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Nearness BCK-Algebras

Dedicated to the memory of Mehmet Sapancı.

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Abstract

In this paper, we consider the problem of how to define nearness BCK-algebra. Also, some properties of nearness BCK-algebras are investigated.

Keywords: *BCK-algebras, near sets, nearness approximation spaces, nearness BCK-algebras.*

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1 Introduction

In 2002, J. F. Peters introduced near set theory as a generalization of rough set theory. Near set theory begins with the selection of probe functions that provide a basis for describing and discerning affinities between objects in distinct perceptual granules. A probe function is a real-valued function representing a feature of physical objects such as images or behaviors of individual biological organisms or collections of artificial organisms such as robot societies. But in this paper, in a more general setting that includes data mining, probe functions φ_i would be defined to allow for non-numerical values, i.e., let $\varphi_i : X \longrightarrow V$, where V is the value set for the range of φ_i [17]. More recent work considers generalized approach theory in the study of the nearness of non-empty sets that resemble each other [15, 16]. This more general definition of $\varphi_i \in \mathcal{F}$ is also better in setting forth the algebra and logic of near sets after the manner of algebra and logic.

In 2012, E. İnan and M. A.Öztürk investigated the concept of nearness groups [2, 3]. Also, in 2013, M. A. Öztürk at all introduced near group of weak cosets on nearness approximation spaces [9]. Moreover, in 2015, M. A. Öztürk and E. İnan established nearness semigroups and nearness rings [8, 5]. Also in 2013, M. A. Öztürk and E. İnan combined the soft sets approach with near set theory, which gives rise to the new concepts of soft nearness approximation spaces (SNAS), soft lower and upper approximations [12].

A BCK-algebra is an important class of logical algebras introduced by K. Iseki and was extensively investigated by several researchers [1]. This concept arises from two different topics as set theory and mathematical. In set theory, intersection, union and difference operations is defined by L. Kantoroviç and E. Livenson. As is well known, there is a close relationship between the notions of the set difference in the set theory and the implication functor in logical systems. The aim of this paper is to construct relationship between BCK-algebra and near set theory. we consider the problem of how to define nearness BCK-algebra that is defining BCK-algebra on nearness approximation space. Especially, nearness BCK-algebra was introduced, the some properties of nearness BCK-algebra was investigated, and several examples are given about nearness BCK-algebras.

2 Preliminaries

In this section, we refer to basic concepts from near set theory [11].

Objects are known by their descriptions. An object description is defined by means of a tuple of function values $\Phi(x)$ associated with an object $x \in X$. The important thing to notice is the choice of functions $\varphi_i \in B$ used to describe an object of interest. Assume that $B \subseteq \mathcal{F}$ is a given set of functions representing features of sample objects $X \subseteq \mathcal{O}$. Let $\varphi_i \in B$, where $\varphi_i : \mathcal{O} \longrightarrow \mathbb{R}$. In combination, the functions representing object features provide a basis for an object description $\Phi : \mathcal{O} \longrightarrow \mathbb{R}^L$, a vector containing measurements (returned values) associated with each functional value $\varphi_i(x)$, where the description length $|\Phi| = L$.

Object Description: $\Phi(x) = (\varphi_1(x), \varphi_2(x), \varphi_3(x), ..., \varphi_i(x), ..., \varphi_L(x))$.

The intuition underlying a description $\Phi(x)$ is a recording of measurements from sensors, where each sensor is modelled by a function φ_i.

Sample objects $X \subseteq \mathcal{O}$ are near each if and only if the objects have similar descriptions. Recall that each φ defines a description of an object. Then let Δ_{φ_i} denote

$$\Delta_{\varphi_i} = |\varphi_i(x') - \varphi_i(x)|$$

where $x, x' \in \mathcal{O}$. The difference Δ_φ leads to a definition of the indiscernibility relation "$\sim_B$" introduced by Z. Pawlak [10].

Definition 2.1. *Let $x, x' \in \mathcal{O}$, $B \subseteq \mathcal{F}$.*

$$\sim_B = \{(x, x') \in \mathcal{O} \times \mathcal{O} \mid \forall \varphi_i \in B \ , \ \Delta_{\varphi_i} = 0\}$$

is called the indiscernibility relation on $\mathcal{O}$, where description length $i \leq |\Phi|$.

Definition 2.2. *Let $B \subseteq \mathcal{F}$ be a set of functions representing features of objects $x, x' \in \mathcal{O}$. Objects x, x' are called minimally near each other if there exists $\varphi_i \in B$ such that $x \sim_{\varphi_i} x'$, $\Delta_{\varphi_i} = 0$. We call it the "Nearness Description Principle - NDP" [11].*

The basic idea in the near set approach to object recognition is to compare object descriptions. Sets of objects X, X' are considered near each other if the sets contain objects with at least partial matching descriptions.

Definition 2.3. *Let $X, X' \subseteq \mathcal{O}$, $B \subseteq \mathcal{F}$. Set X is called near X' if there exists $x \in X$, $x' \in X'$, $\varphi_i \in B$ such that $x \sim_{\varphi_i} x'$.*

Symbol	*Interpretation*
B	$B \subseteq \mathcal{F}$,
B_r	$r \leq \lvert B \rvert$ probe functions in B,
$\sim_{B_r}$	Indiscernibility relation defined using B_r,
$[x]_{B_r}$	$[x]_{B_r} = \{x' \in \mathcal{O} \mid x \sim_{B_r} x'\}$, equivalence class,
$\mathcal{O}/\sim_{B_r}$	$\mathcal{O}/\sim_{B_r} = \left\{[x]_{B_r} \mid x \in \mathcal{O}\right\}$, quotient set,
$\xi_{\mathcal{O},B_r}$	Partition $\xi_{\mathcal{O},B_r} = \mathcal{O}/\sim_{B_r}$,
r	$\binom{\lvert B \rvert}{r}$, i.e. $\lvert B \rvert$ probe functions $\varphi_i \in B$ taken r at a time,
$N_r(B)$	$N_r(B) = \{\xi_{\mathcal{O},B_r} \mid B_r \subseteq B\}$, set of partitions,
ν_{N_r}	$\nu_{N_r} : \wp(\mathcal{O}) \times \wp(\mathcal{O}) \longrightarrow [0,1]$, overlap function,
$N_r(B)_* X$	$N_r(B)_* X = \bigcup\limits_{[x]_{B_r} \subseteq X} [x]_{B_r}$, lower approximation,
$N_r(B)^* X$	$N_r(B)^* X = \bigcup\limits_{[x]_{B_r} \cap X \neq \varnothing} [x]_{B_r}$, upper approximation,
$Bnd_{N_r(B)}(X)$	$N_r(B)^* X \setminus N_r(B)_* X = \{x \in N_r(B)^* X \mid x \notin N_r(B)_* X\}$.

Table 1. *Nearness Approximation Space Symbols*

A nearness approximation space is a tuple $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ where the approximation space is defined with a set of perceived objects $\mathcal{O}$, set of probe functions $\mathcal{F}$ representing object features, indiscernibility relation "$\sim_B$" defined relative to $B_r \subseteq B \subseteq \mathcal{F}$, collection of partitions (families of neighbourhoods) $N_r(B)$ and neighbourhood overlap function ν_{N_r}.

We need the notion of nearness between sets, and so we consider the concept of the descriptively near sets. In 2007, descriptively near sets were introduced as a means of solving classification and pattern recognition problems arising from disjoint sets that resemble each other [11, 13].

A set of objects $A \subseteq O$ is characterized by the unique description of each object in the set.

Definition 2.4. *(Set Description, [7]) Let $\mathcal{O}$ be a set of perceptual objects, Φ an object description and A a subset of $\mathcal{O}$. Then the set description of A is defined as*

$$\mathcal{Q}(A) = \{\Phi(a) \mid a \in A\}.$$

Definition 2.5. *(Descriptive Set Intersection, [7, 14]) Let $\mathcal{O}$ be a set of perceptual objects, A and B any two subsets of $\mathcal{O}$. Then the descriptive (set) intersection of A and B is defined as*

$$A \underset{\Phi}{\cap} B = \{x \in A \cup B \mid \Phi(x) \in \mathcal{Q}(A) \text{ and } \Phi(x) \in \mathcal{Q}(B)\}.$$

Definition 2.6. *[12] Let $\mathcal{O}$ be a set of perceptual objects, A and B any two subsets of $\mathcal{O}$. If $\mathcal{Q}(A) \cap \mathcal{Q}(B) \neq \emptyset$, then A is called descriptively near B and denoted by $A\delta_\Phi B$.*

Definition 2.7. *(Descriptive Nearness Collections, [12]) Let $\mathcal{O}$ be a set of perceptual objects and A any subset of $\mathcal{O}$. Then the descriptive nearness collection $\xi_\Phi(A)$ is defined by*

$$\xi_\Phi(A) = \{B \in \mathcal{P}(\mathcal{O}) \mid A\delta_\Phi B\}.$$

Theorem 2.8. *[12] Let Φ be an object description, A any subset of $\mathcal{O}$ and $\xi_\Phi(A)$ a descriptive nearness collections. Then $A \in \xi_\Phi(A)$.*

An algebra $(X; \oplus, 0)$ of type $(2, 0)$ is called a *BCI-algebra* for all $x, y, z \in X$ if it satisfies the following conditions:

1. $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$,
2. $(x \oplus (x \oplus y)) \oplus y = 0$,
3. $x \oplus x = 0$,
4. $x \oplus y = 0,\ y \oplus x = 0 \Rightarrow x = y$,
5. $0 \oplus x = 0$.

Then, X is called a *BCK-algebra.* In a *BCK-algebra* X, $\forall x, y, z \in X$, the following identity holds: $(x \oplus y) \oplus z = (x \oplus z) \oplus y$ [6]. A nonempty subset S of a *BCK-algebra* X is called a *subalgebra* of X if $x \cdot y \in S$ for all $x, y \in S$. A *BCK-algebra* X is said to be *positive implicative* if it satisfies the following identity:

$$((x \oplus y) \oplus z) = (x \oplus y) \oplus (x \oplus z).$$

A positive implicative BCK-algebra will be written by piBCK-algebra for short. A BCK-algebra X is said to be *commutative* if $x \oplus (x \oplus y) = y \oplus (y \oplus x)$ for all $x, y \in X$. A commutative BCK-algebra will be written by cBCK-algebra for short. We refer the reader to the book [6] for further information regarding BCK-algebras.

3 Main results

In this section, we introduce nearness BCK-algebras.

Definition 3.1. *Let* $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ *be a nearness approximation space;* $\emptyset \neq X \subseteq \mathcal{O}$*, "*$\oplus$ *" a binary operation defined on* $\mathcal{O}$ *and* 0 *a constant on* $\mathcal{O}$*. A subset* X *of the set* $\mathcal{O}$ *is called* BCK-Algebra on nearness approximation space *or* nearness BCK-Algebra *for short if the following properties are satisfied for all* $x, y, z \in X$

$(YBCI-1)$ $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$ *property holds in* $N_r(B)^* X$*,*

$(YBCI-2)$ $(x \oplus (x \oplus y)) \oplus y = 0$ *property holds in* $N_r(B)^* X$*,*

$(YBCI-3)$ $(x \oplus x) = 0$ *property holds in* $N_r(B)^* X$*,*

$(YBCI-4)$ $0 \oplus x = 0$ *property holds in* $N_r(B)^* X$*,*

$(YBCI-5)$ *If* $x \oplus y = 0$ *and* $y \oplus x = 0$ *,* $x = y$ *property holds in* X*.*

"$\leq$ " relation defined on $\mathcal{O}$ for all $x, y, z \in \mathcal{O}$

$$x \leq y :\Leftrightarrow x \oplus y = 0$$

BCK-algebra again can be defined on $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ with this relation:

$(YBCI-1')$ $(x \oplus y) \oplus (x \oplus z) \leq (z \oplus y)$ property holds in $N_r(B)^* X$,

$(YBCI-2')$ $x \oplus (x \oplus y) \leq y$ property holds in $N_r(B)^* X$,

$(YBCI-3')$ $x \leq x = 0$ property holds in $N_r(B)^* X$,

$(YBCI-4')$ $0 \leq x = 0$ property holds in $N_r(B)^* X$,

$(YBCI-5')$ If $x \leq y = 0$ and $y \leq x = 0$, $x = y$ property holds in X,

$(YBCI-6')$ $x \leq y = 0 \Leftrightarrow x \oplus y = 0$, $x = y$ property holds in $N_r(B)^* X$.

Example 3.2. *Let* $\mathcal{O} = \{0, a, b, c, d\}$ *be a set of perceptual objects and* $B = \{\varphi_1, \varphi_2, \varphi_3\}$ *a set of probe functions. Values of the probe functions*

$$\varphi_1 : \mathcal{O} \longrightarrow V_1 = \{\alpha_1, \alpha_2, \alpha_3\},$$
$$\varphi_2 : \mathcal{O} \longrightarrow V_2 = \{\alpha_1, \alpha_2\},$$
$$\varphi_3 : \mathcal{O} \longrightarrow V_3 = \{\alpha_1, \alpha_3, \alpha_4\}$$

are given in $Table\ 2$*.*

	φ_1	φ_2	φ_3
0	α_2	α_1	α_3
a	α_3	α_2	α_1
b	α_2	α_1	α_3
c	α_2	α_2	α_3
d	α_1	α_1	α_4

Table 2.

Let "$\oplus$" be a binary operation of perceptual objects on $\mathcal{O}$ with the following table:

$\oplus$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	a	0
c	c	c	c	0	0
d	d	d	d	d	0

Table 3.

Then, $\mathcal{O}$ set of perceptual objects is a BCK-algebra *with the operation "$\oplus$"*
.

Let $X = \{a, c\}$ be a subset of perceptual objects and "$\oplus$" be a operation of perceptual objects on $X \subseteq \mathcal{O}$ with the following table:

$\oplus$	a	c
a	0	0
c	c	0

Table 4.

$$\begin{aligned}[0]_{\varphi_1} &= \{x' \in \mathcal{O} \mid \varphi_1(x') = \varphi_1(0) = \alpha_2\} \\ &= \{0, b, c\} \\ &= [b]_{\varphi_1} = [c]_{\varphi_1},\end{aligned}$$

$$\begin{aligned}[a]_{\varphi_1} &= \{x' \in \mathcal{O} \mid \varphi_1(x') = \varphi_1(a) = \alpha_3\} \\ &= \{a\},\end{aligned}$$

$$[d]_{\varphi_1} = \{x' \in \mathcal{O} \mid \varphi_1(x') = \varphi_1(d) = \alpha_1\} = \{d\},$$

Hence we have that $\xi_{\varphi_1} = \left\{[0]_{\varphi_1}, [a]_{\varphi_1}, [d]_{\varphi_1}\right\}$.

$$\begin{aligned}[0]_{\varphi_2} &= \{x' \in \mathcal{O} \mid \varphi_2(x') = \varphi_2(0) = \alpha_1\} \\ &= \{0, b, d\} = [b]_{\varphi_2} = [c]_{\varphi_2},\end{aligned}$$

$$\begin{aligned}[a]_{\varphi_2} &= \{x' \in \mathcal{O} \mid \varphi_2(x') = \varphi_2(a) = \alpha_2\} \\ &= \{a, c\} = [c]_{\varphi_2},\end{aligned}$$

Thus we obtain that $\xi_{\varphi_2} = \left\{[0]_{\varphi_2}, [a]_{\varphi_2}\right\}$.

$$\begin{aligned}[0]_{\varphi_3} &= \{x' \in \mathcal{O} \mid \varphi_3(x') = \varphi_3(0) = \alpha_3\} \\ &= \{0, b, c\} \\ &= [b]_{\varphi_3} = [c]_{\varphi_3},\end{aligned}$$

$$[a]_{\varphi_3} = \{x' \in \mathcal{O} \mid \varphi_3(x') = \varphi_3(a) = \alpha_1\} = \{a\},$$

$$[d]_{\varphi_3} = \{x' \in \mathcal{O} \mid \varphi_3(x') = \varphi_3(d) = \alpha_4\} = \{d\}$$

So we get that $\xi_{\varphi_3} = \left\{[0]_{\varphi_3}, [a]_{\varphi_3}, [d]_{\varphi_3}\right\}$.

Therefore, for $r = 1$, *a set of partitions of* $\mathcal{O}$ *is* $N_1(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$.
In this case, we can write

$$\begin{aligned}N_1(B)^* X &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{0, b, c\} \cup \{a\} \cup \{a, c\} = \{0, a, b, c\}.\end{aligned}$$

Thus, the following properties are true:

$(YBCI-1)$ $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$ *property holds in* $N_r(B)^* X$ *for all* $x, y, z \in X$,

$(YBCI-2)$ $(x \oplus (x \oplus y)) \oplus y = 0$ *property holds in* $N_r(B)^* X$ *for all* $x, y \in X$,

$(YBCI-3)$ $(x \oplus x) = 0$ *property holds in* $N_r(B)^* X$ *for all* $x \in X$,

$(YBCI-4)$ $0 \oplus x = 0$ *property holds in* $N_r(B)^* X$ *for all* $x \in X$,

$(YBCI-5)$ *If* $x \oplus y = 0$ *and* $y \oplus x = 0$, $x = y$ *property holds in* X *for all* $x, y, \in X$.

And so, a subset of perceptual objects X is a nearness BCK-algebra.

Example 3.3. *Let $\mathcal{O} = \{0, a, b, c\}$ be a set of perceptual objects and $B = \{\varphi_1, \varphi_2, \varphi_3\}$ a set of probe functions. Values of the probe functions*

$$\varphi_1 : \mathcal{O} \longrightarrow V_1 = \{\alpha_1, \alpha_2, \alpha_3\},$$
$$\varphi_2 : \mathcal{O} \longrightarrow V_2 = \{\alpha_1, \alpha_2, \alpha_3\},$$
$$\varphi_3 : \mathcal{O} \longrightarrow V_3 = \{\alpha_2, \alpha_3\}$$

are given in Table 5.

	φ_1	φ_2	φ_3
0	α_1	α_2	α_3
a	α_2	α_1	α_3
b	α_3	α_3	α_2
c	α_2	α_2	α_3

Table 5.

Let "$\oplus$" be a binary operation of perceptual objects on $\mathcal{O}$ with the following table:

$\oplus$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	a	0
c	c	c	c	0

Table 6.

Then, $\mathcal{O}$ set of perceptual objects is not a BCK-algebra. *Since $b \oplus b = a \neq 0$ for $b \in \mathcal{O}$ and so* BCI-3 *is not satisfied.*

Let $X = \{0, c\}$ be a subset of perceptual objects and "$\oplus$" be a operation of perceptual objects on $X \subseteq \mathcal{O}$ with the following table:

$\oplus$	0	c
0	0	0
c	c	0

Table 7.

Thus, X is a BCK-algebra. *Now, we will show that X is a* nearness BCK-algebra. *Hence we have that*

$\xi_{\varphi_1} = \left\{ [0]_{\varphi_1}, [a]_{\varphi_1}, [b]_{\varphi_1} \right\}$, $\xi_{\varphi_2} = \left\{ [0]_{\varphi_2}, [a]_{\varphi_2}, [b]_{\varphi_2} \right\}$, $\xi_{\varphi_3} = \left\{ [0]_{\varphi_3}, [b]_{\varphi_3} \right\}$.
Therefore, for $r = 1$, *a set of partitions of* $\mathcal{O}$ *is* $N_1(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$.
In this case, we can write

$$\begin{aligned} N_1(B)^* X &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{0\} \cup \{a, c\} \cup \{0, a, c\} \cup \{0, c\} = \{0, a, c\}. \end{aligned}$$

Therefore, $N_r(B)^* X$ *is a* BCK-algebra. *Thus, the following properties are true:*

$(YBCI-1)$ $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$ *property holds in* $N_r(B)^* X$ *for all* $x, y, z \in X$,

$(YBCI-2)$ $(x \oplus (x \oplus y)) \oplus y = 0$ *property holds in* $N_r(B)^* X$ *for all* $x, y \in X$,

$(YBCI-3)$ $(x \oplus x) = 0$ *property holds in* $N_r(B)^* X$ *for all* $x \in X$,

$(YBCI-4)$ $0 \oplus x = 0$ *property holds in* $N_r(B)^* X$ *for all* $x \in X$,

$(YBCI-5)$ *If* $x \oplus y = 0$ *and* $y \oplus x = 0$, $x = y$ *property holds in* X *for all* $x, y, \in X$.

And so, a subset of perceptual objects X *is a* nearness BCK-algebra.

Example 3.4. *Let* $\mathcal{O} = \{0, a, b, c, d\}$ *be a set of perceptual objects and* $B = \{\varphi_1, \varphi_2, \varphi_3\}$ *a set of probe functions. Values of the probe functions*

$$\begin{aligned} \varphi_1 &: \mathcal{O} \longrightarrow V_1 = \{\alpha_1, \alpha_2, \alpha_3\}, \\ \varphi_2 &: \mathcal{O} \longrightarrow V_2 = \{\alpha_1, \alpha_3, \alpha_4\}, \\ \varphi_3 &: \mathcal{O} \longrightarrow V_3 = \{\alpha_1, \alpha_2, \alpha_3\} \end{aligned}$$

are given in $Table$ 8.

	φ_1	φ_2	φ_3
0	α_1	α_3	α_3
a	α_2	α_4	α_3
b	α_1	α_3	α_2
c	α_1	α_1	α_2
d	α_3	α_3	α_1

Table 8.

Let "$\oplus$*" be a binary operation of perceptual objects on* $\mathcal{O}$ *with the following table:*

$\oplus$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	b
b	b	a	0	0	0
c	c	a	a	0	0
d	d	c	b	a	0

Table 9.

Then, $\mathcal{O}$ set of perceptual objects is not a BCK-algebra. *Since* $((a \oplus d) \oplus (a \oplus c)) \oplus (c \oplus d) = (b \oplus 0) \oplus 0 = b \oplus 0 = b \neq 0$ *for all* $a, c, d \in \mathcal{O}$ *and so* BCI-1 *is not satisfied.*

Let $X = \{a, b, c\}$ *be a subset of perceptual objects and "$\oplus$" be a operation of perceptual objects on* X *with the following table:*

$\oplus$	a	b	c
a	0	0	0
b	a	0	0
c	a	a	0

Table 10.

X *is not a* BCK-algebra *since* $0 \notin X$. *Hence we have that* $\xi_{\varphi_1} = \left\{ [0]_{\varphi_1}, [a]_{\varphi_1}, [d]_{\varphi_1} \right\}$, $\xi_{\varphi_2} = \left\{ [0]_{\varphi_2}, [a]_{\varphi_2}, [c]_{\varphi_2} \right\}$, $\xi_{\varphi_3} = \left\{ [0]_{\varphi_3}, [a]_{\varphi_3}, [d]_{\varphi_3} \right\}$.

Therefore, for $r = 1$, *a set of partitions of* $\mathcal{O}$ *is* $N_1(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$.

In this case, we can write

$$\begin{aligned} N_1(B)^* X &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{0, b, c\} \cup \{a\} \cup \{0, b, d\} \cup \{c\} = \{0, a, b, c, d\} = \mathcal{O}. \end{aligned}$$

As $\mathcal{O}$ is not BCK-algebra *so* $N_r(B)^* X$ *also is not a* BCK-algebra. *Thus, the following properties are true:*

$(YBCI-1)$ $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$ *property holds in* $N_r(B)^* X$ *for all* $x, y, z \in X$,

$(YBCI-2)$ $(x \oplus (x \oplus y)) \oplus y = 0$ *property holds in* $N_r(B)^* X$ *for all* $x, y \in X$,

$(YBCI-3)$ $(x \oplus x) = 0$ *property holds in* $N_r(B)^* X$ *for all* $x \in X$,

$(YBCI-4)$ $0 \oplus x = 0$ *property holds in* $N_r(B)^* X$ *for all* $x \in X$,

$(YBCI-5)$ *If* $x \oplus y = 0$ *and* $y \oplus x = 0$, $x = y$ *property holds in* X *for all* $x, y, \in X$.

And so, a subset of perceptual objects X is a nearness BCK-algebra.

Remark 3.5. *Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ be a nearness approximation space; $\emptyset \neq X \subseteq \mathcal{O}$, "$\oplus$" a binary operation defined on $\mathcal{O}$ and 0 be a constant on $\mathcal{O}$. Then, X is a* BCK-algebra *so that $0 \in N_r(B)^* X$ and "$\oplus$" should be binary operation on $0 \in N_r(B)^* X$.*

Now, we give essential properties of *nearness BCK-algebra*.

Theorem 3.6. *Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ be a nearness approximation space; "$\oplus$" a binary operation defined on $\mathcal{O}$, X be a* BCK-algebra *and $0 \in N_r(B)^* X$. For all $x, y, z \in X$*
i) if $x \leq y$, $z \oplus x \leq z \oplus y$,
ii)if $x \leq y$ and $y \leq z$, $x \leq z$.

Proof. i) Let $x \leq y$. Then, it follows by *YBCI-1* that $((z \oplus y) \oplus (z \oplus x)) \leq x \oplus y$. Since $x \leq y \Leftrightarrow x \oplus y = 0$, $(z \oplus y) \oplus (z \oplus x) \leq 0$ and by *YBCI-4'*, we have that $0 \leq (z \oplus y) \oplus (z \oplus x)$. Hence, we get $(z \oplus y) \oplus (z \oplus x)$. Thus, by *YBCI-6*, we obtain that $(z \oplus y) \leq (z \oplus x)$.
ii) The result here follows as (i). □

Theorem 3.7. *Let X be a* nearness BCK-algebra *and $0 \in N_r(B)^* X$. Then, $(x \oplus y) \oplus z = (x \oplus z) \oplus y$ for all $x, y, z \in X$.*

Proof. By *YBCI-2'*, $0 \leq x \oplus (x \oplus z) \leq z$. Therefore, it follows from Theorem 3.1.6-(i) $(x \oplus y) \oplus z \leq (x \oplus y) \oplus (x \oplus (x \oplus z))$. By *YBCI-1'*, $(x \oplus y) \oplus z \leq (x \oplus z) \oplus y$. Since $x, y, z \in X$ are arbitrary elements, if y and z are changed on the last expression, it implies that $(x \oplus z) \oplus y \leq (x \oplus y) \oplus z$. By *YBCI-5'*, we get that $(x \oplus y) \oplus z = (x \oplus z) \oplus y$. □

Theorem 3.8. *Let X be a* nearness BCK-algebra *and $0 \in N_r(B)^* X$. Then the following assertions hold for all $x, y, z \in X$:*
i)if $x \oplus y \leq z \Rightarrow x \oplus z \leq y$,
ii)$(x \oplus z) \oplus (y \oplus z) \leq x \oplus y$,
iii) $x \leq y \Rightarrow x \oplus z \leq y \oplus z$,
iv)$x \oplus y \leq x$.

Theorem 3.9. *Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ be a nearness approximation space and X be a* nearness BCK-algebra*. Then, the followings are true for all $x, y, z \in X$*
i) since $x \oplus y = 0$, $x \neq y \Rightarrow y \oplus x \neq 0$,
ii)$x \oplus y = z \Rightarrow z \oplus x = 0$.

Definition 3.10. *Let* $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ *be a nearness approximation space, "*$\oplus$*" be a binary operation defined on* $\mathcal{O}$ *and* 0 *be a constant on* $\mathcal{O}$*.* X *is a* nearness BCK-algebra *and* H *is a nonempty subset of* X*. If* $x \oplus y \in N_r(B)^* H$ *and* $0 \in N_r(B)^* H$*,* H *is called a* sub BCK-algebra *of* X *on nearness approximation space or just* subnearness BCK-algebra*.*

Let X be *subnear BCK-algebra.* 0 is not may be *subnearness BCK-algebra* of X. If $0 \in X$, then 0 and X itself are *subnearness BCK-algebra* of X.

Example 3.11. *Let* $\mathcal{O} = \{0, a, b, c\}$ *be a set of perceptual objects and* $B = \{\varphi_1, \varphi_2, \varphi_3\}$ *a set of probe functions. Values of the probe functions*

$$\varphi_1 : \mathcal{O} \longrightarrow V_1 = \{\alpha_1, \alpha_2\},$$
$$\varphi_2 : \mathcal{O} \longrightarrow V_2 = \{\alpha_2, \alpha_3\},$$
$$\varphi_3 : \mathcal{O} \longrightarrow V_3 = \{\alpha_1\alpha_2, \alpha_3\}$$

are given in Table 11*.*

	φ_1	φ_2	φ_3
0	α_1	α_2	α_1
a	α_2	α_3	α_3
b	α_2	α_3	α_2
c	α_1	α_2	α_2

Table 11.

Let "$\oplus$*" be a binary operation of perceptual objects on* $\mathcal{O}$ *with the following table:*

$\oplus$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	a
c	c	c	c	0

Table 12.

Then, $\mathcal{O}$ *set of perceptual objects is a* BCK-algebra *with the binary operation "*$\oplus$*" . Let* $A = \{0, a, c\}$ *be a subset of perceptual objects and "*$\oplus$*" be a operation of perceptual objects on* A *with the following table:*

$\oplus$	0	a	c
0	0	0	0
a	a	0	a
c	c	c	0

Table 13.

Hence we have that $\xi_{\varphi_1} = \left\{[0]_{\varphi_1}, [a]_{\varphi_1}\right\}$, $\xi_{\varphi_2} = \left\{[0]_{\varphi_2}, [a]_{\varphi_2}\right\}$, $\xi_{\varphi_3} = \left\{[0]_{\varphi_3}, [a]_{\varphi_3}, [b]_{\varphi_3}\right\}$.

Therefore, for $r = 1$, *a set of partitions of* $\mathcal{O}$ *is* $N_1(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$.

In this case, we can write

$$\begin{aligned} N_1(B)^* A &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{0, c\} \cup \{a, b\} \cup \{0\} \cup \{a\} \cup \{b, c\} = \{0, a, b, c\} = \mathcal{O}. \end{aligned}$$

Thus, the following properties are true:

$(YBCI-1)$ $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$ *property holds in* $N_r(B)^* A$ *for all* $x, y, z \in A$,

$(YBCI-2)$ $(x \oplus (x \oplus y)) \oplus y = 0$ *property holds in* $N_r(B)^* A$ *for all* $x, y \in A$,

$(YBCI-3)$ $(x \oplus x) = 0$ *property holds in* $N_r(B)^* A$ *for all* $x \in A$,

$(YBCI-4)$ $0 \oplus x = 0$ *property holds in* $N_r(B)^* A$ *for all* $x \in A$,

$(YBCI-5)$ *If* $x \oplus y = 0$ *and* $y \oplus x = 0$, $x = y$ *property holds in* A *for all* $x, y, \in A$.

And so, a subset of perceptual objects A *is a* nearness BCK-algebra. *Let* $H = 0, a$ *and* $K = a, c$ *be subset of* nearness BCK-algebra $A = \{0, a, c\}$ *and* "$\oplus$" *be a operation of perceptual objects on* H *and* K *with the tables 14 and 15, respectively:*

$\oplus$	0	a
0	0	0
a	a	0

Tablo 14

$\oplus$	a	c
a	0	a
c	c	0

Tablo 15

In this case, we can write

$$\begin{aligned} N_1(B)^* H &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{0, c\} \cup \{a, b\} \cup \{0\} \cup \{a\} = \{0, a, b, c\} = \mathcal{O}. \end{aligned}$$

Then, $x \oplus y \in N_1(B)^* H$ *implies that* H *is a* subnearness BCK-algebra *of* A *for all* $x, y \in H$. *Similarly,*

$$\begin{aligned} N_1(B)^* K &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{0, c\} \cup \{a, b\} \cup \{0\} \cup \{a\} \cup \{b, c\} = \{0, a, b, c\} = \mathcal{O}. \end{aligned}$$

Then, $x \oplus y \in N_1(B)^* K$ *implies that* K *is a* subnearness BCK-algebra *of* A *for all* $x, y \in K$.

Proposition 3.12. *Let H and K be two different non-empty subsets of* nearness BCK-algebra X. *Then the following assertions hold:*
i)$(N_r(B)^* H) \oplus (N_r(B)^* K) \subseteq N_r(B)^* (H \oplus K)$,
ii) if "$\sim_{B_r}$ " is complete indiscernibility relation, then $(N_r(B)^* H) \oplus (N_r(B)^* K) \subseteq N_r(B)^* (H \oplus K)$.

Proof. Suppose that $z \in (N_r(B)^* H) \oplus (N_r(B)^* K)$. Then, $x \in (N_r(B)^* H)$ and $y \in (N_r(B)^* K)$ imply that $z = x \oplus y$. Hence, there exist $h \in H$ and $k \in K$ such that $h \in [x]_{B_r} \bigcap H$ and $k \in [y]_{B_r} \bigcap K$. Since $h \in [x]_{B_r}$ and $k \in [y]_{B_r}$, $h \oplus k \in [x]_{B_r} \oplus [y]_{B_r}$. From indiscernibility relation "$\sim_{B_r}$" , it implies that $[x]_{B_r} \oplus [y]_{B_r} \subseteq [x \oplus y]_{B_r}$ for all $x, y \in X$. Thus, it follows that $h \oplus k \in [x \oplus y]_{B_r}$ and so $h \oplus k \in [x \oplus y]_{B_r} \bigcap (H \oplus K)$. Then, $z = x \oplus y \in N_r(B)^* (H \oplus K)$.
ii)The result here follows as (i). □

4 Some Nearness BCK-Algebras

Definition 4.1. *Let* $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ *be a nearness approximation space;* $\emptyset \neq X \subseteq \mathcal{O}$*, "$\oplus$ " a binary operation defined on* $\mathcal{O}$ *and* 0 *a constant on* $\mathcal{O}$*. If* $(x \oplus z) \oplus (y \oplus z) = (x \oplus y) \oplus z$ *equality holds for all* $x, y, z \in X$*,* X *is called* positive meaning BCK-algebra *on nearness approximation space or just* nearness positive meaning BCK-algebra.

Lemma 4.2. *Let* $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ *be a nearness approximation space; "$\oplus$ " a binary operation defined on* $\mathcal{O}$*,* X *a* nearness BCK-algebra *and* $0 \in N_r(B)^* X$*. For all* $x, y, z \in X$

$$(x \oplus (x \oplus y)) \oplus (y \oplus x) \leq x \oplus (x \oplus (y \oplus (y \oplus x))).$$

Example 4.3. *Let* $\mathcal{O} = \{0, a, b, c\}$ *be a set of perceptual objects and* $B = \{\varphi_1, \varphi_2, \varphi_3\}$ *a set of probe functions. Values of the probe functions*

$$\varphi_1 : \mathcal{O} \longrightarrow V_1 = \{\alpha_1, \alpha_2, \alpha_3\},$$
$$\varphi_2 : \mathcal{O} \longrightarrow V_2 = \{\alpha_1, \alpha_2\},$$
$$\varphi_3 : \mathcal{O} \longrightarrow V_3 = \{\alpha_1, \alpha_3\},$$

are given in Table 16.

	φ_1	φ_2	φ_3
0	α_1	α_2	α_3
a	α_1	α_1	α_3
b	α_2	α_1	α_1
c	α_3	α_2	α_3

Table 16.

Let "$\oplus$" be a binary operation of perceptual objects on $\mathcal{O}$ with the following table:

$\oplus$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	c	c	0

Table 17.

Then, $\mathcal{O}$ set of perceptual objects is a positive meaning BCK-algebra*.*

Let $A = \{a, b, c\}$ be a subset of perceptual objects and "$\oplus$" be a operation of perceptual objects on $A \subseteq \mathcal{O}$ with the following table:

$\oplus$	a	b	c
a	0	a	0
b	b	0	0
c	c	c	0

Table 18.

Hence we have that $\xi_{\varphi_1} = \left\{[0]_{\varphi_1}, [b]_{\varphi_1}, [c]_{\varphi_1}\right\}$, $\xi_{\varphi_2} = \left\{[0]_{\varphi_2}, [a]_{\varphi_2}\right\}$, $\xi_{\varphi_3} = \left\{[0]_{\varphi_3}, [b]_{\varphi_3}\right\}$.

Therefore, for $r = 1$, a set of partitions of $\mathcal{O}$ is $N_1(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$.

In this case, we can write

$$\begin{aligned} N_1(B)^* A &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{0, a\} \cup \{b\} \cup \{c\} \cup \{0, c\} \cup \{a, b\} \cup \{0, a, c\} = \{0, a, b, c\} = \mathcal{O}. \end{aligned}$$

Thus, the following properties are true:

$(YBCI-1)$ $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$ *property holds in $N_r(B)^* A$ for all $x, y, z \in A$,*

$(YBCI-2)$ $(x \oplus (x \oplus y)) \oplus y = 0$ *property holds in $N_r(B)^* A$ for all $x, y \in A$,*

$(YBCI-3)$ $(x \oplus x) = 0$ *property holds in $N_r(B)^* A$ for all $x \in A$,*

$(YBCI-4)$ $0 \oplus x = 0$ *property holds in $N_r(B)^* A$ for all $x \in A$,*

$(YBCI-5)$ *If $x \oplus y = 0$ and $y \oplus x = 0$, $x = y$ property holds in A for all $x, y, \in A$.*

And so, a subset of perceptual objects A is a nearness positive meaning BCK-algebra.

Definition 4.4. *Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ be a nearness approximation space; $\emptyset \neq K \subseteq \mathcal{O}$ and K a* nearness BCK-algebra. *If $(x \wedge z) = (y \wedge y)$ for all $x, y \in K$, i.e $y \oplus (y \oplus x) = x \oplus (x \oplus y)$, K is called* commutative BCK-algebra *on nearness approximation space or just* nearness commutative BCK-algebra.

Example 4.5. *Let $\mathcal{O} = \{0, a, b, c, d\}$ be a set of perceptual objects and $B = \{\varphi_1, \varphi_2, \varphi_3\}$ a set of probe functions. Values of the probe functions*

$$\varphi_1 : \mathcal{O} \longrightarrow V_1 = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\},$$
$$\varphi_2 : \mathcal{O} \longrightarrow V_2 = \{\alpha_2, \alpha_3, \alpha_4\},$$
$$\varphi_3 : \mathcal{O} \longrightarrow V_3 = \{\alpha_3, \alpha_4\},$$

are given in Table 19.

	φ_1	φ_2	φ_3
0	α_1	α_2	α_3
a	α_1	α_2	α_3
b	α_2	α_3	α_4
c	α_4	α_4	α_4
d	α_3	α_2	α_3

Table 19.

Let "$\oplus$" be a binary operation of perceptual objects on $\mathcal{O}$ with the following table:

$\oplus$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	0	0
c	c	a	a	0	0
d	d	c	b	a	0

Table 20.

$\mathcal{O}$ set of perceptual objects is a not a commutative BCK-algebra *with the operation "$\oplus$". Since, $b \wedge c = c \oplus (c \oplus b) = c \oplus a = a$,$c \wedge b = b \oplus (b \oplus c) = b \oplus 0 = b$ but $b \wedge c \neq c \wedge b$.*

Let $K = \{0, a\}$ be a subset of perceptual objects and "$\oplus$" a operation of perceptual objects on $K \subseteq \mathcal{O}$ with the following table:

$\oplus$	0	a
0	0	0
a	a	0

Table 21.

Hence we have that $\xi_{\varphi_1} = \left\{ [0]_{\varphi_1}, [b]_{\varphi_1}, [c]_{\varphi_1}, [d]_{\varphi_1} \right\}$, $\xi_{\varphi_2} = \left\{ [0]_{\varphi_2}, [b]_{\varphi_2}, [c]_{\varphi_1} \right\}$, $\xi_{\varphi_3} = \left\{ [0]_{\varphi_3}, [b]_{\varphi_3} \right\}$.

Therefore, for $r = 1$, *a set of partitions of* $\mathcal{O}$ *is* $N_1(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$.

In this case, we can write

$$\begin{aligned} N_1(B)^* K &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{0, a\} \cup \{0, a, d\} = \{0, a, d\} = \mathcal{O}. \end{aligned}$$

Thus, the following properties are true:

$(YBCI-1)$ $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$ *property holds in* $N_r(B)^* K$ *for all* $x, y, z \in K$,

$(YBCI-2)$ $(x \oplus (x \oplus y)) \oplus y = 0$ *property holds in* $N_r(B)^* K$ *for all* $x, y \in K$,

$(YBCI-3)$ $(x \oplus x) = 0$ *property holds in* $N_r(B)^* K$ *for all* $x \in K$,

$(YBCI-4)$ $0 \oplus x = 0$ *property holds in* $N_r(B)^* K$ *for all* $x \in K$,

$(YBCI-5)$ *If* $x \oplus y = 0$ *and* $y \oplus x = 0$, $x = y$ *property holds in* K *for all* $x, y, \in K$.

And so, a subset of perceptual objects K *is a* nearness BCK-algebra. *Since* $x \wedge y = y \wedge x$ *for all* $x, y \in K$, K *is a* nearness commutative BCK-algebra.

Definition 4.6. *Let* $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ *be a nearness approximation space;* $\emptyset \neq X \subseteq \mathcal{O}$ *and* X *be a* BCK-algebra *on nearness approximation space. If* $x = x \oplus (y \oplus x)$ *for all* $x, y \in X$, X *is called* meaning BCK-algebra *on nearness approximation space or just* nearness meaning BCK-algebra.

Example 4.7. *Let* $\mathcal{O} = \{0, a, b, c\}$ *be a set of perceptual objects and* $B = \{\varphi_1, \varphi_2, \varphi_3\}$ *a set of probe functions. Values of the probe functions*

$$\begin{aligned} \varphi_1 &: \mathcal{O} \longrightarrow V_1 = \{\alpha_1, \alpha_3, \alpha_4\}, \\ \varphi_2 &: \mathcal{O} \longrightarrow V_2 = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}, \\ \varphi_3 &: \mathcal{O} \longrightarrow V_3 = \{\alpha_2, \alpha_3, \alpha_4\} \end{aligned}$$

are given in Table 22.

	φ_1	φ_2	φ_3
0	α_1	α_2	α_3
a	α_3	α_2	α_4
b	α_4	α_4	α_2
c	α_1	α_3	α_2
d	α_4	α_1	α_2

Table 22.

Let "$\oplus$" be a binary operation of perceptual objects on $\mathcal{O}$ with the following table:

$\oplus$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	a	0
c	c	c	c	0	0
d	d	d	d	d	0

Table 23.

$\mathcal{O}$ set of perceptual objects is a BCK-algebra *with the operation "$\oplus$" .*

Let $X = \{a\}$ be a subset of perceptual objects and "$\oplus$" be a operation of perceptual objects on $X \subseteq \mathcal{O}$ with the following table:

$\oplus$	a
a	0

Table 24.

Since $0 \notin X$*, X is not a* BCK-algebra.

Hence we have that $\xi_{\varphi_1} = \left\{[0]_{\varphi_1}, [a]_{\varphi_1}, [b]_{\varphi_1}\right\}$, $\xi_{\varphi_2} = \left\{[0]_{\varphi_2}, [b]_{\varphi_2}, [c]_{\varphi_2}, [d]_{\varphi_2}\right\}$, $\xi_{\varphi_3} = \left\{[0]_{\varphi_3}, [a]_{\varphi_3}, [b]_{\varphi_2}\right\}$.

Therefore, for $r = 1$, a set of partitions of $\mathcal{O}$ is $N_1(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$.

In this case, we can write

$$\begin{aligned} N_1(B)^* X &= \bigcup_{[x]_{\varphi_i} \cap G \neq \varnothing} [x]_{\varphi_i} \\ &= \{a\} \cup \{0, a\} = \{0, a\} = \mathcal{O}. \end{aligned}$$

Thus, the following properties are true:

$(YBCI-1)$ $((x \oplus y) \oplus (x \oplus z)) \oplus (z \oplus y) = 0$ *property holds in* $N_r(B)^* X$ *for all* $x, y, z \in X$,

$(YBCI-2)$ $(x \oplus (x \oplus y)) \oplus y = 0$ *property holds in* $N_r(B)^* X$ *for all* $x, y \in X$,

$(YBCI-3)$ $(x \oplus x) = 0$ *property holds in* $N_r(B)^* X$ *for all* $x \in X$,

$(YBCI-4)$ $0 \oplus x = 0$ *property holds in* $N_r(B)^* X$ *for all* $x \in X$,

$(YBCI-5)$ *If* $x \oplus y = 0$ *and* $y \oplus x = 0$, $x = y$ *property holds in* X *for all* $x, y, \in X$.

And so, a subset of perceptual objects X *is a* nearness BCK-algebra. *Since* $x = x \oplus (x \oplus y)$ *for all* $x, y \in X$, X *is a* nearness meaning BCK-algebra.

5 Open Problem

We have studied the problem of how to define nearness BCK-algebra. Also, approach is to explain some properties of nearness BCK-algebras. One can consider others types of algebra like BCC-algebra, BCH-algebra, PU-algebra, subtraction algebra and etc. on nearness approximation spaces.

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