}M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_i,\tilde{A}\right)\right)$$

Now by considering $\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\bigcup_{i=1}^{k}\left(\xi_i,\tilde{A}\right)\right)$ as a test set and using the fact that $\left(\xi_k,\tilde{A}\right)$ is soft measurable, we have

$$M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\bigcup_{i=1}^{k}\left(\xi_i,\tilde{A}\right)\right)\right)$$

$$=M^*\left(\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\bigcup_{i=1}^{k}\left(\xi_i,\tilde{A}\right)\right)\right)\tilde{\cap}\left(\xi_k,\tilde{A}\right)\right)+M^*\left(\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\bigcup_{i=1}^{k}\left(\xi_i,\tilde{A}\right)\right)\right)\tilde{\cap}\left(\xi_k,\tilde{A}\right)^{\tilde{c}}\right)$$

Thus $M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\bigcup_{i=1}^{k}\left(\xi_i,\tilde{A}\right)\right)\right)=\sum_{i=1}^{k}M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_i,\tilde{A}\right)\right)$. Hence by the principle of mathematical induction the theorem follows.

Note: For $\left(\zeta,\tilde{A}\right)=\Re_{soft}$, we get $M^*\left(\bigcup_{i=1}^{n}\left(\xi_i,\tilde{A}\right)\right)=\sum_{i=1}^{n}M^*\left(\xi_i,\tilde{A}\right)$.

Theorem 4.8 If $\left(\xi_1,\tilde{A}\right)$ *and* $\left(\xi_2,\tilde{A}\right)$ are soft measurable, then $M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)+M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)\right)=M^*\left(\xi_1,\tilde{A}\right)+M^*\left(\xi_2,\tilde{A}\right)$.

Proof:-Let us choose $\left(\zeta,\tilde{A}\right)$ as a test soft set. Then as $\left(\xi_1,\tilde{A}\right)$ is soft measurable, we have

$$M^*\left(\zeta,\tilde{A}\right)=M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_1,\tilde{A}\right)\right)+M^*\left(\left(\zeta,\tilde{A}\right)\tilde{\cap}\left(\xi_1,\tilde{A}\right)^{\tilde{c}}\right)\text{............(4.8.1)}$$

Now choosing $\left(\zeta,\tilde{A}\right)=\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)$ and adding $M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)\right)$ to the both sides of (4.8.1), we get

$$M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)+M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)\right)$$

$$=M^*\left(\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_1,\tilde{A}\right)\right)+M^*\left(\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_1,\tilde{A}\right)^{\tilde{c}}\right)+M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)\right)$$

$$=M^*\left(\xi_1,\tilde{A}\right)+M^*\left(\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_1,\tilde{A}\right)^{\tilde{c}}\right)+M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)\right)$$

Since each of $\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_1,\tilde{A}\right)^{\tilde{c}}$ and $\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)$ is soft measurable, then using *the facts that* $\left(\xi_2,\tilde{A}\right)=\left[\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_1,\tilde{A}\right)^{\tilde{c}}\right]\tilde{\cup}\left[\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)\right]$ *and*

$\left[\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_1,\tilde{A}\right)^{\tilde{c}}\right]\tilde{\cap}\left[\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)\right]=\phi_{soft}$,

We obtain

$$M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)\right)+M^*\left(\left(\xi_1,\tilde{A}\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)\right)=M^*\left(\xi_1,\tilde{A}\right)+M^*\left(\xi_2,\tilde{A}\right).$$

Theorem 4.9 If $\left(\xi_1,\tilde{A}\right)$ *and* $\left(\xi_2,\tilde{A}\right)$ are soft measurable such that $\left(\xi_1,\tilde{A}\right)\supset\left(\xi_2,\tilde{A}\right)$ *and*

$M^*\left(\xi_2,\tilde{A}\right)<\infty$, *then* $M^*\left(\left(\xi_1,\tilde{A}\right)\Theta\left(\xi_2,\tilde{A}\right)\right)=M^*\left(\xi_1,\tilde{A}\right)-M^*\left(\xi_2,\tilde{A}\right)$.

Proof:-It follows from the fact that $\left(\xi_1,\tilde{A}\right)=\left(\left(\xi_1,\tilde{A}\right)\Theta\left(\xi_2,\tilde{A}\right)\right)\tilde{\cup}\left(\xi_2,\tilde{A}\right)$ and each of $\left(\xi_1,\tilde{A}\right)\Theta\left(\xi_2,\tilde{A}\right)$ *and* $\left(\xi_2,\tilde{A}\right)$ is soft measurable with $\left(\left(\xi_1,\tilde{A}\right)\Theta\left(\xi_2,\tilde{A}\right)\right)\tilde{\cap}\left(\xi_2,\tilde{A}\right)=\phi_{soft}$.

Theorem 4.10 If $\left\{\left(\xi_n,\tilde{A}\right)\right\}_n$ be a sequence of disjoint measurable soft sets, then

$$M^*\left(\bigcup_{i=1}^{\infty}\left(\xi_i,\tilde{A}\right)\right)=\sum_{i=1}^{\infty}M^*\left(\xi_i,\tilde{A}\right).$$

Proof:-Putting $\left(\zeta,\tilde{A}\right)=\Re_{soft}$ in theorem (4.7) and using the fact that $\bigcup_{i=1}^{\infty}\left(\xi_i,\tilde{A}\right)\supset\bigcup_{i=1}^{n}\left(\xi_i,\tilde{A}\right)$ *for all* $n\in N$, the theorem follows.

Theorem 4.11 If $\left\{\left(\xi_i,\tilde{A}\right)\right\}_{i=1}^{\infty}$ be an infinite monotone sequence of measurable soft sets, then

$$M^*\left(\bigcup_{i=1}^{\infty}\left(\xi_i,\tilde{A}\right)\right)=\lim_{n\to\infty}M^*\left(\xi_n,\tilde{A}\right).$$

Proof:-Case- I: Let $\left\{\left(\xi_i,\tilde{A}\right)\right\}$ be an infinite increasing sequence of measurable soft sets.

If $M^*\left(\xi_p,\tilde{A}\right)=\infty,$ for some $p\in N,$ then the result is trivially true since

$$M^*\left(\bigcup_{i=1}^{\infty}\left(\xi_i,\tilde{A}\right)\right)\geq M^*\left(\xi_p,\tilde{A}\right)=\infty \textit{ and } M^*\left(\xi_n,\tilde{A}\right)=\infty \textit{ for all } n\geq p.$$

Let us assume that $M^*\left(\xi_i,\tilde{A}\right)<\infty$ *for each* $i\in N$. Also let $\left(\xi,\tilde{A}\right)=\bigcup_{i=1}^{\infty}\left(\xi_i,\tilde{A}\right)$ *and* $\left(\zeta_i,\tilde{A}\right)=\left(\xi_{i+1},\tilde{A}\right)\Theta\left(\xi_i,\tilde{A}\right)$. Then the soft sets $\left(\zeta_i,\tilde{A}\right)$ are measurable and pair wise disjoint.

Then as

$$\left(\xi,\tilde{A}\right)=\left(\xi_1,\tilde{A}\right)\tilde{\cup}\left(\left(\xi_2,\tilde{A}\right)\Theta\left(\xi_1,\tilde{A}\right)\right)\tilde{\cup}\left(\left(\xi_3,\tilde{A}\right)\Theta\left(\xi_2,\tilde{A}\right)\right)\tilde{\cup}.........;\textit{we have}$$

$$\left(\xi,\tilde{A}\right)\Theta\left(\xi_1,\tilde{A}\right)=\bigcup_{i=1}^{\infty}\left(\zeta_i,\tilde{A}\right)$$

$$\Rightarrow M^*\left(\left(\xi,\tilde{A}\right)\Theta\left(\xi_1,\tilde{A}\right)\right)=M^*\left(\bigcup_{i=1}^{\infty}\left(\zeta_i,\tilde{A}\right)\right)=\sum_{i=1}^{\infty}M^*\left(\zeta_i,\tilde{A}\right)=\sum_{i=1}^{\infty}M^*\left(\left(\xi_{i+1},\tilde{A}\right)\Theta\left(\xi_i,\tilde{A}\right)\right)$$

$$\Rightarrow M^*\left(\xi,\tilde{A}\right)-M^*\left(\xi_1,\tilde{A}\right)$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(M^*\left(\xi_{i+1},\tilde{A}\right)-M^*\left(\xi_i,\tilde{A}\right)\right)=\lim_{n\to\infty}\left(M^*\left(\xi_{n+1},\tilde{A}\right)-M^*\left(\xi_1,\tilde{A}\right)\right)$$

$$\Rightarrow M^*\left(\xi,\tilde{A}\right)=M^*\left(\bigcup_{i=1}^{\infty}\left(\xi_i,\tilde{A}\right)\right)=\lim_{n\to\infty}M^*\left(\xi_n,\tilde{A}\right).$$

This completes the proof.

Case-II:_Let $\left\{\left(\xi_i,\tilde{A}\right)\right\}_{i=1}^{\infty}$ be an infinite decreasing sequence of measurable soft sets. Then proceeding as in case- I, the result follows.

4 Conclusion and future work

Soft set theory is a tool for solving problems with uncertainty. In the present paper we extend the concept of outer measure and measurability in soft set theory context. We have also made an attempt to explain the equivalent version of some theorems on outer measure and measurability in the background of soft sets. All these concepts are basic supporting structures for research with the combination of measure theory and soft set theory.

5 Open Problem

With the ideas presented in this paper one can think of measurable functions and lebesgue integral in soft set theory context.

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